Lectures: Condensed Matter II

1 – Electronic Transport in Quantum dots

2 – Kondo effect: Intro/theory.

3 – Kondo effect in nanostructures

Luis Dias – UT/ORNL

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➡ 1 – Electronic Transport in Quantum dots

2 – Kondo effect: Intro/theory.

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Lecture 1: Outline

- Introduction: From atoms to “artificial atoms”.
- What are Quantum Dots?
- Confinement regimes.
- Transport in QDs: General aspects.
- Transport in QDs: Coulomb blockade regime.
- Transport in QDs: Peak Spacing.

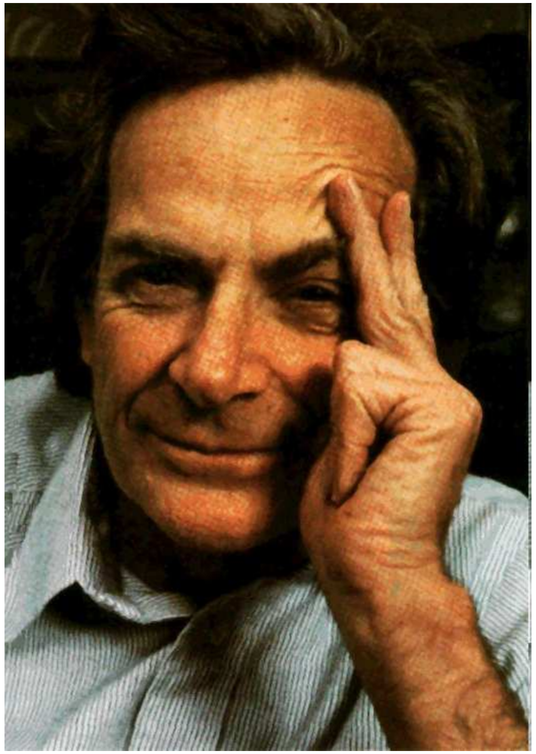
“More is Different”

“ The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of simple extrapolation of the properties of a few particles.

Instead, at each level of complexity entirely new properties appear and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other.“

Phillip W. Anderson, “More is Different”,
Science **177** 393 (1972)

“More is Different?”



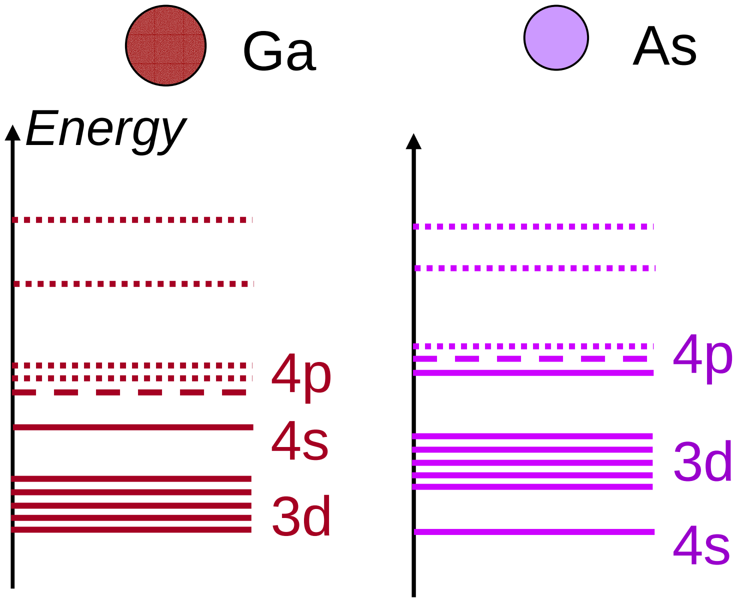
“If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words?

I believe it is the *atomic hypothesis* that ***All things are made of atoms-little particles that that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.***“

In that one sentence, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

R. P. Feynman – *The Feynman Lectures*

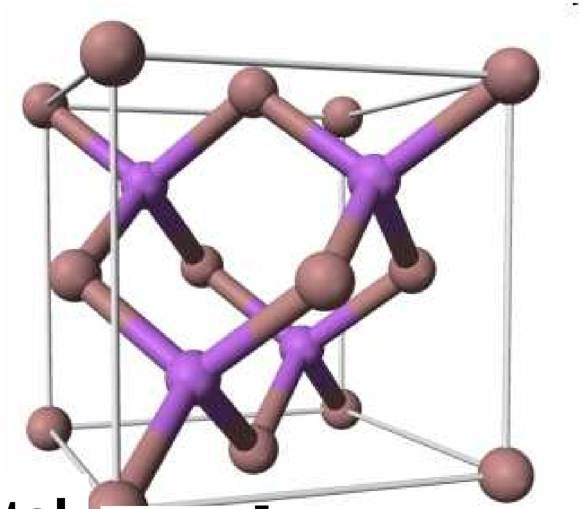
Can you make “atoms” out of atoms?



$[Ar]\ 3d^{10}\ 4s^{2}\ 4p^{1}$ $[Ar]\ 4s^{2}\ 3d^{10}\ 4p^{3}$

Atomic Energy levels

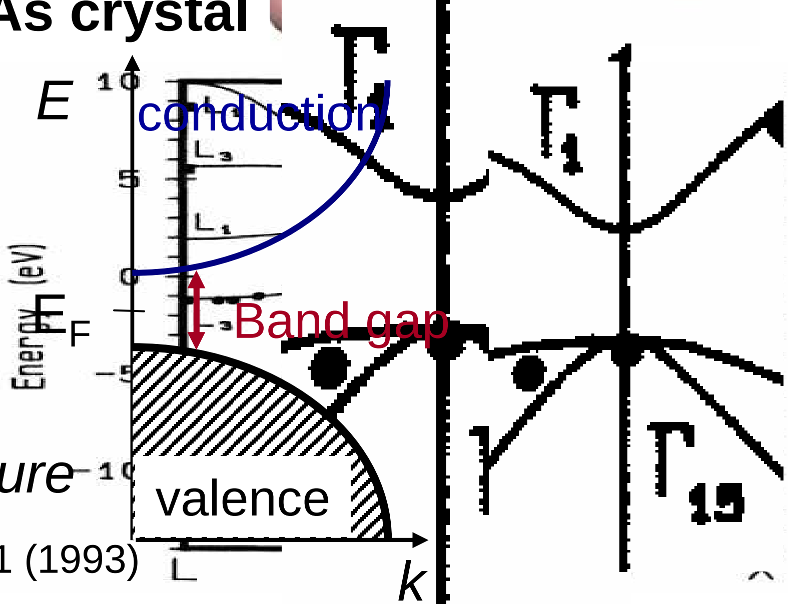
Many Atoms!



GaAs crystal

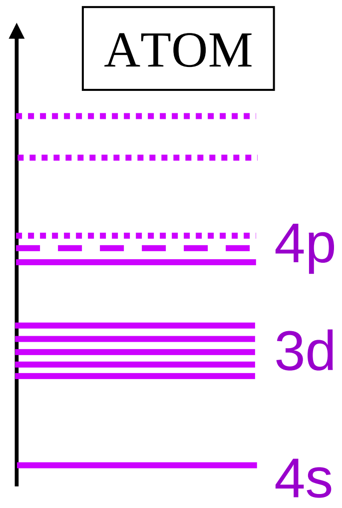
Many Atoms!

Band structure

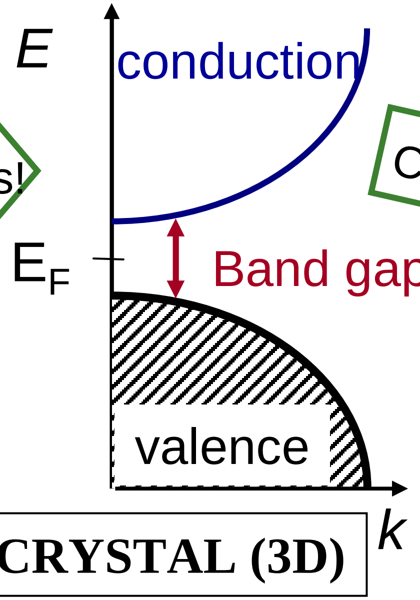


M. Rohlfing et al. PRB **48** 17791 (1993)

Making “artificial atoms”(?) out of atoms

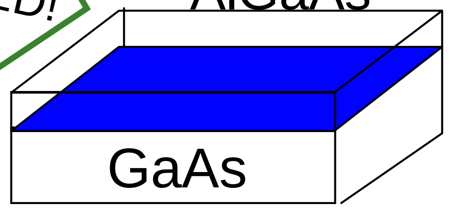


Many Atoms!

