DMRG Study of Spinless Fermionic Ladder in the presence of a Magnetic Field

Rahul Soni[1,2] (rsoni1@vols.utk.edu)

[1]Department of Physics, University of Tennessee-Knoxville

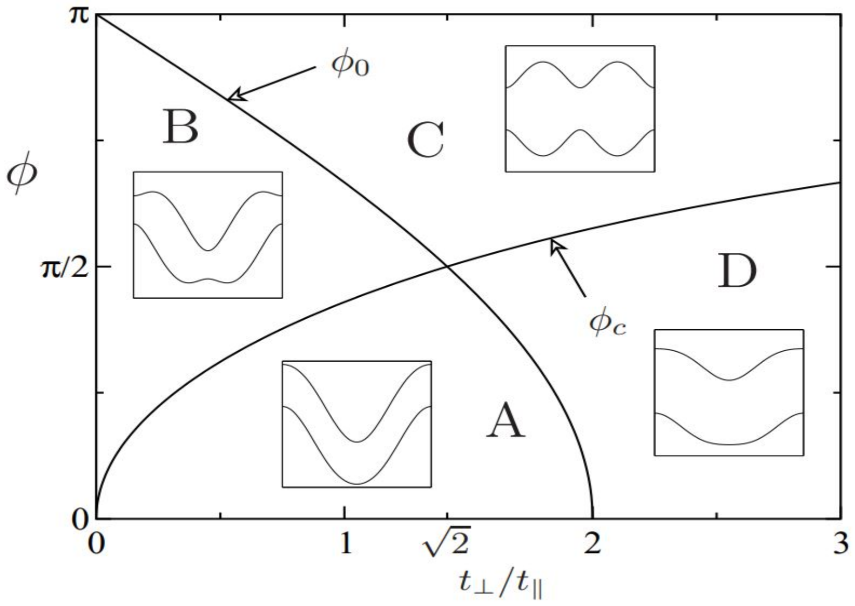
[2]Materials Science and Technology Division, Oak Ridge National Laboratory

Outline

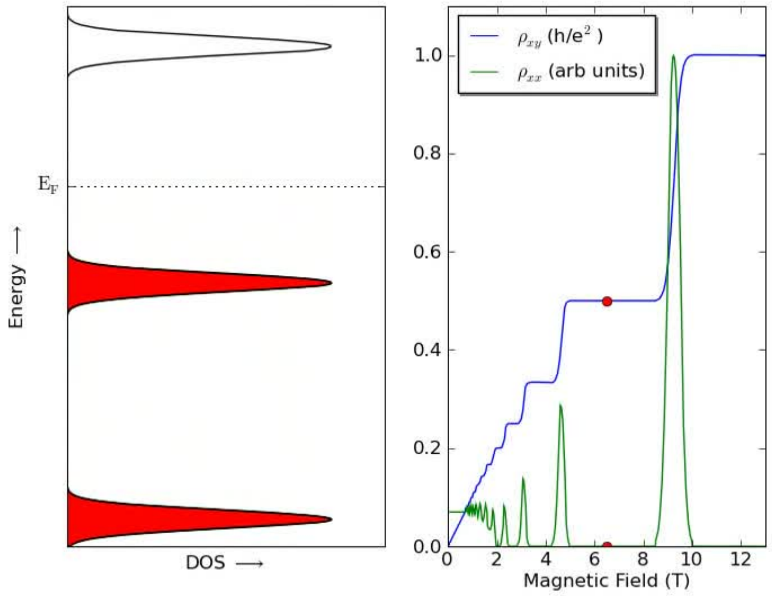
- Overview and Motivation
- Model
- Results for $B = 0$.
- Weak Coupling Regime ($B \neq 0$)
- Summary

Overview [effect of magnetic field]:

- The introduction of magnetic field has a dramatic effect on all properties of the system.
- Even in the absence of interaction, the energy spectrum of a free fermi-gas gets modified[1].
- The most significant consequence of this phenomena is the quantum Hall effect[2] in 2D.



[1] G. Roux, E. Orignac, S. R. White, and D. Poilblanc
Phys. Rev. B 76, 195105 (2007)



[2] K. Klitzing, G. Dorda, and M. Pepper
Phys. Rev. Lett. 45, 494 (1980)

Ultracold atoms in Optical Lattices:

Neutral Atoms ⟷	Valence electrons
Optical Lattices ⟷	Ionic Crystal

Fermions → K^{40}, Li^{6}
Bosons → Rb^{87}, Na^{23}

Interactions between the atoms can also be tuned by Feshbach Resonance.

Markus Greiner and Simon Fölling, *Nature* **453**, 736–738.

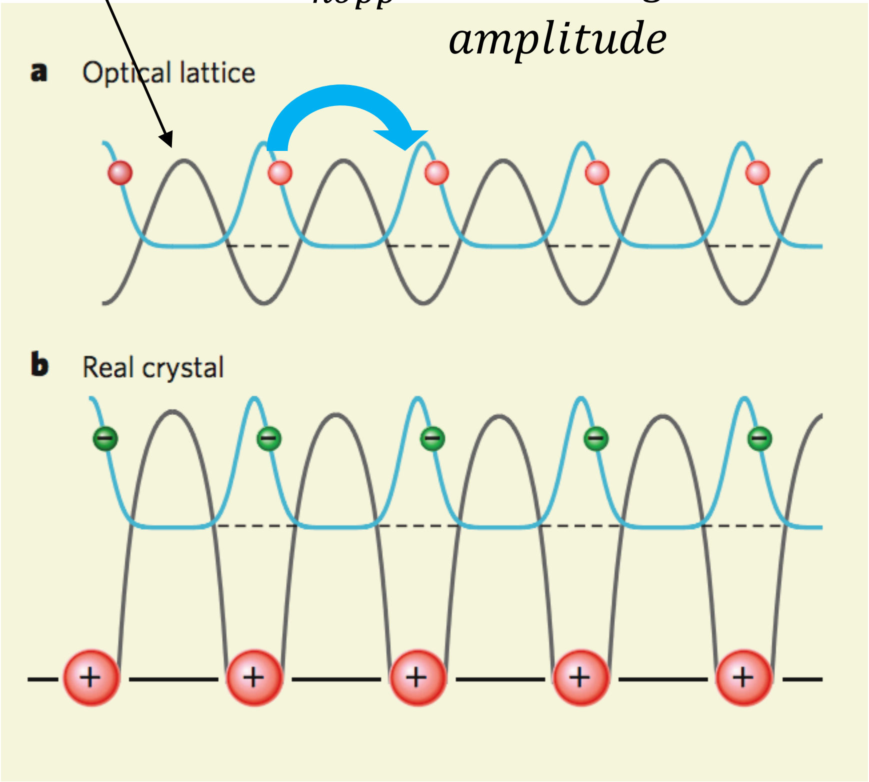
Background Periodic potential:

$$V \propto -V_0 \cos^2(\pi x/a_0)$$

From

$$V = -\overrightarrow{d.E}$$

$t_{hopp} \propto tunneling\ amplitude$



Artificial Gauge Field in Optical Lattices:

Peierls substitution:

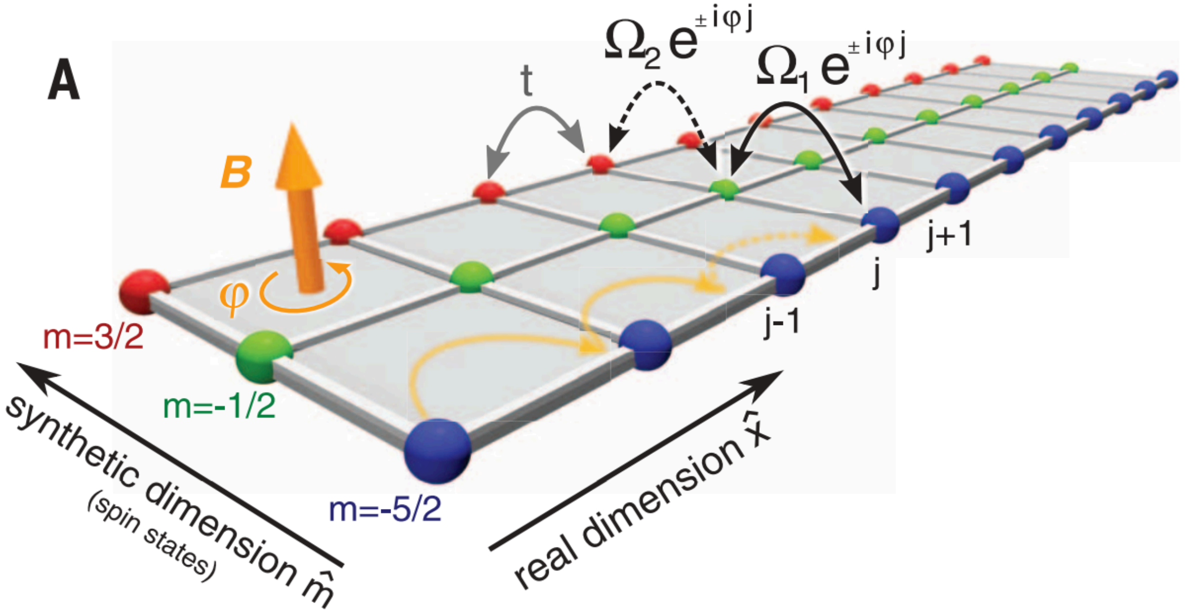
In the presence of an external vector potential ($\vec{A}$), hopping parameter acquires a complex phase factor.

