Topological Insulators a.k.a Quantum Spin Hall Effect

jason Lambert*

Abstract

The recent experimental realization of a the quantum spin hall effect has caused a lot of interest in these topological states of matter. Topological insulators are a new form of condensed matter that are not characterized by a spontaneously broken symmetry. Instead the materials are characterized by topology of the manifold of the occupied bloch states. Topological insulators have the unique property that the surface states are spin polarized based upon the direction the current is flowing. Like the quantum hall effect, the resistance of the surface states is quantized with the quantum unit of resistivity.

*Electronic address: \noindentjlamber9@utk.edu;

I. INTRODUCTION AND HISTORY

Before 1980, it was believed that all states in condensed matter physics could be explained by the principle of spontaneous symmetry breaking[1]. For example, the difference between solid and liquid water can be viewed as a break in translational and rotational symmetry. Liquid water has continuous translation and rotational symmetry but ice, because of the discrete unit cell, has a discrete translational and rotational symmetry. Ferromagnetics break rotational invariance because the introduction of preferred direction from domain alignment. Many other systems have been explained using this principle. The Quantum Hall Effect(QHE) was the first condensed matter phase were no spontaneously broken symmetry existed. The QHE is observed when a semiconductor is placed in a large magnetic field on the order of ~15T and at liquid helium temperature and is characterized by discrete units of conductance $\sigma_{xy} = \frac{-ne^2}{h}$ of the surface states in counterpropagating directions[2]. The hall resistance is given by $R_h = \frac{\mu_0 c}{2\alpha n}$ where n is an integer and α is the fine structure constants. The QHE states on each surface act as dissipationless conduction bands that conduct in counterpropagating directions at the opposite surfaces of the material.

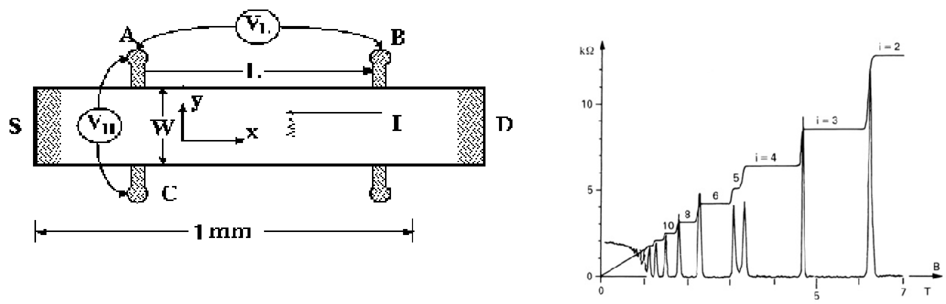

Figure 1: Figure showing the setup for the quantum hall effect measurement[2]. The resistance due to the surface states is quantized as$R_h = \frac{\mu_0 c}{2\alpha n}$ along the y direction. The important thing to notice is that the resistance at the hall resistance does not depend on the material dimensions. The second figure show the resistance in the x direction that varies like a stairstep and the resistance goes to zero for the resistance in the diagonal xy direction.

The electron transport properties on the surface of a material exhibiting the QHE are independent of scattering rates associated. This is why the surface states that characterize the quantum hall effect are extremely robust to scattering impurities in the sample. This

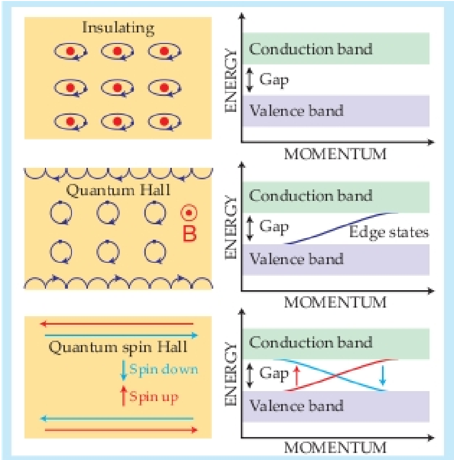

Figure 2: In an insulator the gap between there is a gap between the conduction and valence band. In the quantum hall effect this gap is crossed by the edge states of the system. The bottom diagram depicts the quantum spin hall effect spin polarized conduction bands. This is nessecary to preserve the time reversal invariance nessecary for a topological insulator.