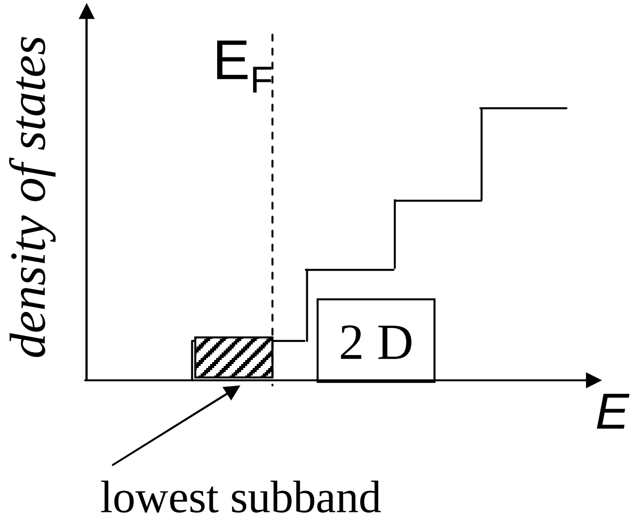


Confine in 2D!

AlGaAs/GaAs heterostructure



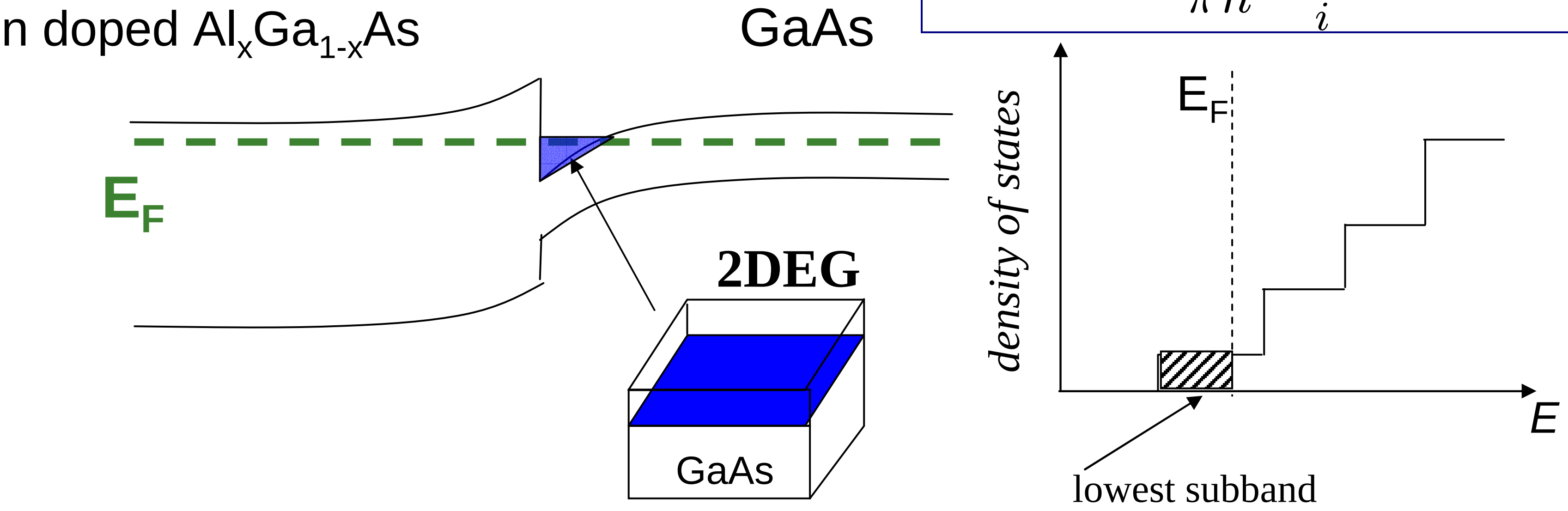
2D Electron gas



Confined in 1 direction: 2D system

If a thin enough 2D plane of material (containing free electrons) is formed the electrons can be confined to be two dimensional in nature. Experimentally this is usually done in semiconductors.

$$\rho_{2D}(E) = \frac{m}{\pi \hbar^2} \sum_i \Theta(E - E_i)$$

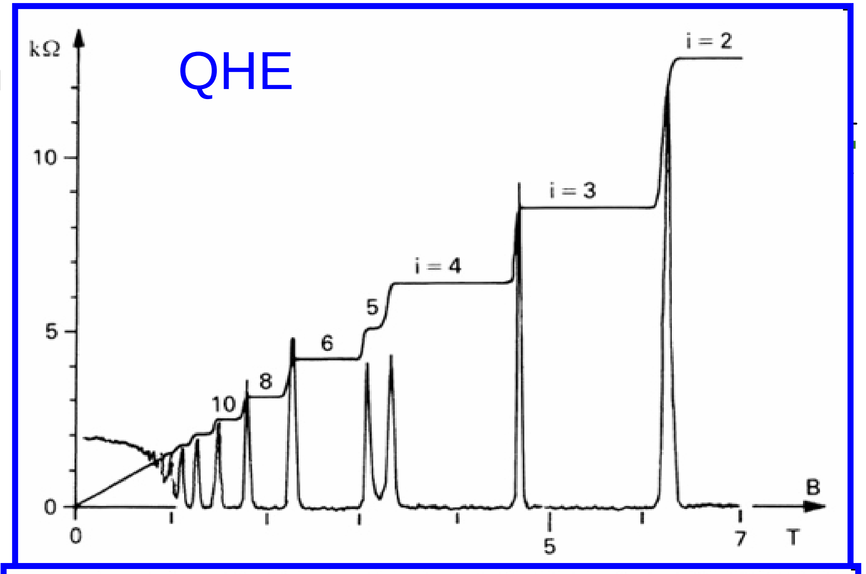


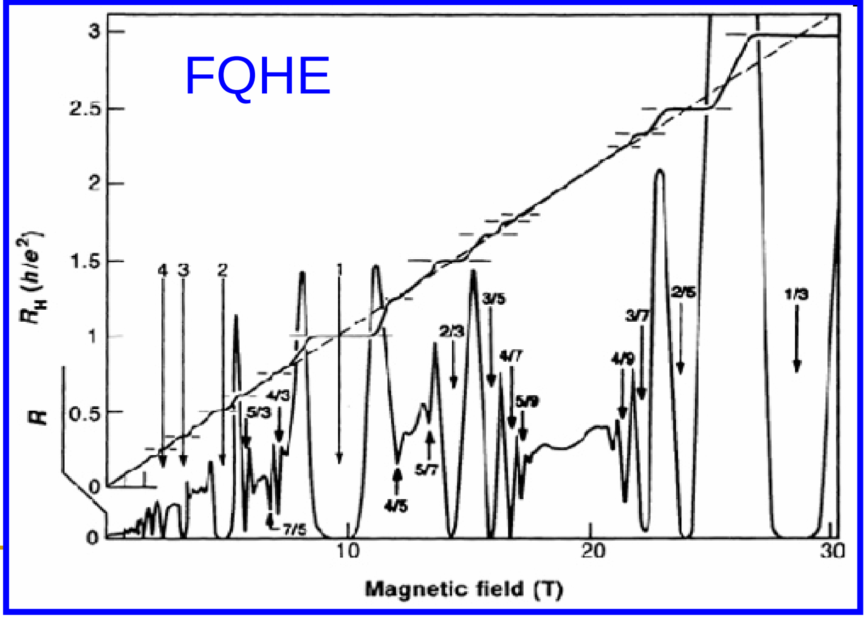
e.g. by growing a large band gap material with a smaller band gap material you can confine a region of electrons to the interface - TWO DIMENSIONAL ELECTRON GAS (2DEG).

More details: Ando, Fowler, Stern *Rev. Mod. Phys.* **54** 437 (1982)

Confined in 1 direction: 2D system

Provided the electrons are confined to the lowest subband the electrons behave exactly as if they are two-dimensional i.e. obey 2D Schrodinger equation etc.





2DEG: Rich source of Physics.

- Nobel Prizes in Physics:
  - in 1985 to von Klitzing for the Quantum Hall Effect (QHE),
  - In 1998 to Tsui, Stormer and Laughlin for the Fractional QHE
- Semiconductor heterostructures, lithography
- Applications (lasers, QHE, etc.)

Side example: electrons confined in 1D

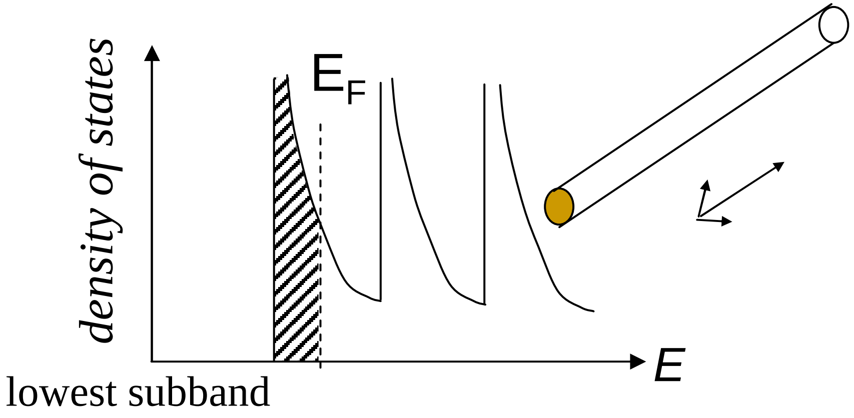
If the confinement length is of the order of the Fermi wavelength **($L \sim \lambda_F$ or $E_1 <\sim E_F$)** then electrons confined in one quantum mechanical state in two directions, but are free to move in the third: 1D confinement.