$$t_{ij} \longrightarrow t_{ij} e^{-i \frac{e}{\hbar} \int_i^j \mathbf{A} \cdot d\mathbf{l}}$$

Artificial Gauge Field in Optical Lattices:

Peierls substitution:
In the presence of an external vector potential ($\vec{A}$), hopping parameter acquires a complex phase factor.

$$t_{ij} \longrightarrow t_{ij} e^{-i\frac{e}{\hbar}\int_i^j \mathbf{A}\cdot d\mathbf{l}}$$

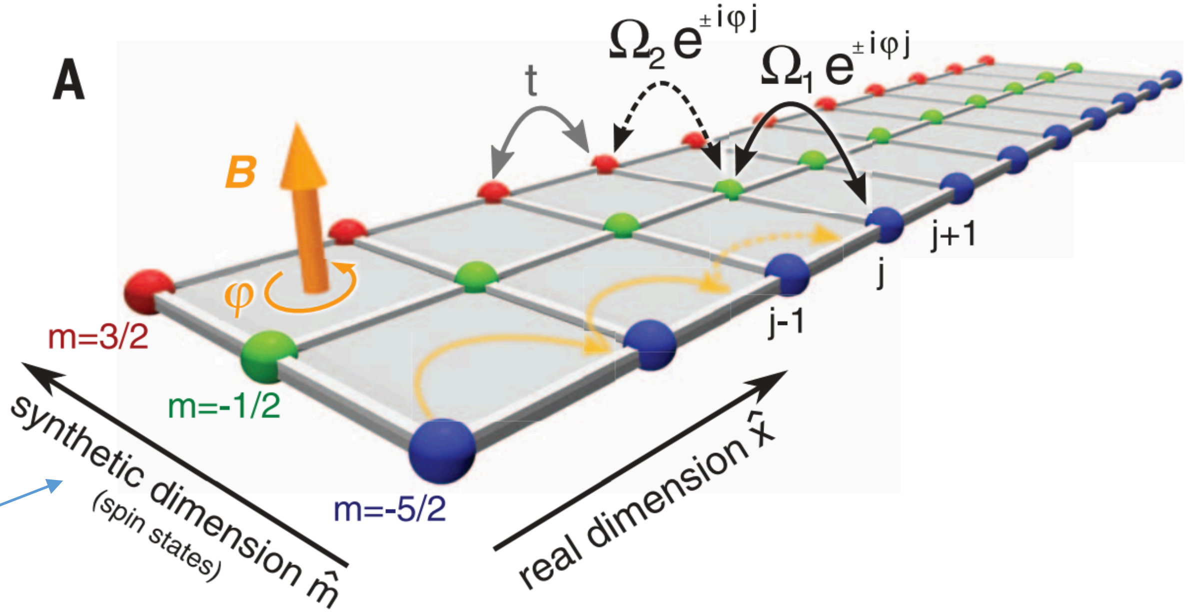


- Artificial Gauge Fields can be constructed in Optical lattices using atom's internal degrees of freedom [proposed by *A. Celi et al., PRL* ***112****, 043001 (2014)*]
- First time realization of Artificial Gauge Fields using above approach in Optical lattices in 2015 [*M. Mancini et al., Science* ***349****, 1510 (2015)*]

Artificial Gauge Field in Optical Lattices:

Peierls substitution:
In the presence of an external vector potential ($\vec{A}$), hopping parameter acquires a complex phase factor.

$$t_{ij} \longrightarrow t_{ij} e^{-i\frac{e}{\hbar}\int_i^j \mathbf{A}\cdot d\mathbf{l}}$$



No. of legs can be tuned in experiment.

- Artificial Gauge Fields can be constructed in Optical lattices using atom's internal degrees of freedom [proposed by *A. Celi et al., PRL **112**, 043001 (2014)*]
- First time realization of Artificial Gauge Fields using above approach in Optical lattices in 2015 [*M. Mancini et al., Science **349**, 1510 (2015)*]

Motivation

- Theoretical studies of low dimensional systems in the presence of a magnetic field provides a guiding direction to cold atom community.

Motivation

- Theoretical studies of low dimensional systems in the presence of a magnetic field provides a guiding direction to cold atom community.
- Minimal geometry to study the effect of magnetic field is two-leg ladder.

Motivation

- Theoretical studies of low dimensional systems in the presence of a magnetic field provides a guiding direction to cold atom community.
- Minimal geometry to study the effect of magnetic field is two-leg ladder.
- Longer-range interactions have also been realized in cold atom experiments [*Peter Schauß et al., Nature **491**, 87–91 (2012).*]

Objective:

In past, using analytical calculations (bosonization), “magnetic flux” vs “U_{onsite}/V_{NN}” phase diagram have been created.

[*Sam T. Carr et al., Phys. Rev. B **73**, 195114 (2006)*]

We aim towards creating the phase diagram in intermediate and strong coupling limit using an exact Density matrix renormalization group technique, where the analytical techniques does not work.

There have many few DMRG studies on similar models, but a comprehensive study is still missing.

[Simone Barbarino et al., Nature communications 6, 8134 (2015)]

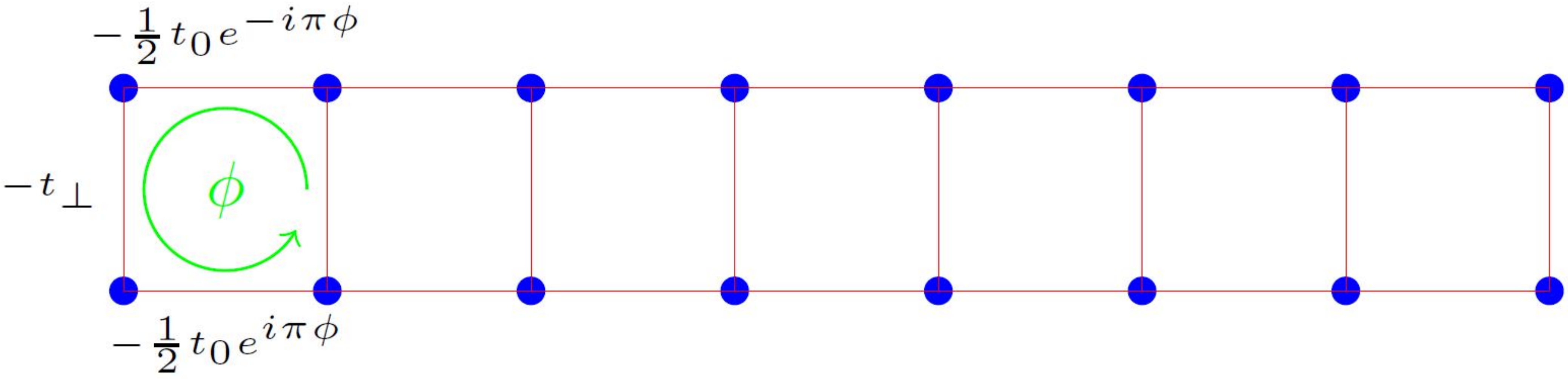
Model

Hamiltonian:

$$H = -\frac{1}{2}\sum_{i,\sigma}\left[t_{\|}(\sigma)c^\dagger_{i,\sigma}c_{i+1,\sigma} + h.c.\right] - t_\perp \sum_i \left[c^\dagger_{i,1}c_{i,2} + h.c.\right]$$
$$+U\sum_i n_{i,1}n_{i,2} + V\sum_{i,\sigma} n_{i,\sigma}n_{i+1,\sigma}$$