form of dissipationless conduction could have numerous applications in the semiconductor industry if the extremes in temperature and magnetic field were not required.
The recent prediction and experimental realization of the quantum spin hall effect represents a new topological state of matter referred to as a topological insulator[3]. Topological insulators are insulating in the bulk but have surface conducting states similar to the quantum hall state where the resistance is quantized on the surface. The quantum spin hall effect permits electrons to flow with quantized conduction but the direction of the flow depends on the spin of the electron. For the sample material Bi_2Te_3 the lattice critical points are given in figure 4.

II. THEORY

The distinction between a conventional insulator and a topological insulator is a property of the topology of the manifold of occupied bloch states. Topological insulators and superconducting phases can be classified based upon the symmetry of the hamiltonian that describe the particular phase. Table one gives the symmetry class of a system and its dimensionality to classify different types of condensed phases as either $\mathbb{Z}$, $\mathbb{Z}_2$, or not topological. The important symmetries for classification of the hamiltonian are time reversal symmetry,

	Symmetry			d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Figure 3: The topological periodic table where the hamiltonian symmetries and the dimensionality of the system determine the topological nature of the material. The important symmetries are time reversal symmetry Θ, particle hole symmetry Ξ, and chiral symmetry Π. The d stands for dimensionality of the system. The value ± 1 is the value of Θ^2 , Ξ^2or Π^2.

particle hole symmetry, and chiral symmetry. The QH state has "A" symmetry and has a dimensionality of 2, therefore it is classified as a $\mathbb{Z}$ class topological state. The QSH state has symmetry AII and is 2 or 3 dimensional. A topological superconductor has of symmetry class D with dimensions 1, and 2. It is fascinating that many of the possible phases of topological matter have not been realized experimentally[4].

It is not a trivial matter to classify a material as a topological insulator based on band theory but this article presents the most tangible method for doing so. If the material under investigation has inversion symmetry then a procedure that looks at the eigenvalue of the parity operator at time invariant points of the lattice has been utilized to determine the topological nature of the material. To determine the topological invariant for material with inversion symmetry, first look at the points in the brillouin zone where the bloch states are time reversal invariant. These occur at the high symmetry points of the brillouin zone where the where the bloch states are eigenvectors of the parity operator. If we consider the product of the eigenvalues for the parity operator at the these points the value of the topological invariant can be determined which determines whether the material is a topological insulator.

The topological invariant can be determined from equations 2 and 3

$$\delta_\alpha = \prod_m \xi_m(\Lambda_\alpha) \tag{1}$$

$$(-1)^\nu = \prod_\alpha \delta_\alpha \tag{2}$$

where ξ_m is the parity eigenvalue of the mth occupied Kramers pair and α corresponds to special point in the brollium zone. ν is the $\mathbb{Z}_2$ topological invariant determines whether the material is ordinary insulator or a topological insulator[4, 5].