A truly 1D DoS *diverges* at some energy values: these are van Hove singularities.

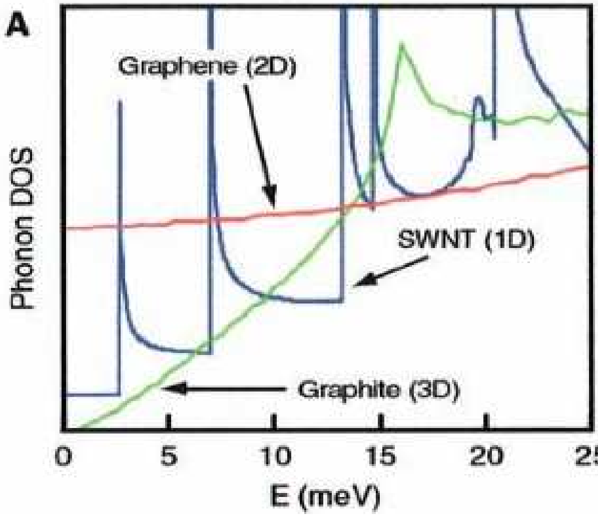
Electrons interact differently in 1D compared to 2D and 3D (e.g. Luttinger liquid physics).

Analogy: think of cars (electrons) moving along a single track lane. They “interact” differently compared to cars on dual lane roads, for instance.

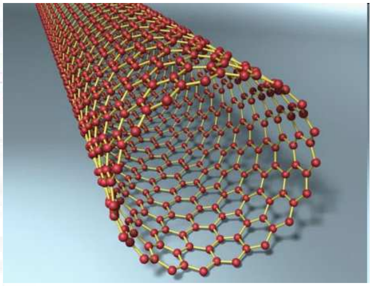
Examples include carbon nanotubes, nanowires, lithographically defined regions of 2DEGs etc.



$$\rho_{1D}(E) = \left(\frac{2m}{\pi^2\hbar^2}\right)^{1/2} \sum_i \frac{n_i\Theta(E-E_i)}{(E-E_i)^{1/2}}$$

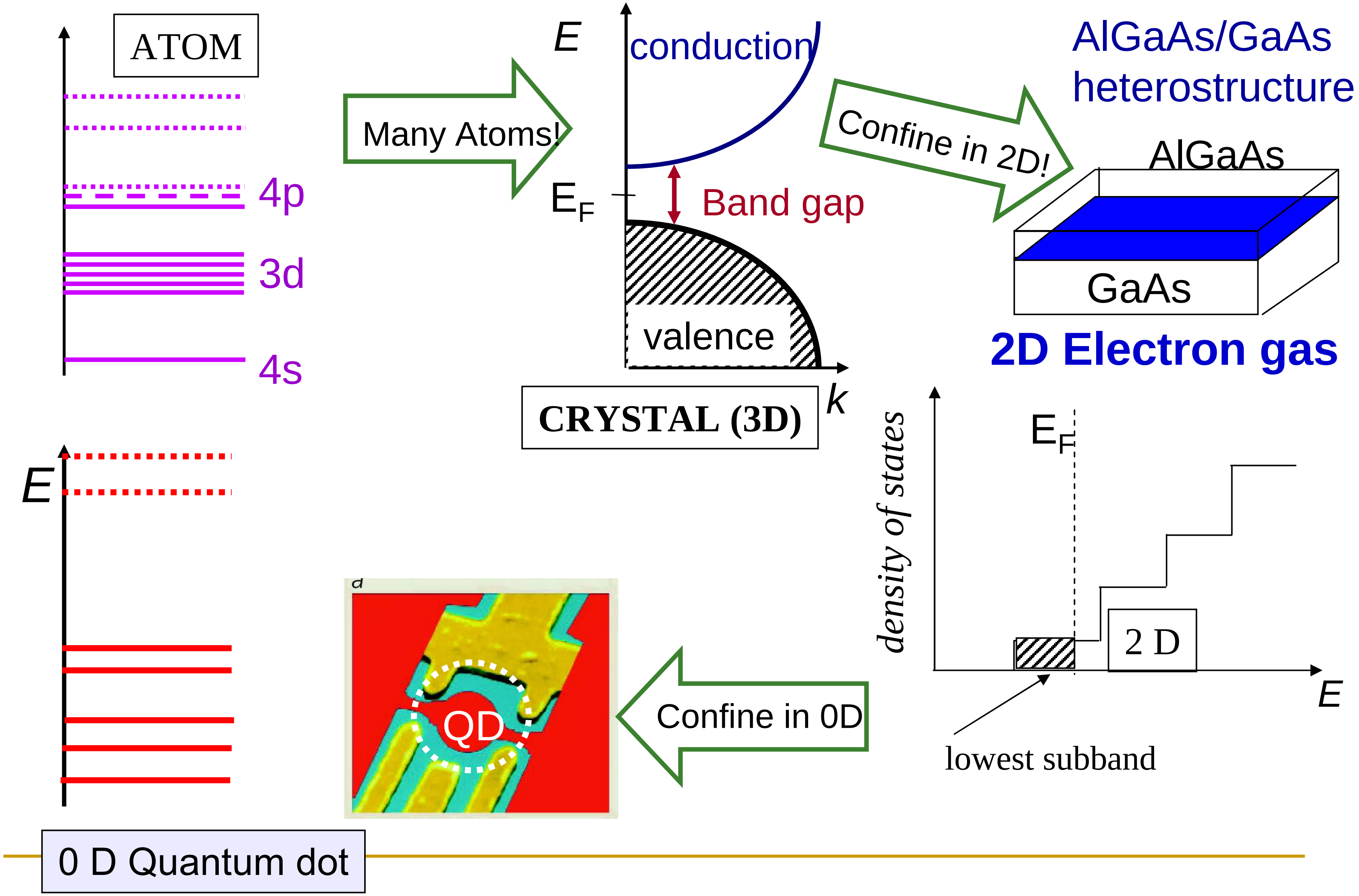


Hone, J. et al., *Quantized Phonon Spectrum of Single-Wall Carbon Nanotubes*, Science, **289**, 1730, (2000).

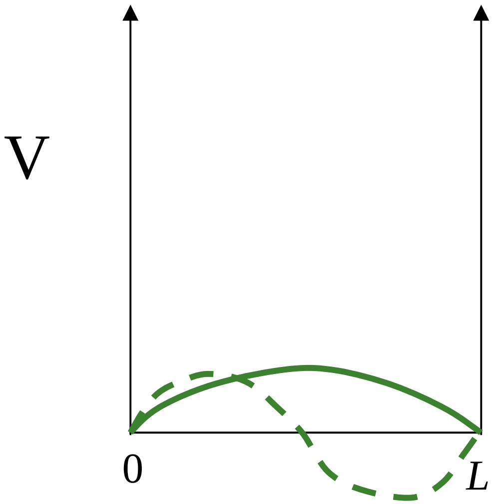


Carbon nanotubes

Making “artificial atoms”(?) out of atoms



Confinement: Particle in a box



1-d box: wavefunction constrained so that

L = N λ / 2 or
k = 2 π / λ = N π / L

Energy of states given by Schrodinger Equation:

$$\hat{H}\Psi = E\Psi$$

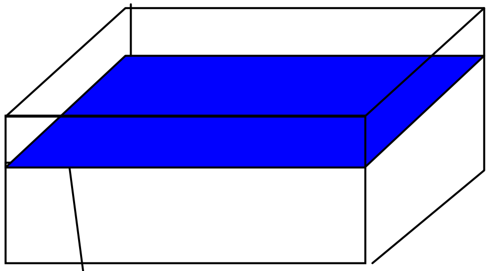
$$\boxed{E_N = \frac{\hbar^2 k_N^2}{2m} = \frac{\hbar^2 \pi^2 N^2}{2mL^2}}$$

Typical semiconductor dots:
L in nm, E in meV range

As the length scale decreases the energy level spacing increases.

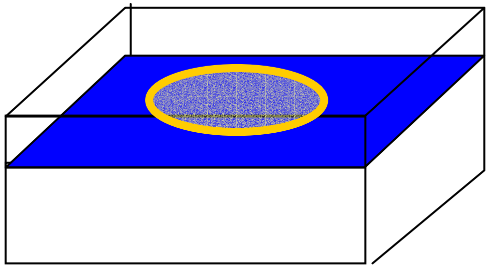
$$\boxed{\Delta E = E_{N+1} - E_N \propto \frac{1}{L^2}}$$

Can you make “atoms” out of atoms?