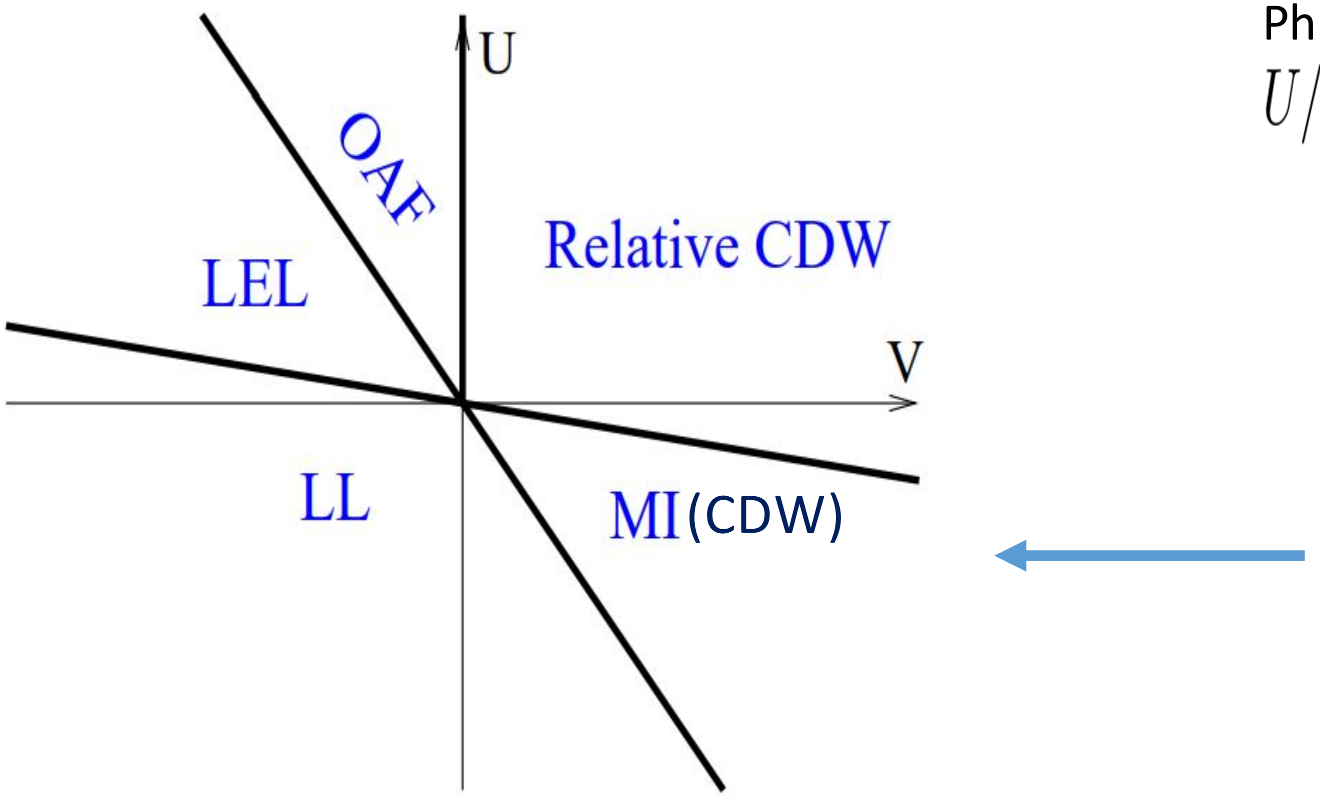
where, $\sigma = 1,2$ and $t_{\|}(1) = t_0 e^{-i\pi\phi}$, $t_{\|}(2) = t_0 e^{i\pi\phi}$, defining $\tau = t_\perp/t_0$.

S. T. Carr, B. N. Narozhny, PRB 73, 195114 (2006)



Phase diagram for ϕ=0 (in the literature)

- At half filling:



Phase boundaries: $V = 0$

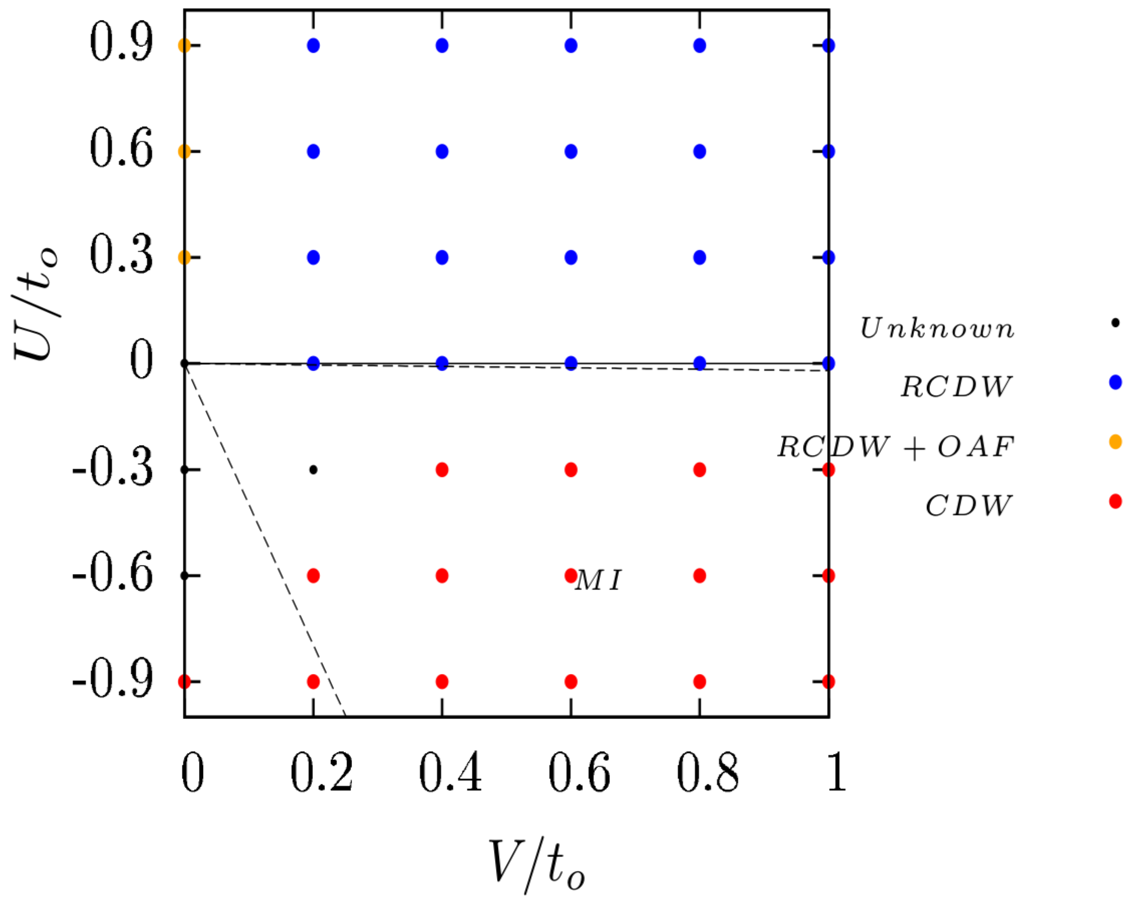
$U/2V = -2 + \tau^2$ and $U/2V = -\tau^2$

Using Bosonization

[1] S. T. Carr, B. N. Narozhny, PRB 73, 195114 (2006)