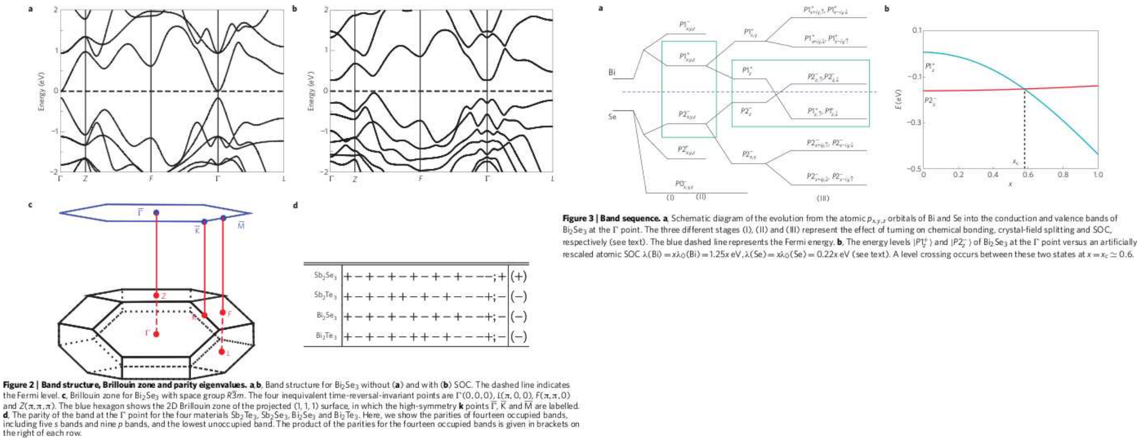

Figure 4: Figure showing the brolluoiun zone of the the set of topological insulators Bi_2Te_3, Sb_2Se_3, and Sb_2Te_3. The band structure of the material is shown in a and b. The parity analysis at the Γ point is presented in figure d. The image on the right shows the hybridization and crystal splitting and the importance of spin orbit coupling to the band inversion in the material. [5]

The topological invariant ν is either 0 if it is not a topological insulator or 1 if it is a topological insulator. The parity analysis presented in figure 3 for Bi_2Te_3, Bi_2Se_3, Sb_2Se_3, and Sb_2Te_3 . This importance of spin orbit coupling is evident from figure 4. The parity eigenvalue remains unchanged at all points except for the Γ point so the Γ point is the only momenta point that needs to be considered. At this point the parity of one band is inverted because spin orbit coupling inverts the lowest unoccupied and highest occupied band.

To understand why time reversal symmetry is important for a topological insulator we consider the surface state characteristics. The surface states of a quantum topological insulator are spin polarized based upon the direction in which current is flowing. Time reversal symmetry acts on the wavevector $\Theta u(\mathbf{k}) \rightarrow -u(-\mathbf{k})$ and the spin $\Theta\chi(\frac{1}{2}) = \chi(-\frac{1}{2})$. The surface states are time reversal invariant because spin and momentum are both negative under time reversal. The fact that the surface states are time reversal invariant makes them robust against perturbations that are time reversal invariant. However, a time reversal symmetry breaking perturbation like a magnetic field will destroy the surface states of a topological insulator.

III. EXPERIMENTAL REALIZATION

In the search for topological insulators there are two key ingredients to consider; the material has to have a bulk insulating gap and spin orbit coupling needs to be present. Using these guidlines the first experimental observation of a topological insulator was reported in HgTe quantum wells[6]. The topological insulating phase in these materials depended on the thickness of the quantum well. The HgTe quantum well highest valence and lowest conduction band invert at a critical thickness of ~ 6.5 nm where it exhibited the quantum spin hall effect.

More recently the quantum spin hall effect has been realized for the pure substances Bi_2Te_3, Bi_2Se_3, and Sb_2Te_3. It was also accurately predicted that the material Sb_2Se_3 does not exhibit the quantum spin hall effect[5]. These so called second generation material show a large amount of promise because the band gap of approximately 0.3 ev is large enough for the topological insulator to exist at room temperature. One essential component of a topological insulator is the dirac cone not mentioned is the fact that it must have an odd number of dirac cones in the momentum energy dispersion diagram. This means that electrons within a certain range of momentum behave like massless dirac fermions.