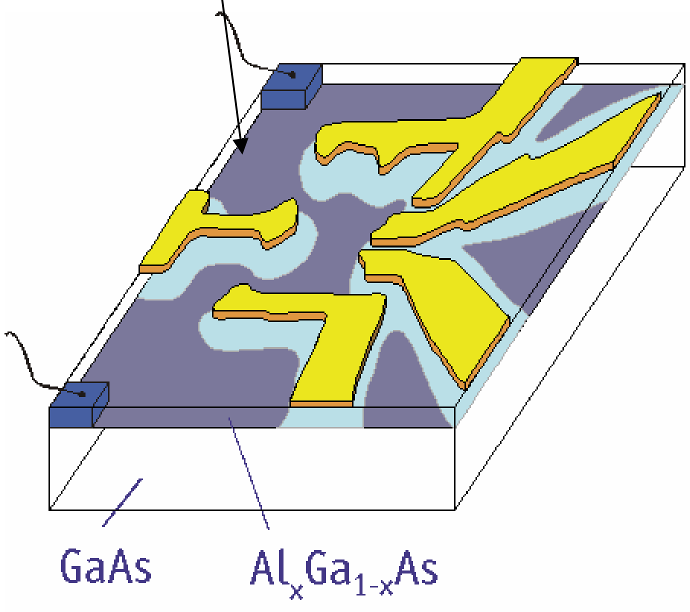


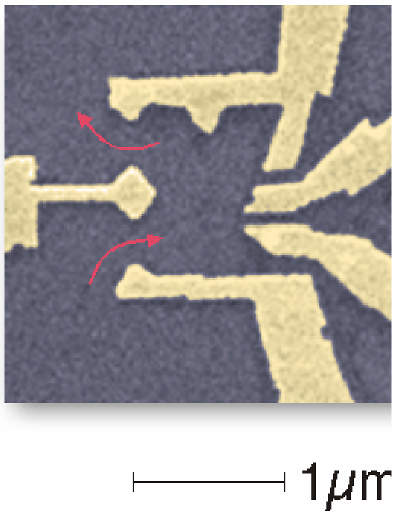
2D Electron gas

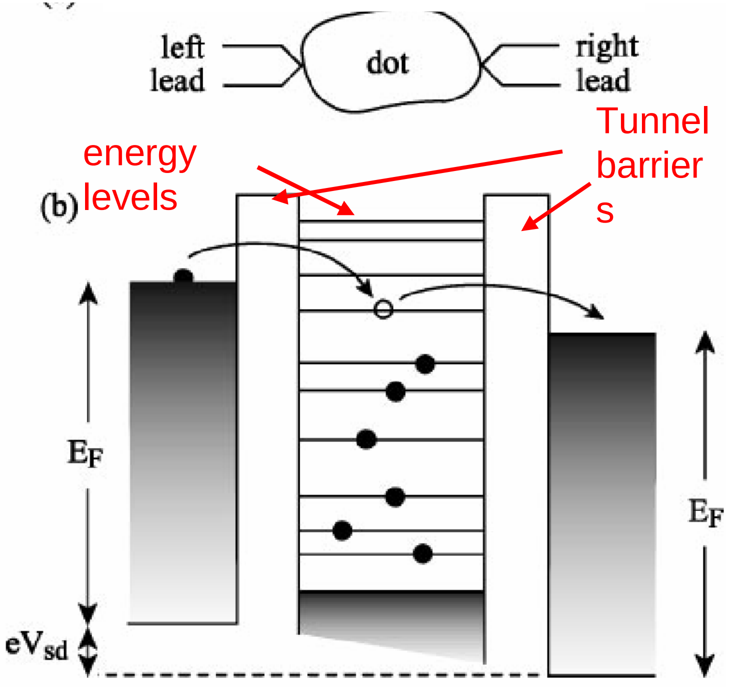
Electrostatically confine electrons within an small (nanometer-size) region.



“Quantum dot”





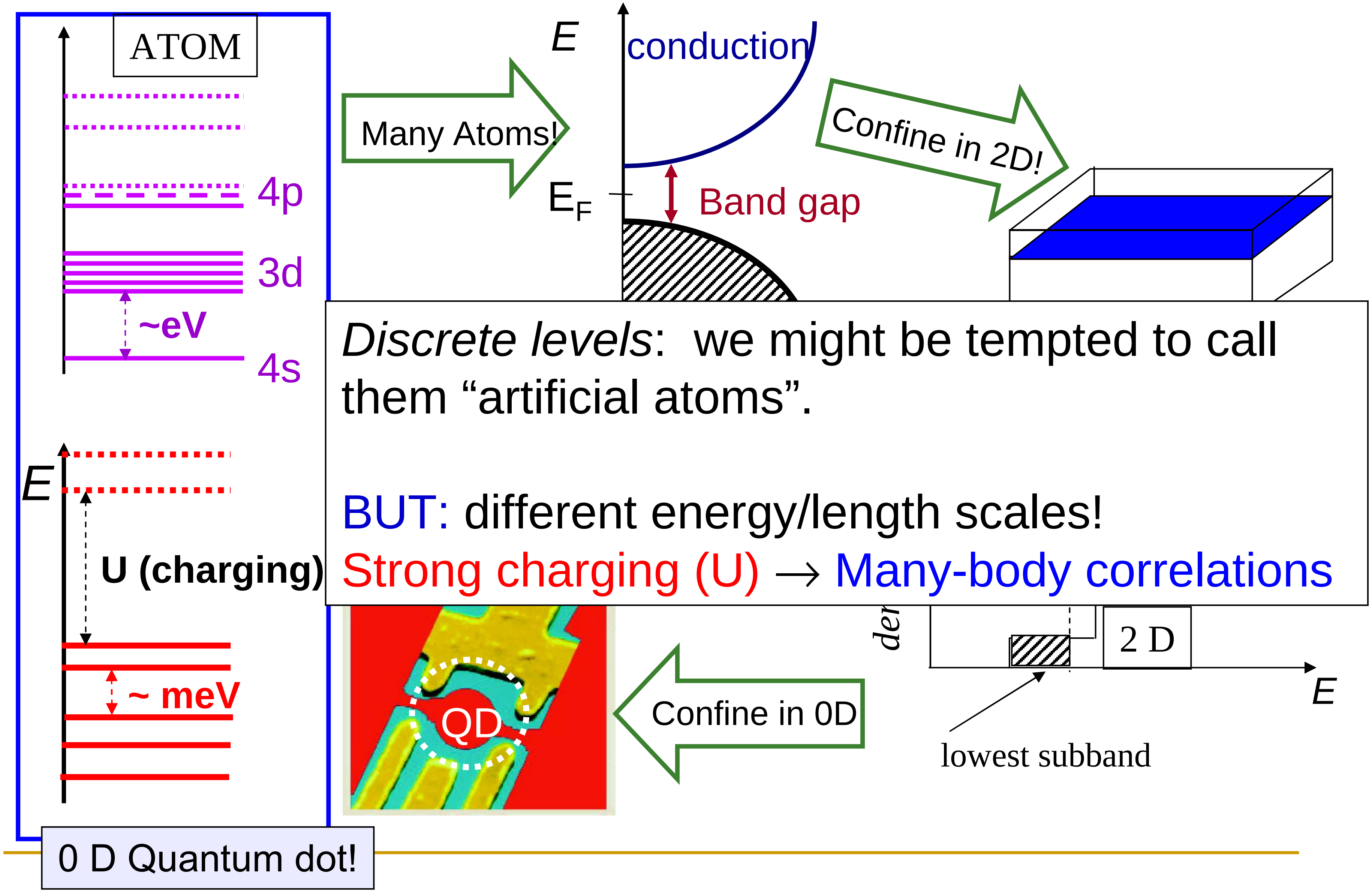


energy levels

Tunnel barriers

from Charlie Marcus’ Lab website (marcuslab.harvard.edu)

Making “artificial atoms”(?) out of atoms

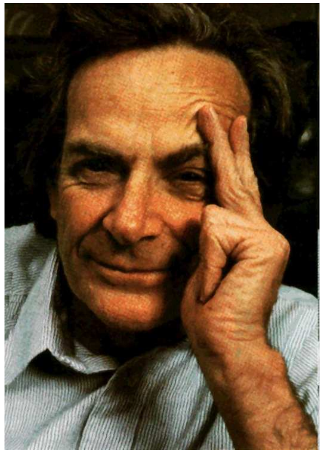


Discrete levels: we might be tempted to call them “artificial atoms”.

BUT: different energy/length scales!