T. Giamarchi, *Quantum Physics in One Dimension*, Oxford University Press, (2004)

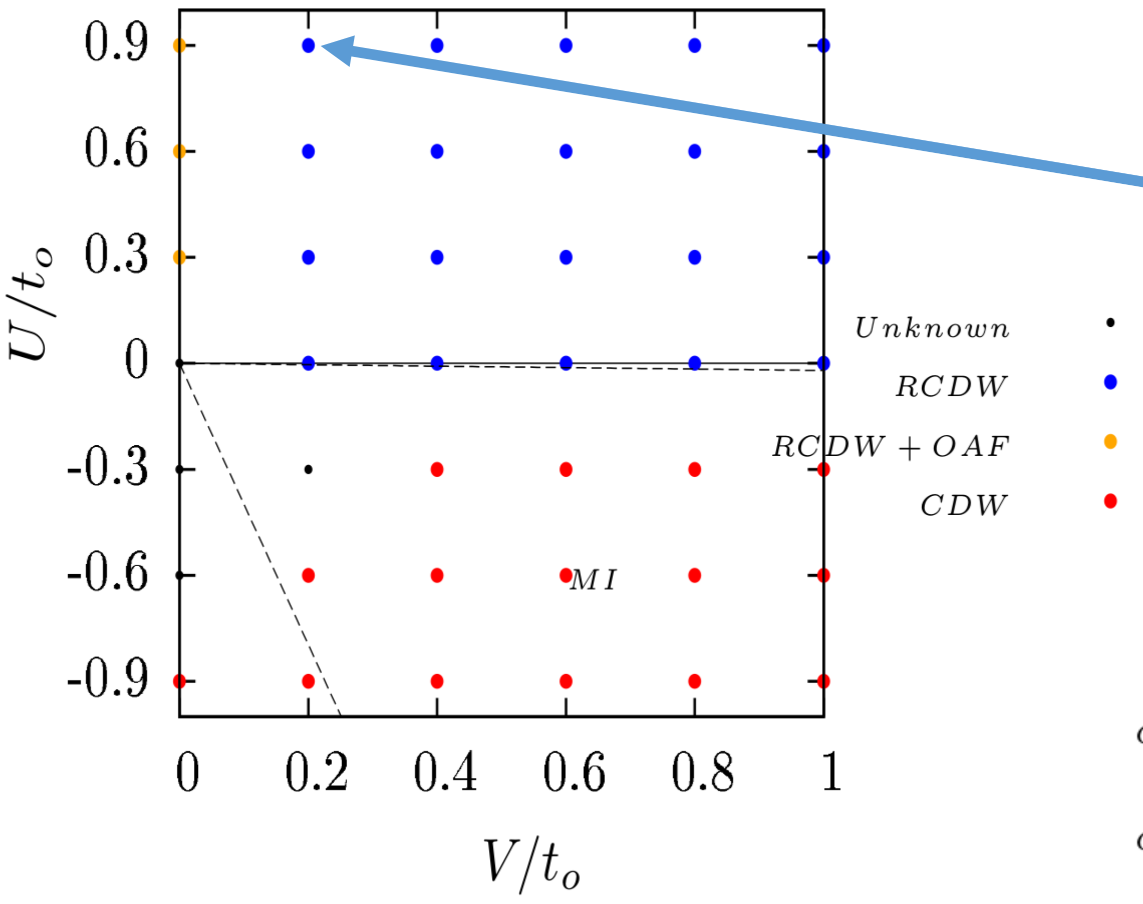
Phase diagram for ϕ=0 at half filling (DMRG study)



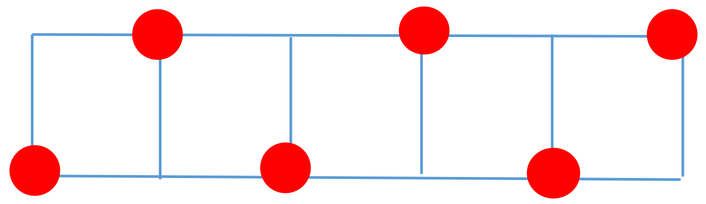
Where, τ=0.1

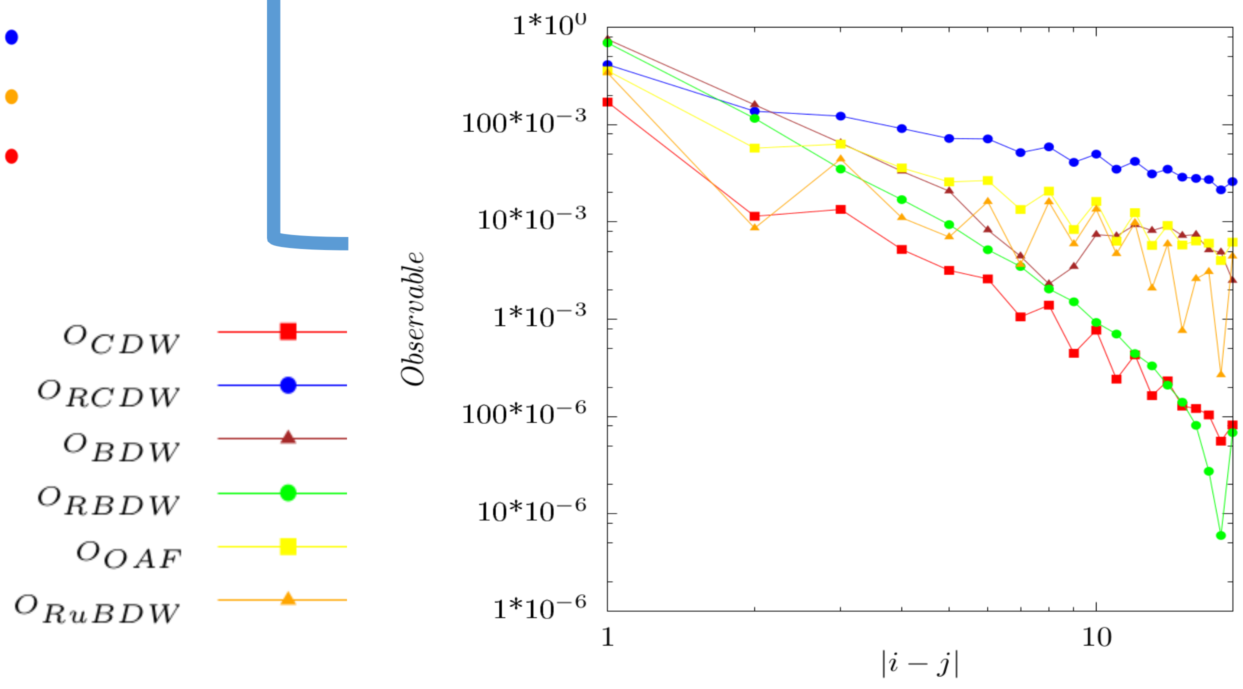
System size=24x2

Phase diagram for ϕ=0 at half filling (DMRG study)



Relative Charge Density wave:



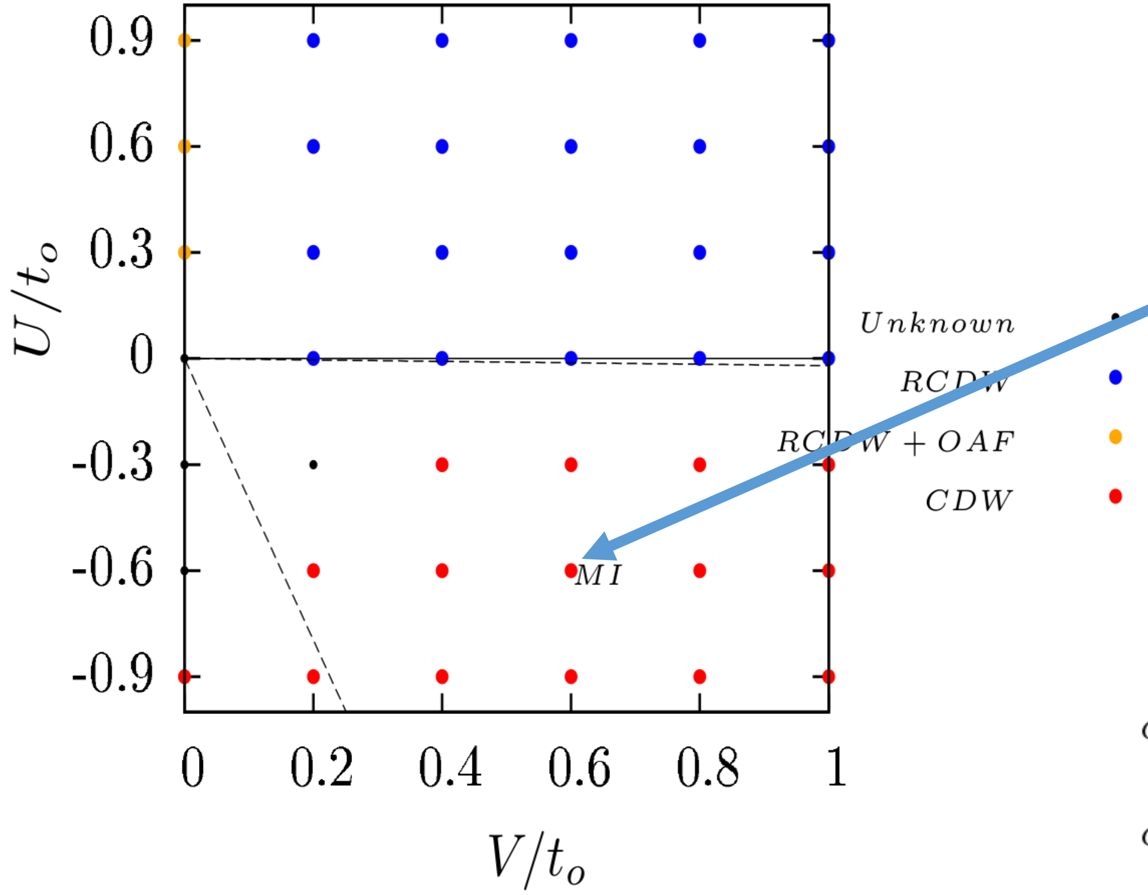


Correlation results for $U = 0.9,\ V = 0.2$

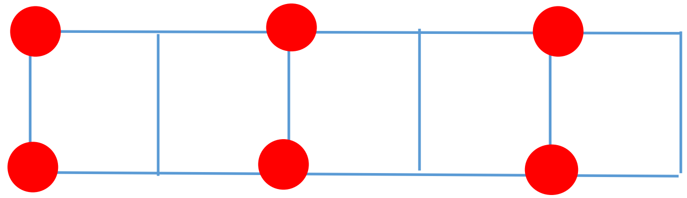
Where, τ=0.1

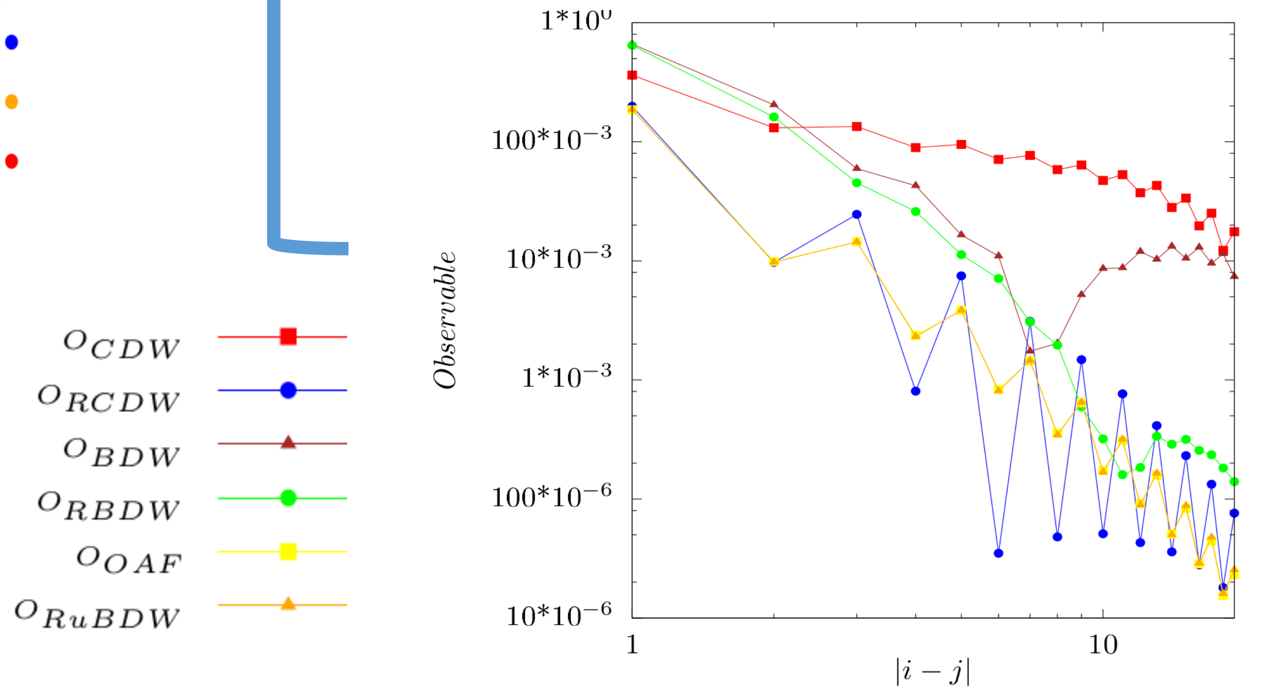
System size=24x2

Phase diagram for ϕ=0 at half filling (DMRG study)



Charge Density wave:

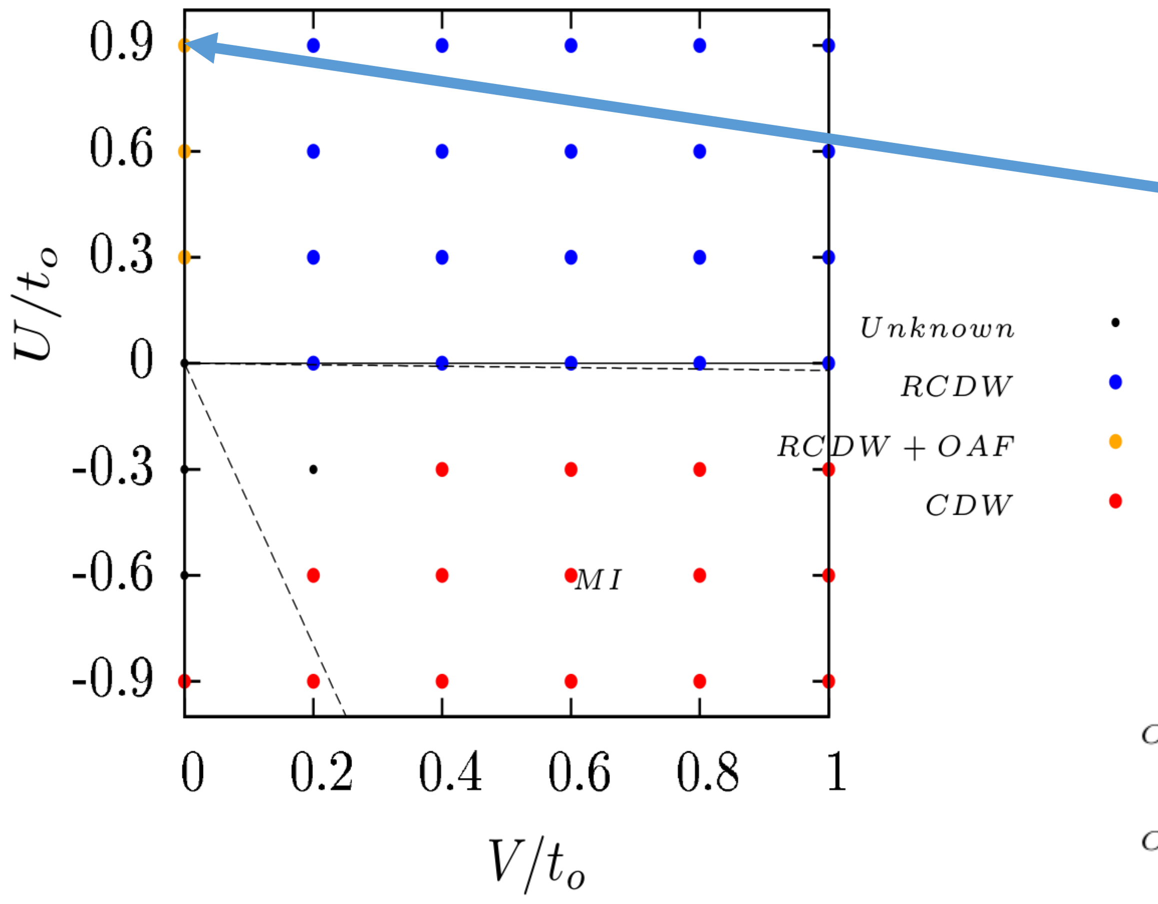




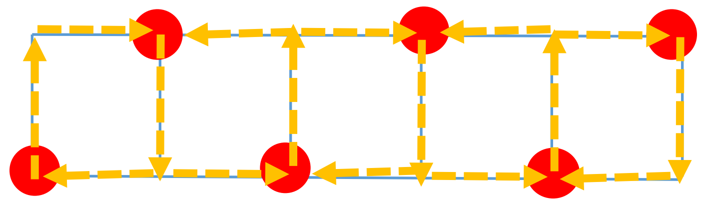
Correlation results $U = -0.6,\ V = 0.6$