IV. POTENTIAL APPLICATIONS AND PHYSICS EXOTICA

The topic of topological insulators has become an extremely popular because of the numerous suggested applications for these materials. The application to spintronics is an obvious one because the fact that surface conduction bands flowing in opposite directions are spin polarized. Also, the flow of electrons across the surface of a topological insulator do not dissipate energy as heat. This dissipationless conduction has obvious application in small electronic devices because the conduction does not depend upon the size of the material. For ohmic materials the resistance scales with length. For very small devices

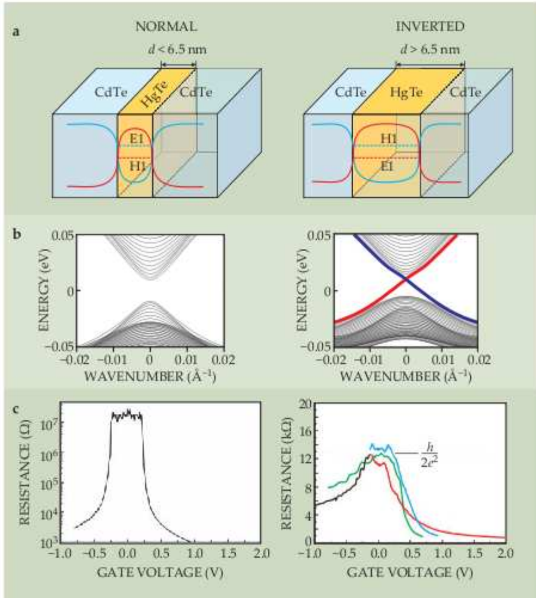

Figure 5: a) The mercury telleride quantum well at thickness greater then and less then the critical thickness. The H1 and E1 valence and conduction bands invert at the critical thickness as a result of the spin orbit coupling. b) The quantum well below the critical thickness has an insulating gap. Above the critical thickness ofapproximately 6.5nm the surface states of opposite spin polarity cross the bandgap. Near the crossing point of the surface states the dispersion is linear, this can be described by the dirac equation for a massless fermion. c) When the material is below the critical thickness, the resistance of the system in nearly infinite at small gate voltage. When the thickness increased above the critical thickness, the material showed a quantized resistance[1].

the ohmic dissipation of energy as heat is very large. On top of the practical applications there are some more exotic properties to considered. for example, it has been predicted that a topological insulator coated with a thin layer of a ferromagnetic material will create an image magnetic monopole on its surface because of the unique electromagnetic effective action inside a topological insulator.[1]. This another interesting prospect predicts that a topological insulator coated with a superconductor will produce majorana fermions. It has been suggested that neutrino's may be majorana fermions. A majorana fermion is a fermion that is its own antiparticle[7]. These particles are predicted to exist at the boundary between an s wave superconductors and a topological insulator. The use of the topological insulators with majorana fermions have been proposed a potential basis for a quantum computer. If the majorana fermions are observed, then topological insulators could produce table top