Strong charging (U) → Many-body correlations

“Nano is Different”



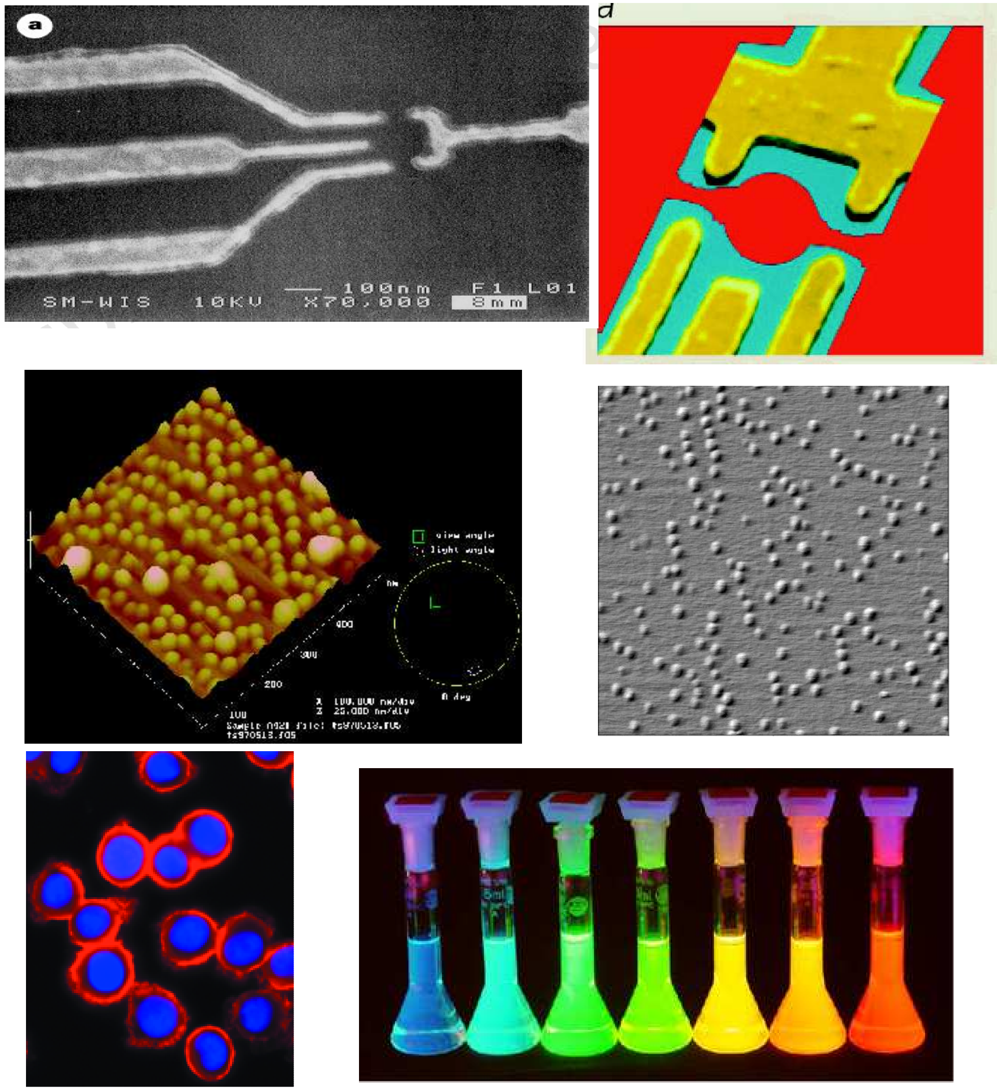
“There is plenty of room at the bottom”.

“This field is not quite the same as the others in that it will not tell us much of the fundamental physics in the sense of, ‘What are the strange particles?’. But it is more like solid state physics in the sense that it might tell us much of the great interest about the strange phenomena that occur in complex situations.”
– *Richard Feynman*

Or, as Anderson might put it:

The rules of the game are different at the bottom.

What are *Quantum Dots*?

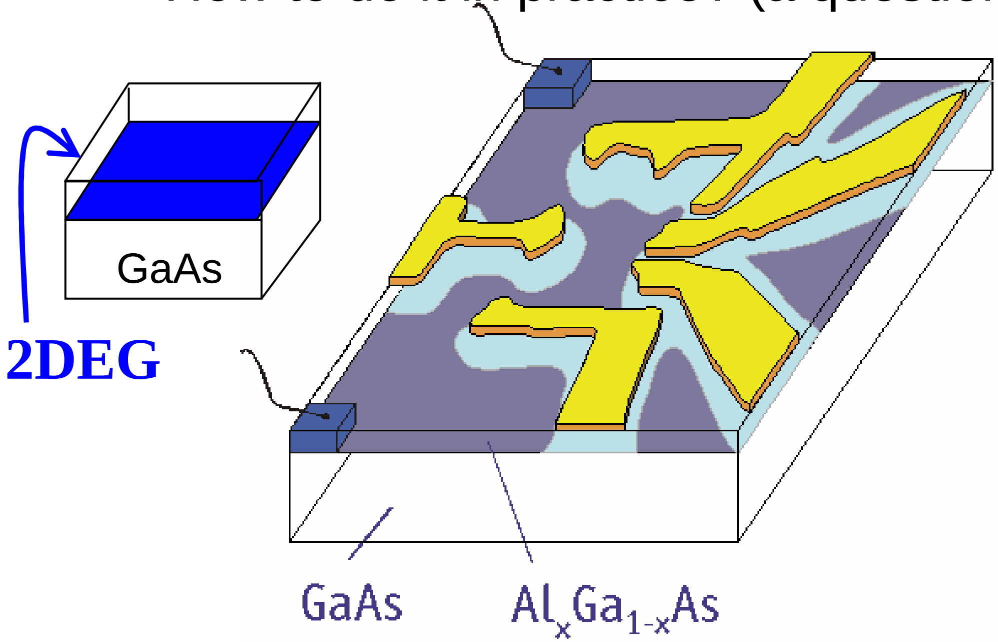


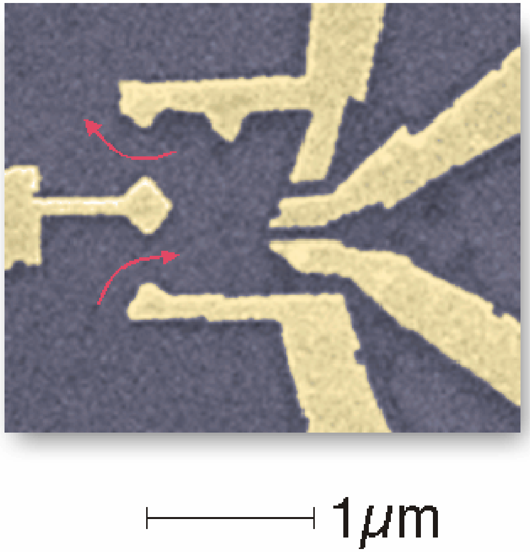
Semiconductor Quantum Dots:

- Devices in which electrons are **confined** in nanometer size volumes.
- Sometimes referred to as “artificial atoms”.
- “Quantum dot” is a generic label: lithographic QDs, self-assembled QDs, colloidal QDs have different properties.

Lithographic Quantum Dots

How to do it in practice? (a question for the experimentalists…)



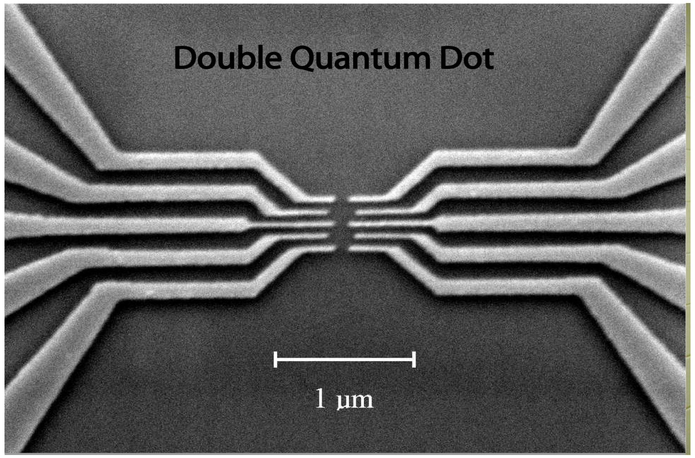


from Charlie Marcus' Lab website (marcuslab.harvard.edu)