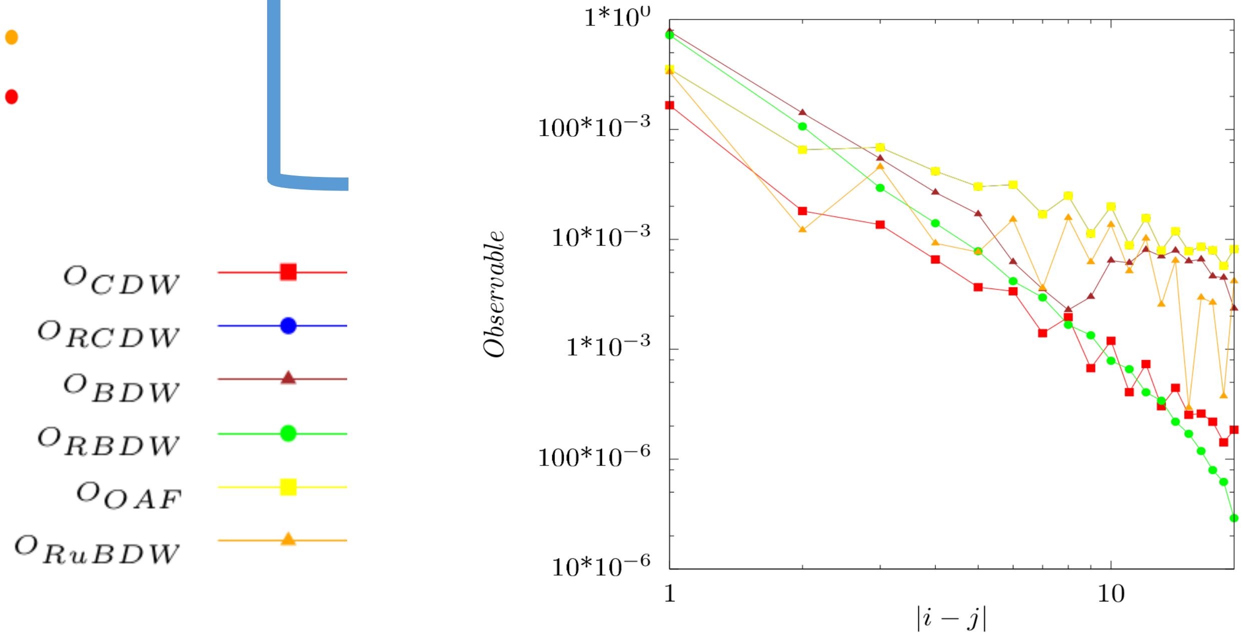
Where, τ=0.1
System size=24x2

Phase diagram for ϕ=0 at half filling (DMRG study)



RCDW +
Orbial Antiferromagnetism:



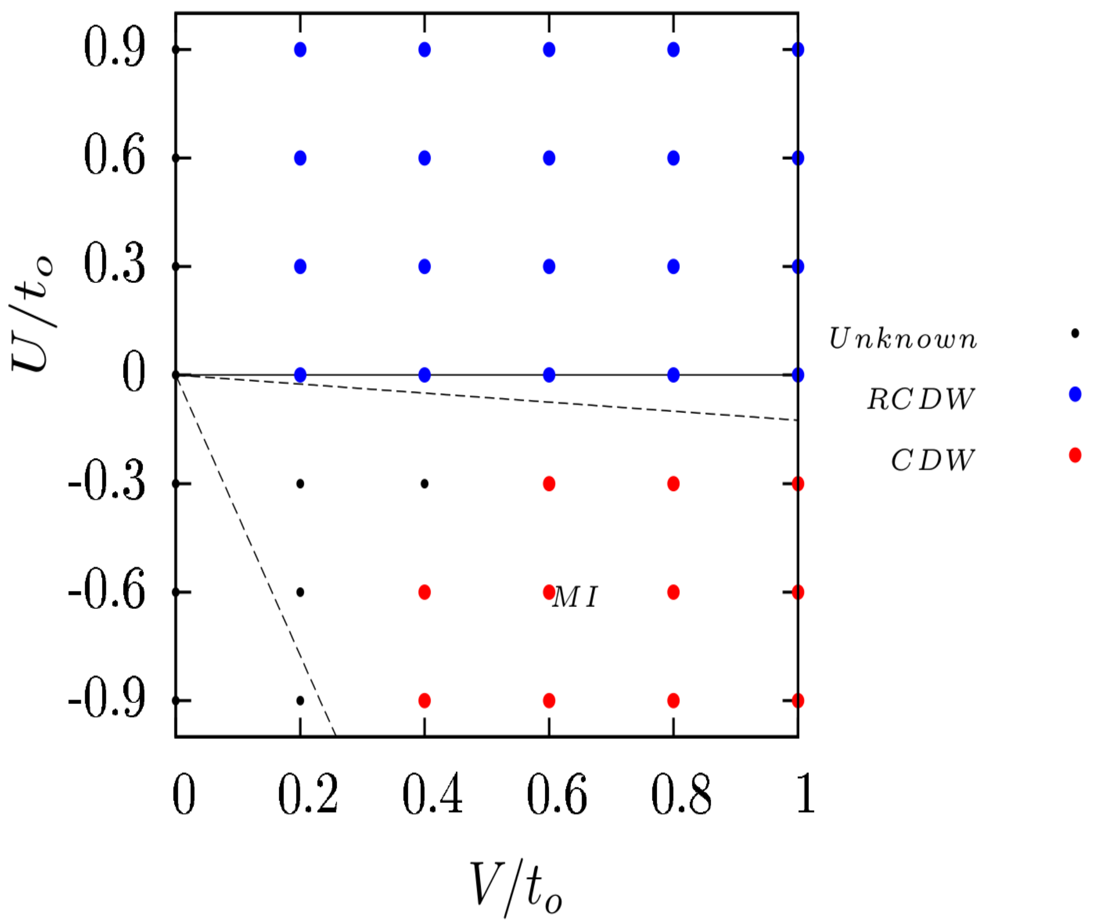


Where, τ=0.1
System size=24x2

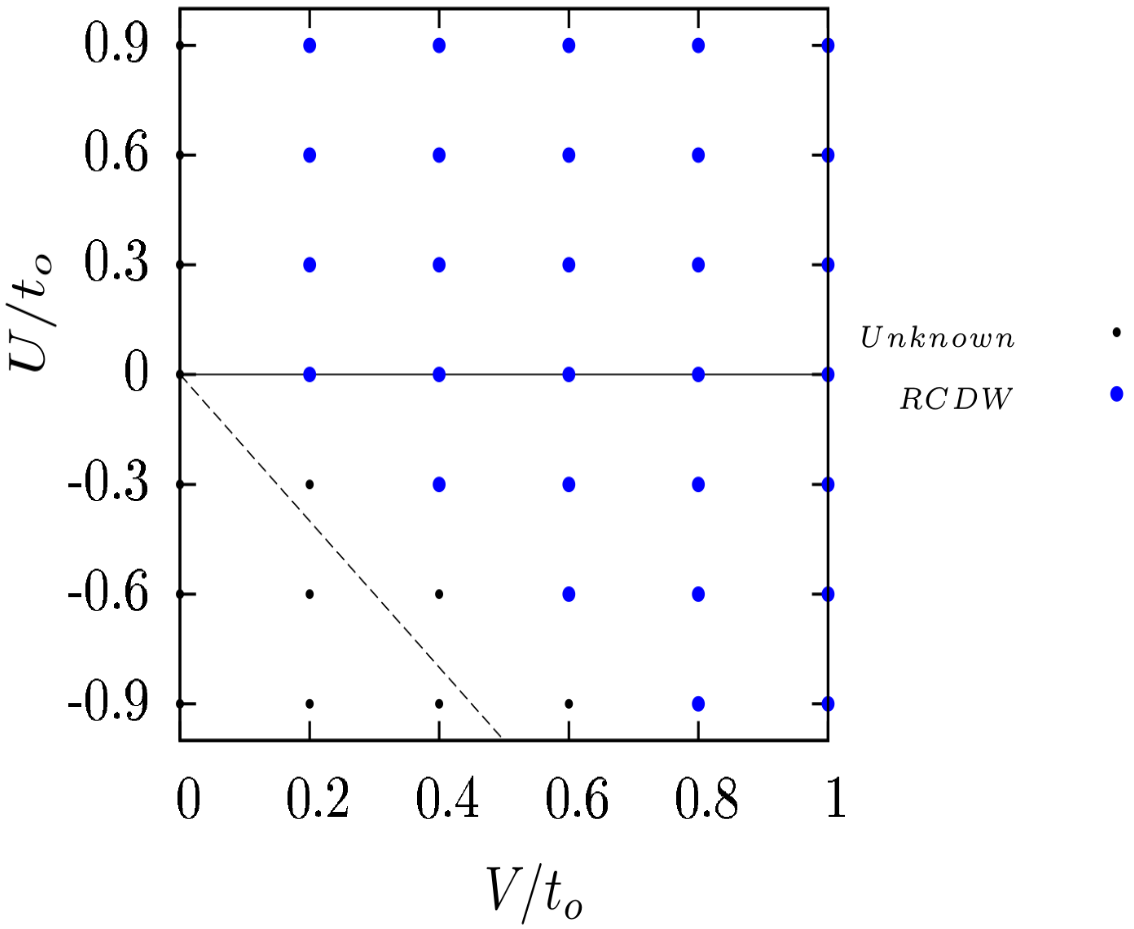
Correlation results for $U = 0.9,\ V = 0$