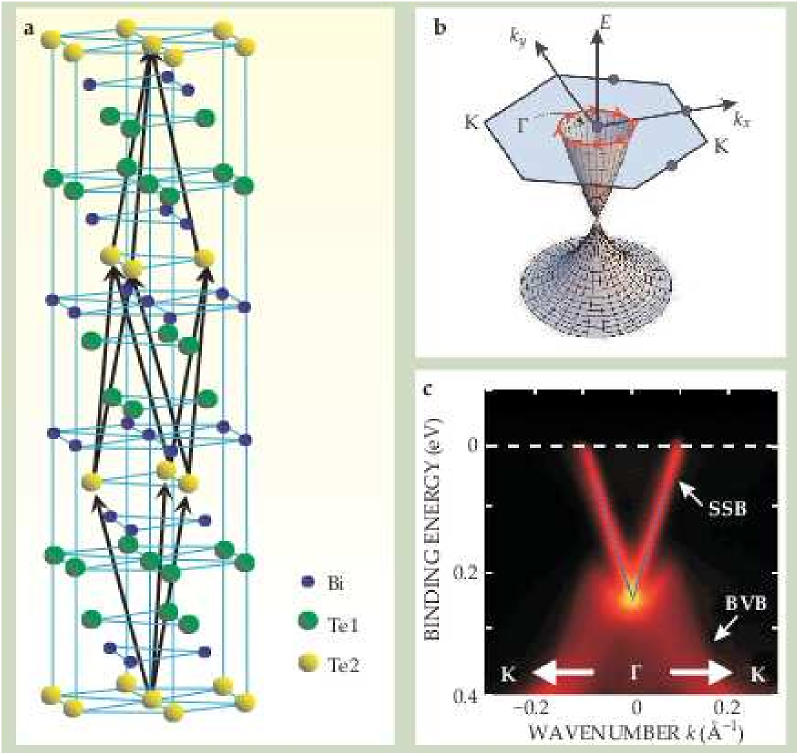

Figure 6: a) The crystalline structure of the Bi_2Te_3, Bi_2Se_3, and Sb_2Te_3. b) The energy versus momentum plot mapped out using ARPES(angle resolved photoelectron spectroscopy). b) The cone in this plot is referred to as a dirac cone, and essential component in topological insulators. The energy versus momentum in a dirac cone is linear like that of massless particles and is the two dimensional equivalent of the one dimension dirac cone in the HgTe quantum well. c) A plot of energy versus momentum from ARPES data. The linearly dispersing surface states are referred to as SSB(surface state band) and the bulk valence insulating states are referred to BVB (Bulk Valence Band).

experiments.

---

[1] S.-C. Z. Xiao-Liang Qi, Physics Today **63**, 33 (2010)

[2] K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (Aug. 1980), `http://link.aps.org/doi/10.1103/PhysRevLett.45.494`

[3] S. Murakami, N. Nagaosa, and S.-C. Zhang, Science **301**, 1348 (2003), `http://www.sciencemag.org/cgi/content/abstract/301/5638/1348`

[4] M. Z. Hasan and C. L. Kane, arXiv(Febuary 2010)

[5] H. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, Nat Phys **5**, 438 (Jun. 2009), ISSN 1745-2473, `http://dx.doi.org/10.1038/nphys1270`

[6] M. KÃ¶nig, S. Wiedmann, C. BrÃŒne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science **318**, 766 (2007)

[7] F. Wilczek, Nat Phys **5**, 614 (Sep. 2009), ISSN 1745-2473, http://dx.doi.org/10.1038/nphys1380

[8] C. Day, Physics Today **61**, 19 (2008)

[9] L. Fu and C. L. Kane, Phys. Rev. Lett. **100**, 096407 (Mar. 2008), http://link.aps.org/doi/10.1103/PhysRevLett.100.096407

[10] L. Fu and C. L. Kane, Phys. Rev. B **76**, 045302 (Jul. 2007), http://link.aps.org/doi/10.1103/PhysRevB.76.045302

[11] L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. **98**, 106803 (Mar. 2007), http://link.aps.org/doi/10.1103/PhysRevLett.98.106803

[12] D. Hsieh, Y. Xia, L. Wray, D. Qian, A. Pal, J. H. Dil, J. Osterwalder, F. Meier, G. Bihlmayer, C. L. Kane, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Science **323**, 919 (Feb. 2009), http://www.sciencemag.org/cgi/content/abstract/323/5916/919

[13] J. Linder, Y. Tanaka, T. Yokoyama, A. Sudbï¿œ, and N. Nagaosa, Phys. Rev. Lett. **104**, 067001 (Feb. 2010), http://link.aps.org/doi/10.1103/PhysRevLett.104.067001

[14] J. Moore, Nat Phys **5**, 378 (Jun. 2009), ISSN 1745-2473, http://dx.doi.org/10.1038/nphys1294

[15] J. C. Y. Teo, L. Fu, and C. L. Kane, Phys. Rev. B **78**, 045426 (Jul. 2008), http://link.aps.org/doi/10.1103/PhysRevB.78.045426

[16] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nat Phys **5**, 398 (Jun. 2009), ISSN 1745-2473, http://dx.doi.org/10.1038/nphys1274