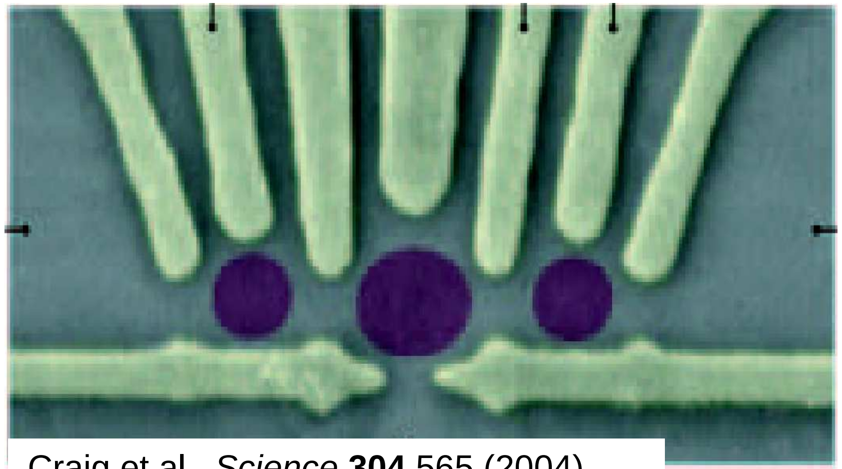
Ingredients:

- Growth of heterostructures to obtain the 2DEG
  - (good quality, large mean free-paths)
- Metallic electrodes electrostatically deplete charge: confinement
- Sets of electrodes to apply bias etc.
- LOW TEMPERATURE! (~100 mK)

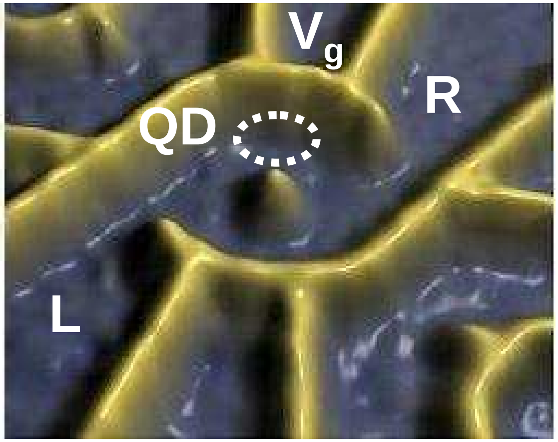
Lithographic Quantum Dots



Jeong, Chang, Melloch *Science* **293** 2222 (2001)



Craig et al., *Science* **304** 565 (2004)



From:
K. Ensslin's group website

Lithography evolved quite a bit in the last decade or so. Allow different patterns: double dots, rings, etc.

Quantum Dots: transport

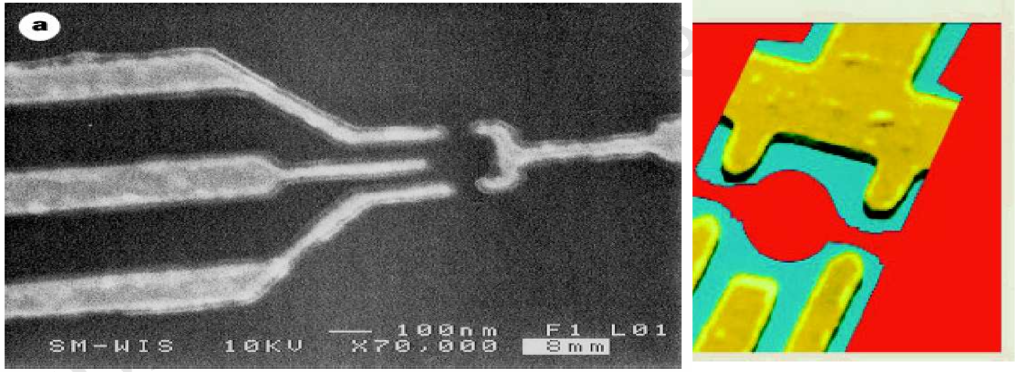
Lithographic Quantum Dots:

- Behave like small capacitors:

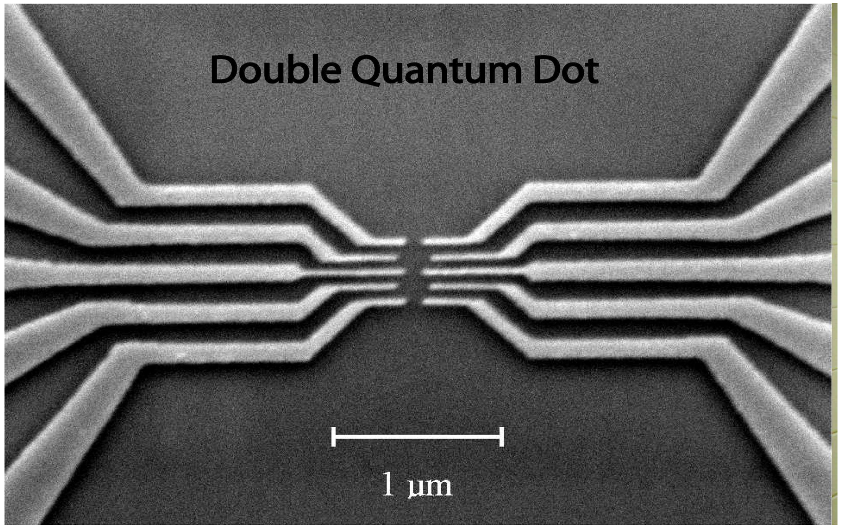
$$\boxed{E_c = \frac{e^2}{C}}$$

- Weakly connected to metallic leads.
- Energy scales: level spacing ΔE; level-broadening Γ.
- E_C is usually largest energy scale:

$$\boxed{E_c \gg \Delta E, \Gamma}$$

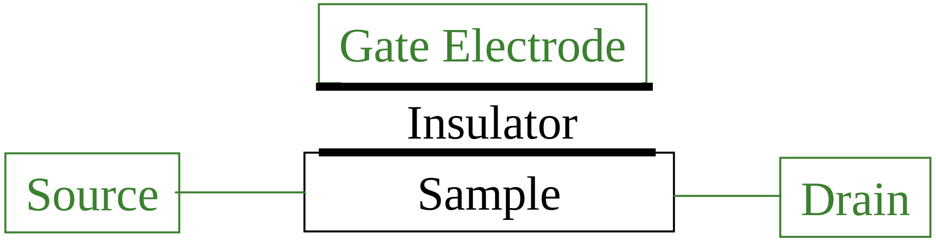


Goldhaber-Gordon *et al. Nature* **391** 156 (1998)



Jeong, Chang, Melloch *Science* **293** 2222 (2001)