Phase diagrams for some other τ's at ϕ=0

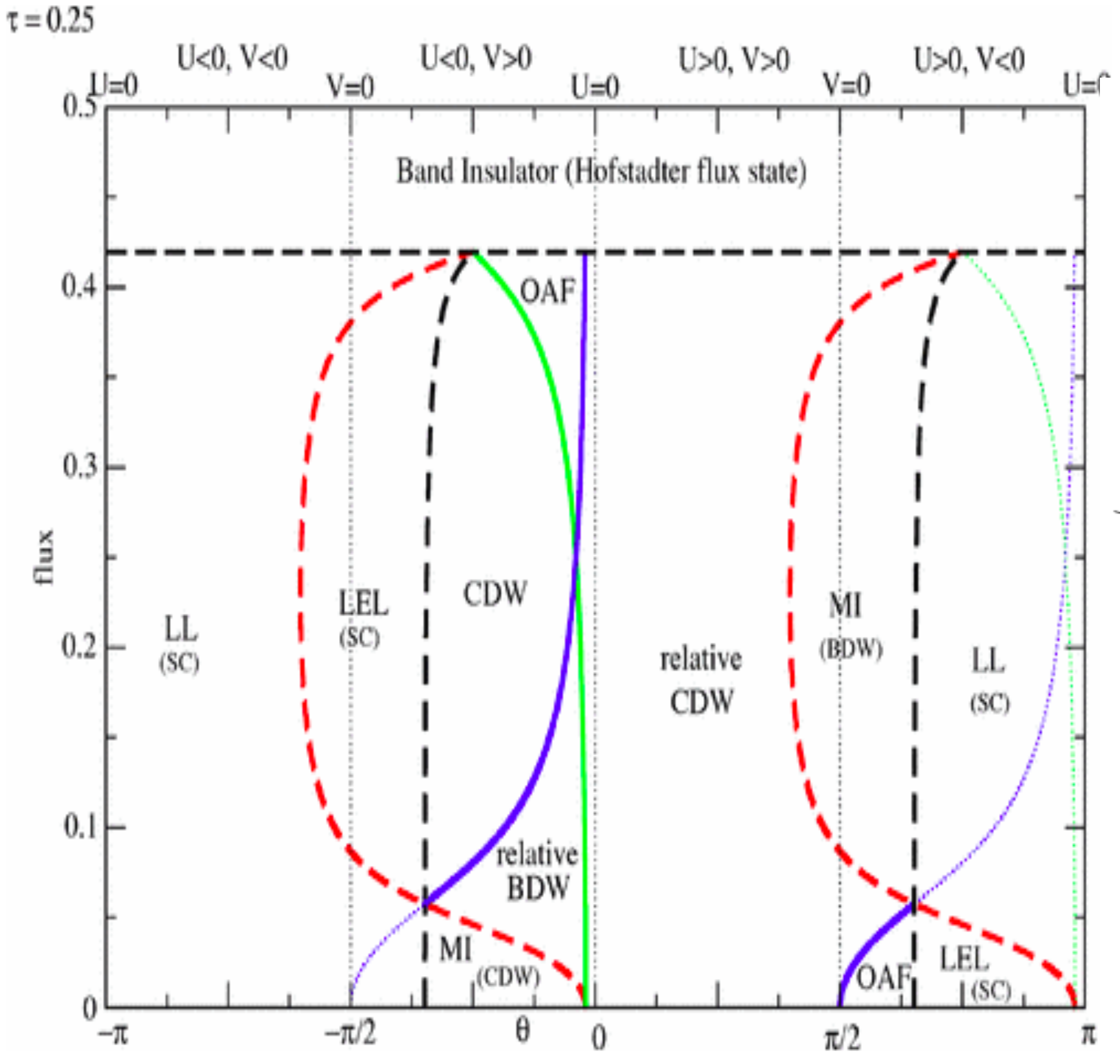
For τ=0.25



For τ=1.0

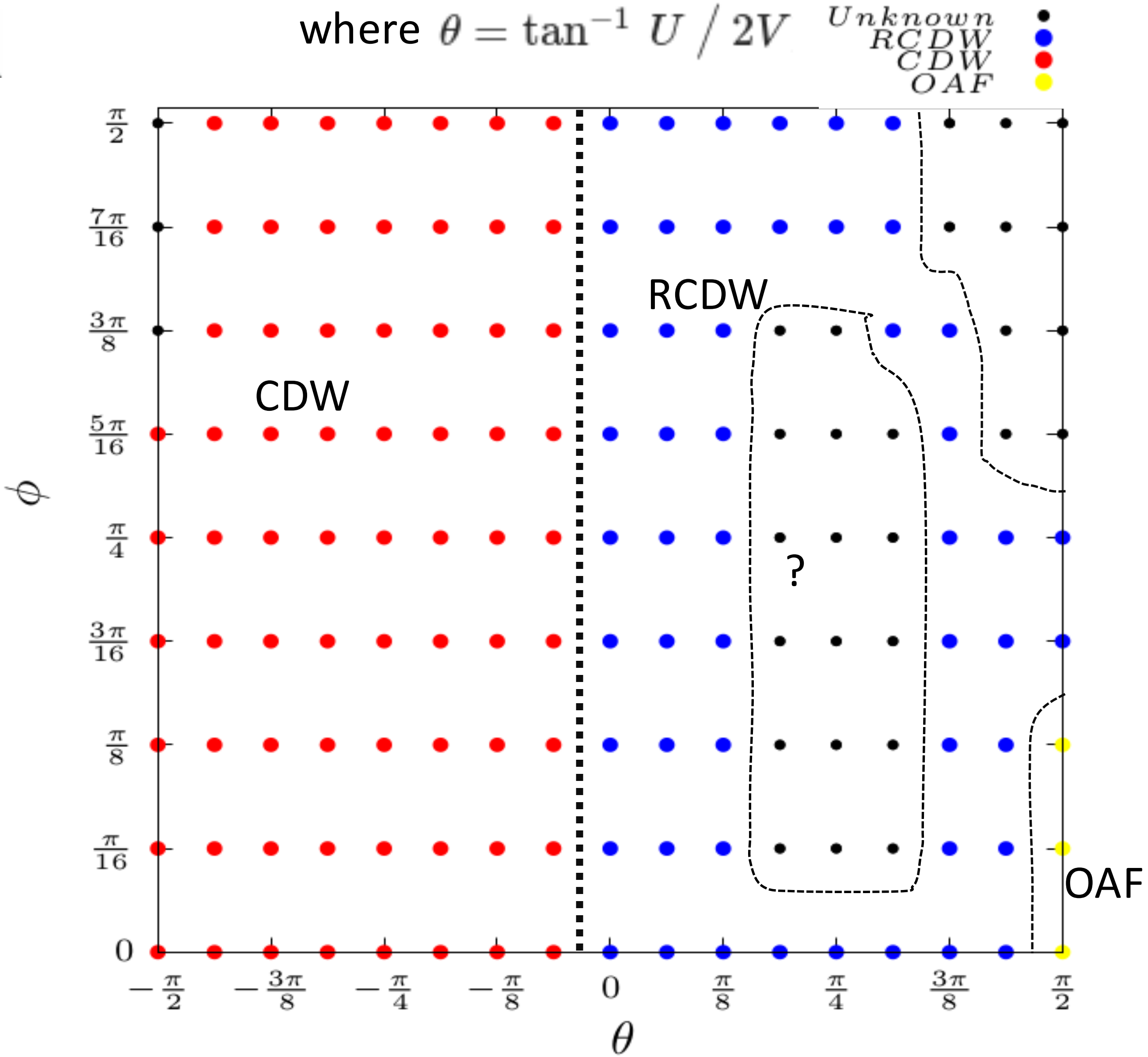


At ϕ≠0 [Weak coupling limit]



Analytical result from Literature

where $\theta = \tan^{-1} U/2V$



Our results so far (DMRG results)

Summary

- We are able to capture similar phases using DMRG in weak coupling limit as in analytical calculations.
- We are interested in performing a similar study in intermediate and strong coupling regime.
- Our results will provide as a reference to understand these systems in Cold atoms, where they can control the parameters of the Hamiltonian using newly emerged techniques.

Collaborators

Nitin Kaushal
University of Tennessee-
Knoxville

Dr. Gonzalo Alvarez
Oak Ridge National Laboratory

Prof. Elbio Dagotto (Supervisor)
University of Tennessee-
Knoxville

Thank you for your patience.