Electrical Transport

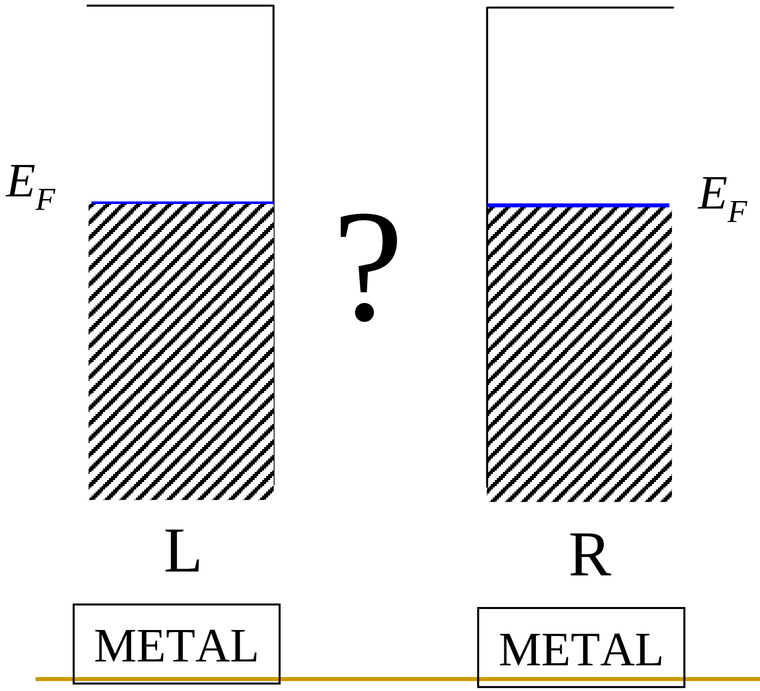

Gate Electrode

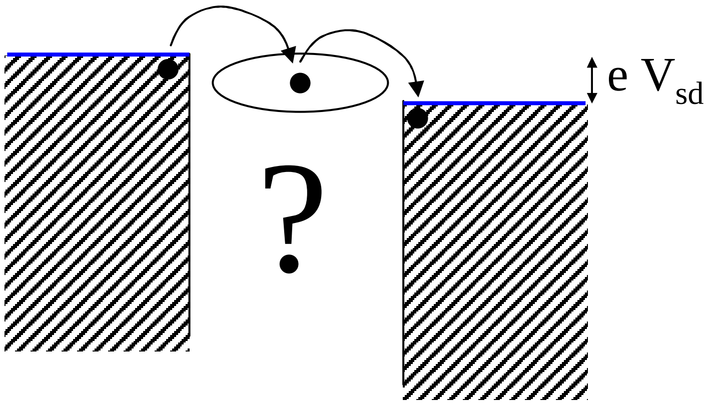
Insulator

Source

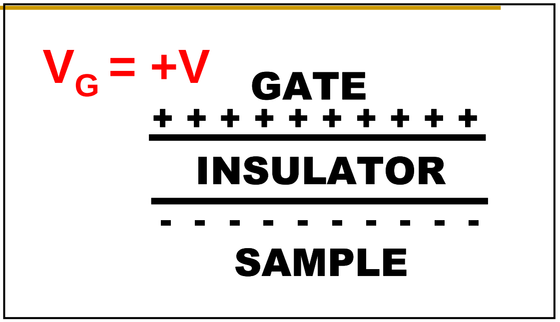
Sample

Drain





Role of the Gate Electrode



$V_G = +V$

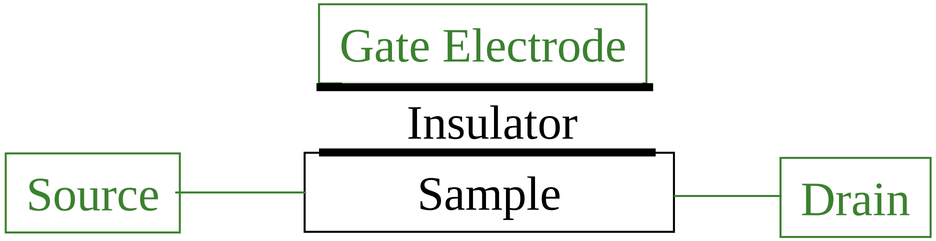
GATE

+ + + + + + + + + +

INSULATOR

- - - - - - - - - -

SAMPLE



Gate Electrode

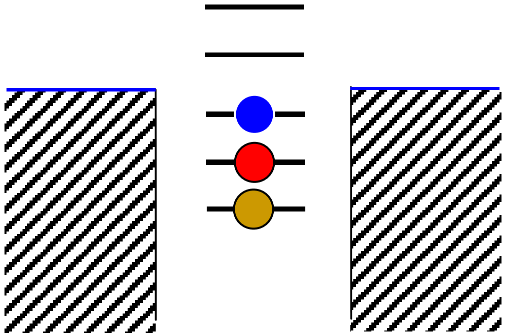
Insulator

Source

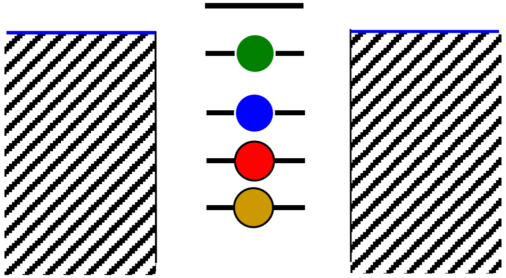
Sample

Drain

$V_G = 0$

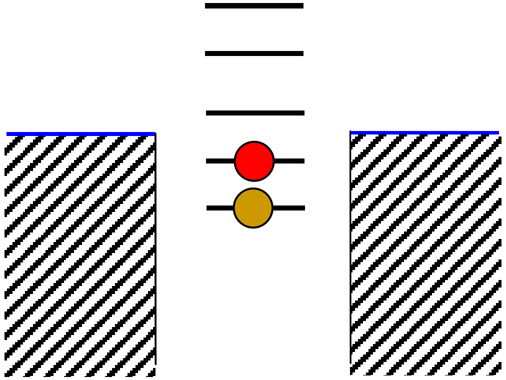


$V_G = +V$



Raise Fermi level – adds electrons

$V_G = -V$



Lower Fermi level – remove electrons

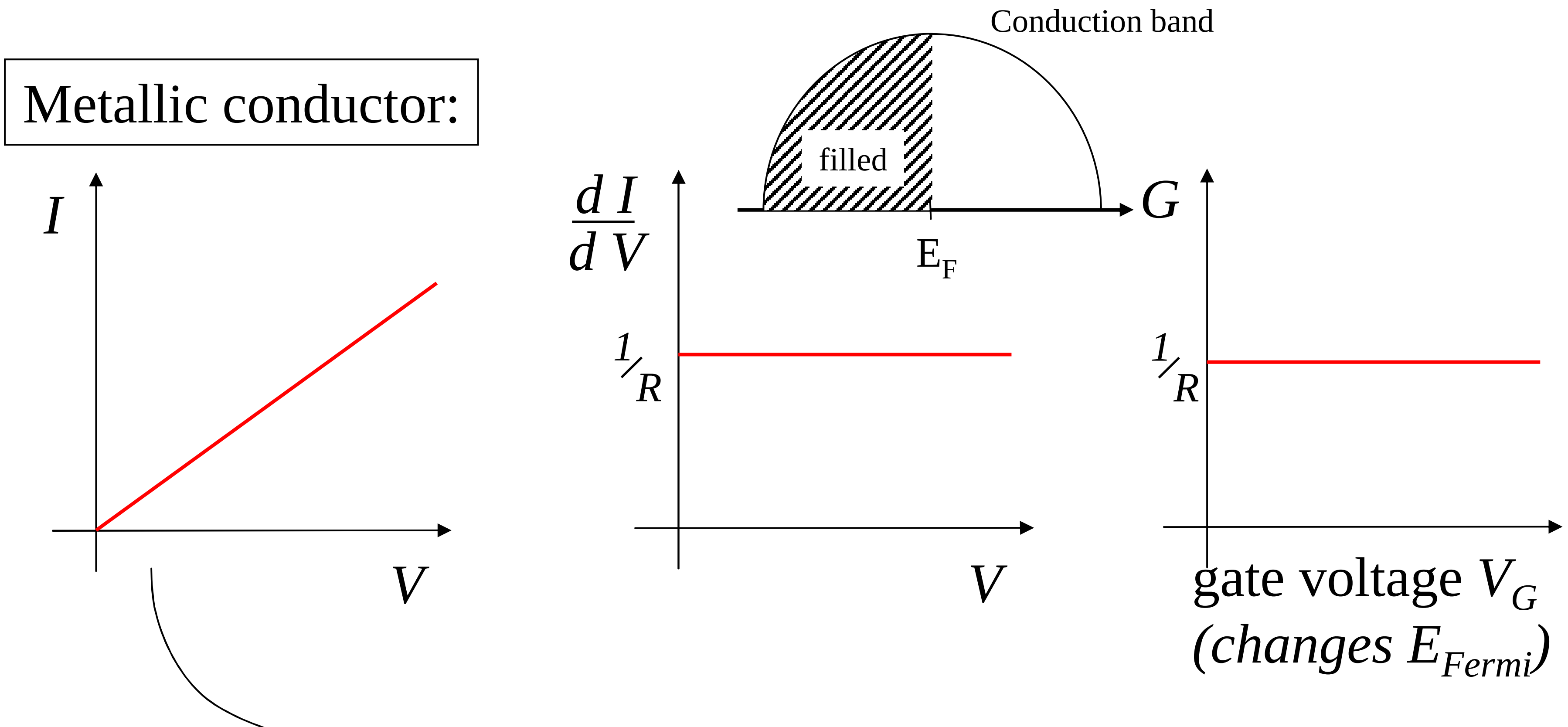
Electrical Transport: Ohm's Law

Ohm's Law holds for metallic conductors => V = I R

We can also define a conductance which can be bias dependent $\frac{dI}{dV}$

The **zero bias conductance**, *G*, is conventionally quoted.

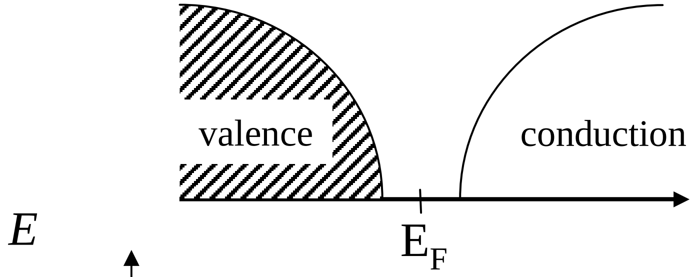
Metallic conductor:

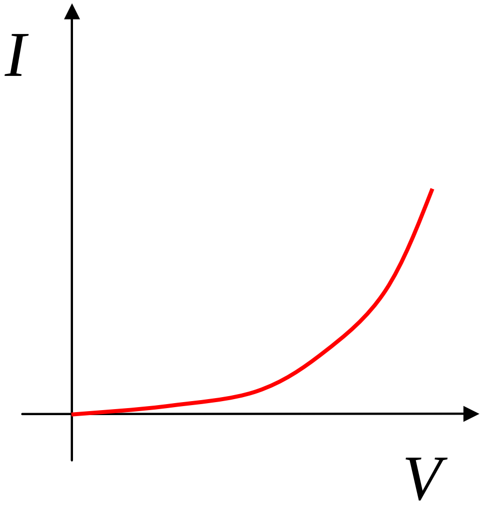


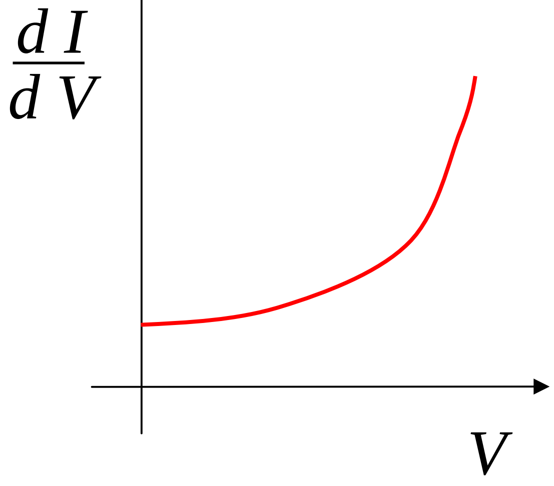
Resistance due to scattering off impurities, mfp ~ 10 nm