Extra slide: Local Operators

Ordered Phase	Equation
Orb. Anti-Ferro. (OAF)	$J_\perp = -it_\perp \left(c_1^\dagger c_2 - c_2^\dagger c_1 \right)$
Charge Den. Wave (CDW)	$\rho_+ = c_1^\dagger c_1 + c_2^\dagger c_2$
Relative CDW (RCDW)	$\rho_- = c_1^\dagger c_1 - c_2^\dagger c_2$
Bond Den. Wave (BDW)	$\rho_{\parallel,+} = e^{i\pi\phi} c_1^\dagger(x_n) c_1(x_{n+1}) + e^{-i\pi\phi} c_2^\dagger(x_n) c_2(x_{n+1}) + H.c.$
Relative BDW (RBDW)	$\rho_{\parallel,-} = e^{i\pi\phi} c_1^\dagger(x_n) c_1(x_{n+1}) - e^{-i\pi\phi} c_2^\dagger(x_n) c_2(x_{n+1}) + H.c.$
Rung BDW (RuBDW)	$\rho_\perp = c_1^\dagger c_2 + c_2^\dagger c_1$

Extra slide: Correlations

$$O_{OAF} = \langle J_\perp^\dagger(i) J_\perp(j) \rangle - \langle J_\perp^\dagger(i) \rangle \langle J_\perp(j) \rangle$$

$$O_{CDW} = \langle \rho_+(i) \rho_+(j) \rangle - \langle \rho_+(i) \rangle \langle \rho_+(j) \rangle$$

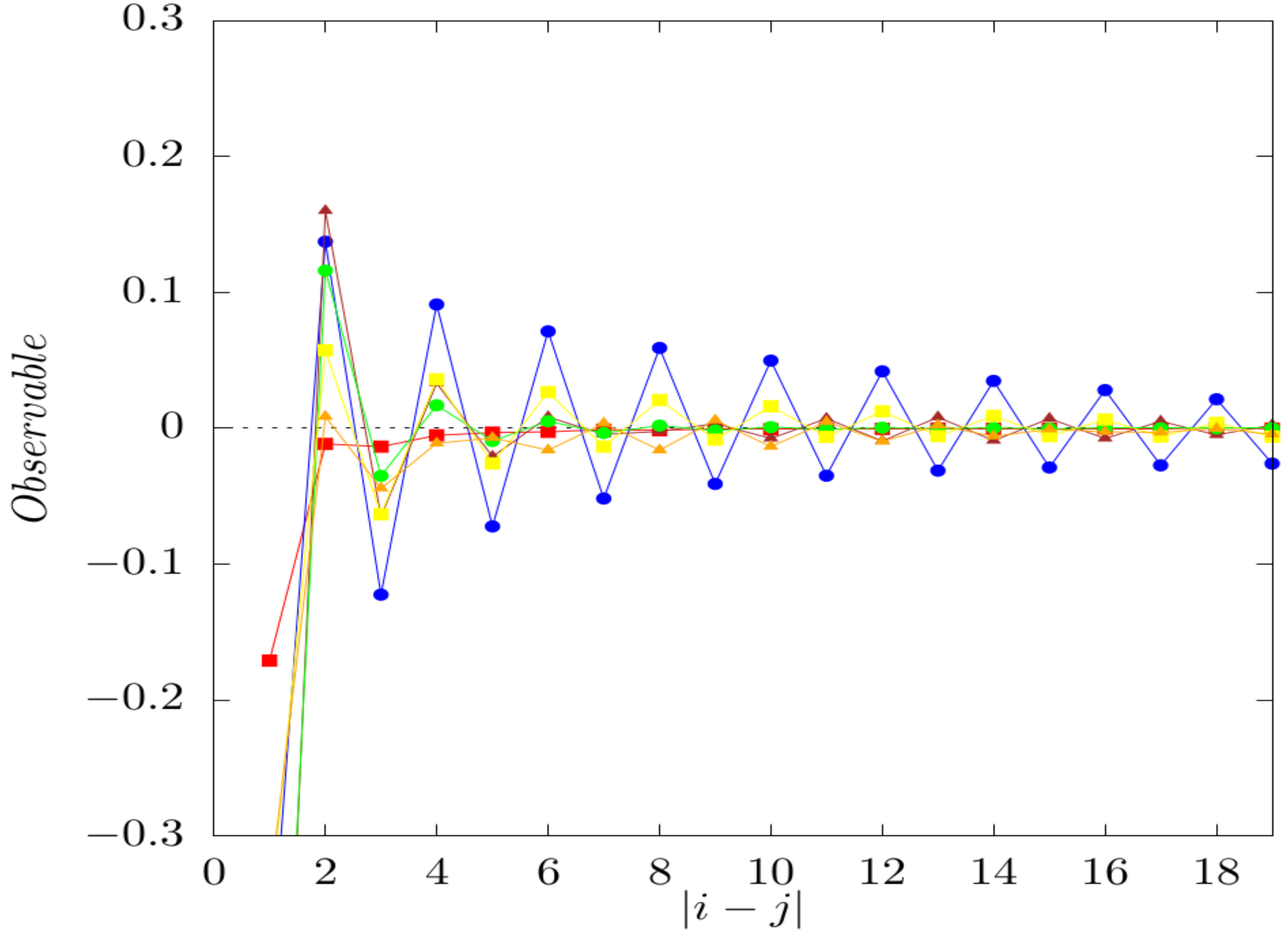
$$O_{RCDW} = \langle \rho_-(i) \rho_-(j) \rangle - \langle \rho_-(i) \rangle \langle \rho_-(j) \rangle$$

$$O_{BDW} = \langle \rho_{\|,+}(i) \rho_{\|,+}(j) \rangle - \langle \rho_{\|,+}(i) \rangle \langle \rho_{\|,+}(j) \rangle$$

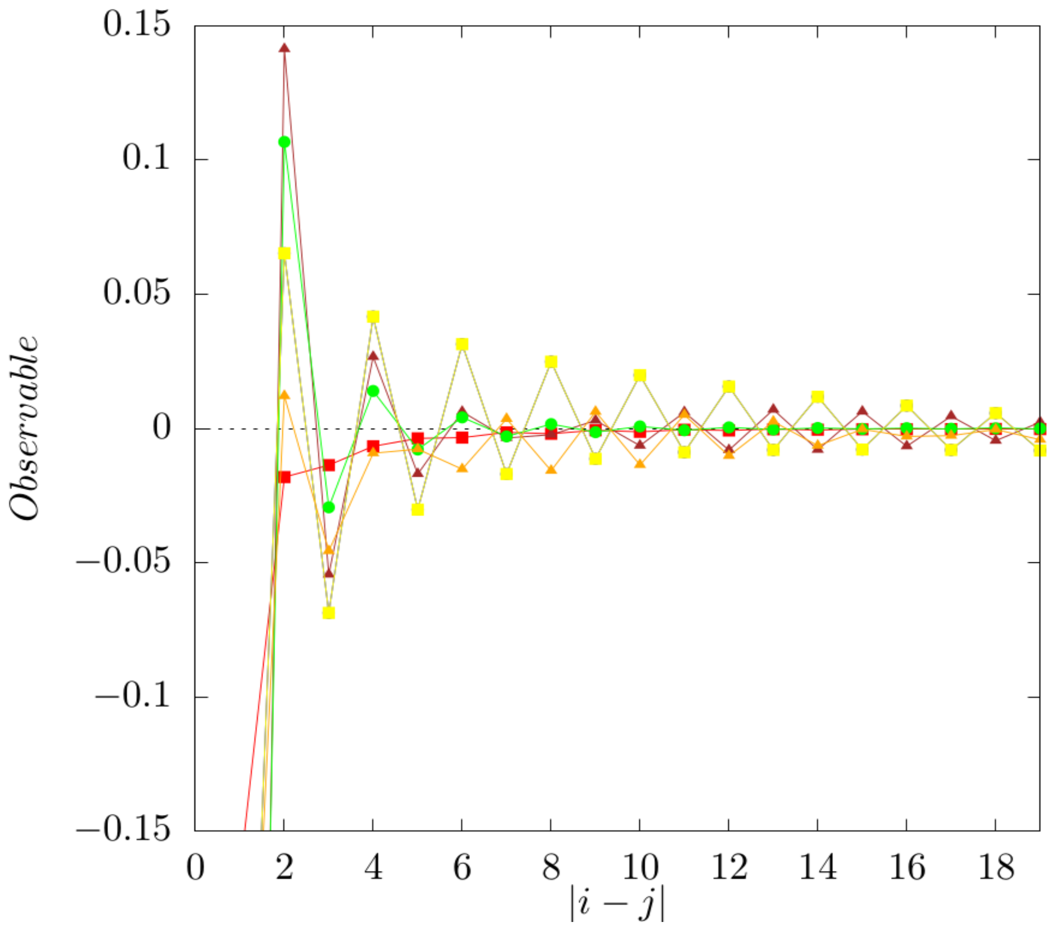
$$O_{RBDW} = \langle \rho_{\|,-}(i) \rho_{\|,-}(j) \rangle - \langle \rho_{\|,-}(i) \rangle \langle \rho_{\|,-}(j) \rangle$$

$$O_{RuBDW} = \langle \rho_\perp(i) \rho_\perp(j) \rangle - \langle \rho_\perp(i) \rangle \langle \rho_\perp(j) \rangle$$

Staggering in RCDW



Staggering in RCDW+OAF



Staggering in CDW