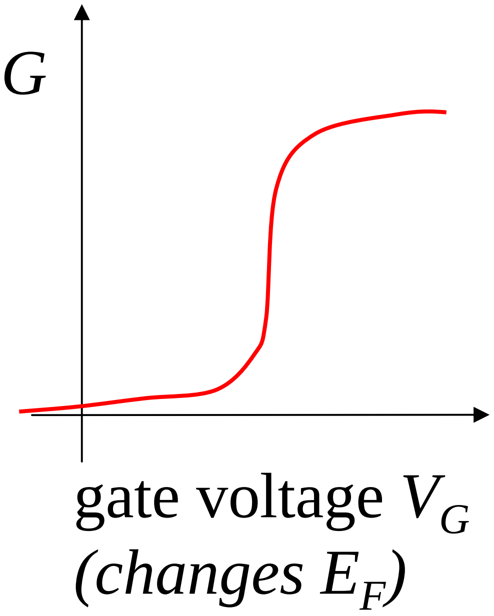
Electrical Transport: semiconductors

Semiconductor:









Semiconductor - nonlinear I - V response

→ tunneling through Schottky barrier
or out of band gap.

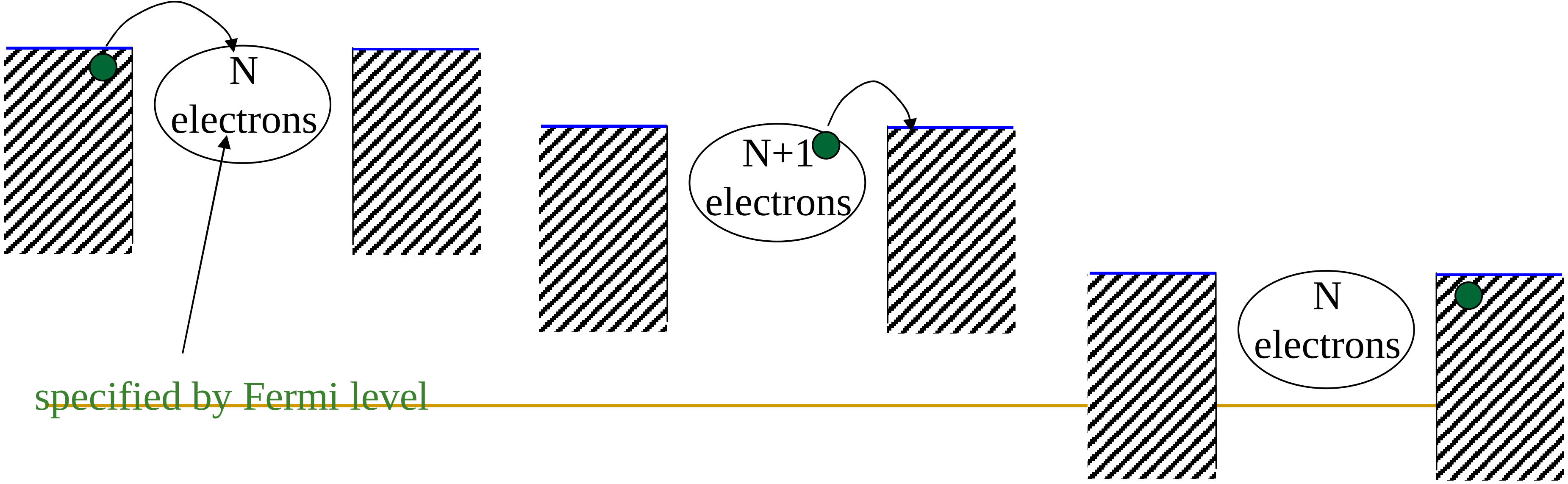
Conductance through quantum dot

Quantum dots contain an integer number of electrons.

Adding an electron to the QD changes its energy => electrostatic charging energy $\frac{Q^2}{2C}$

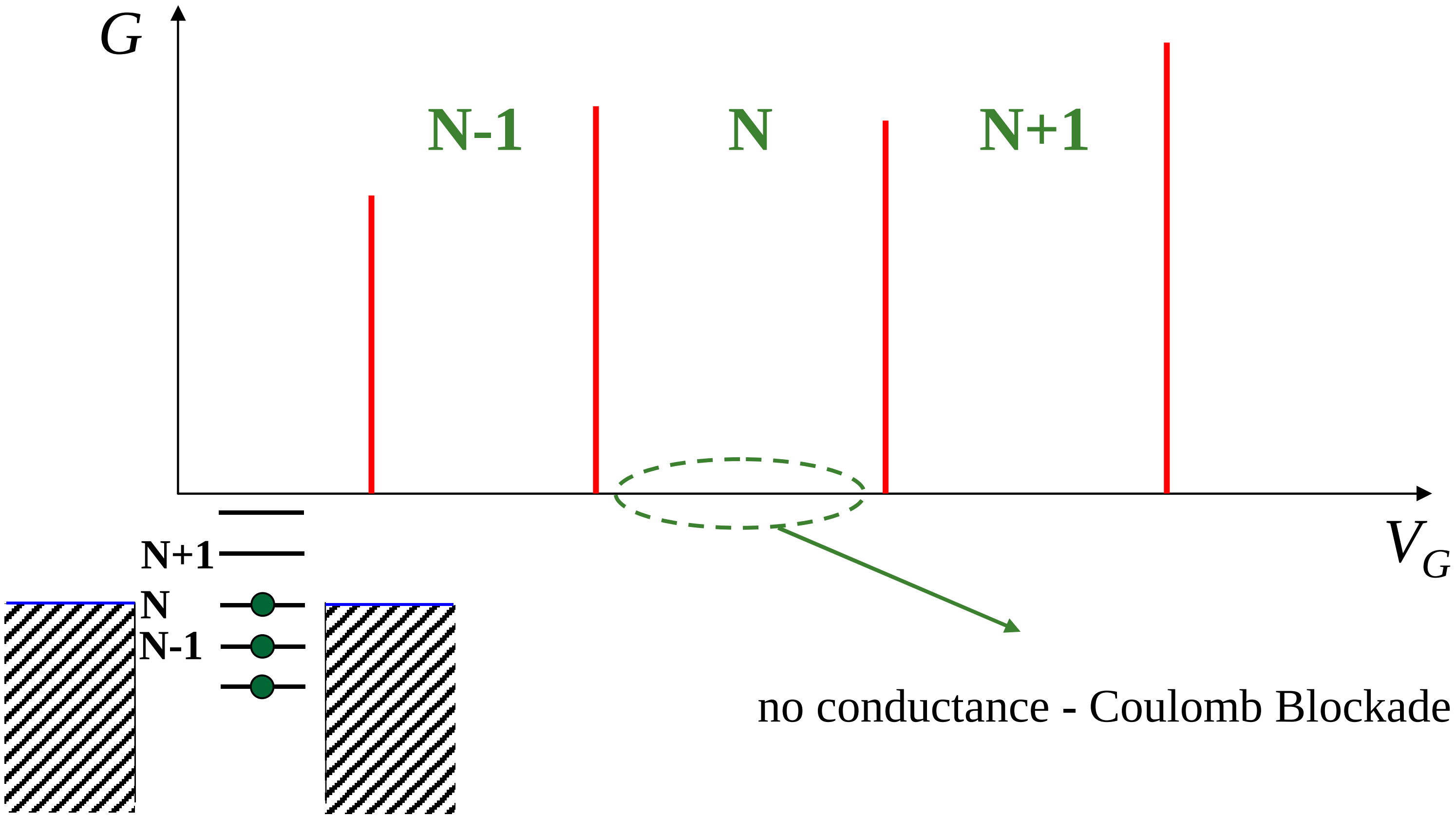
In order for a current to pass an electron must tunnel onto the dot, and an electron must tunnel off the dot.

For **conduction at zero bias** this requires the energy of the dot with *N* electrons must equal the energy with *N+1* electrons. i.e. charging energy balanced by gate potential.



Coulomb blockade

The addition of an electron to the dot is blocked by the charging energy as well as the level spacing => **Coulomb blockade**

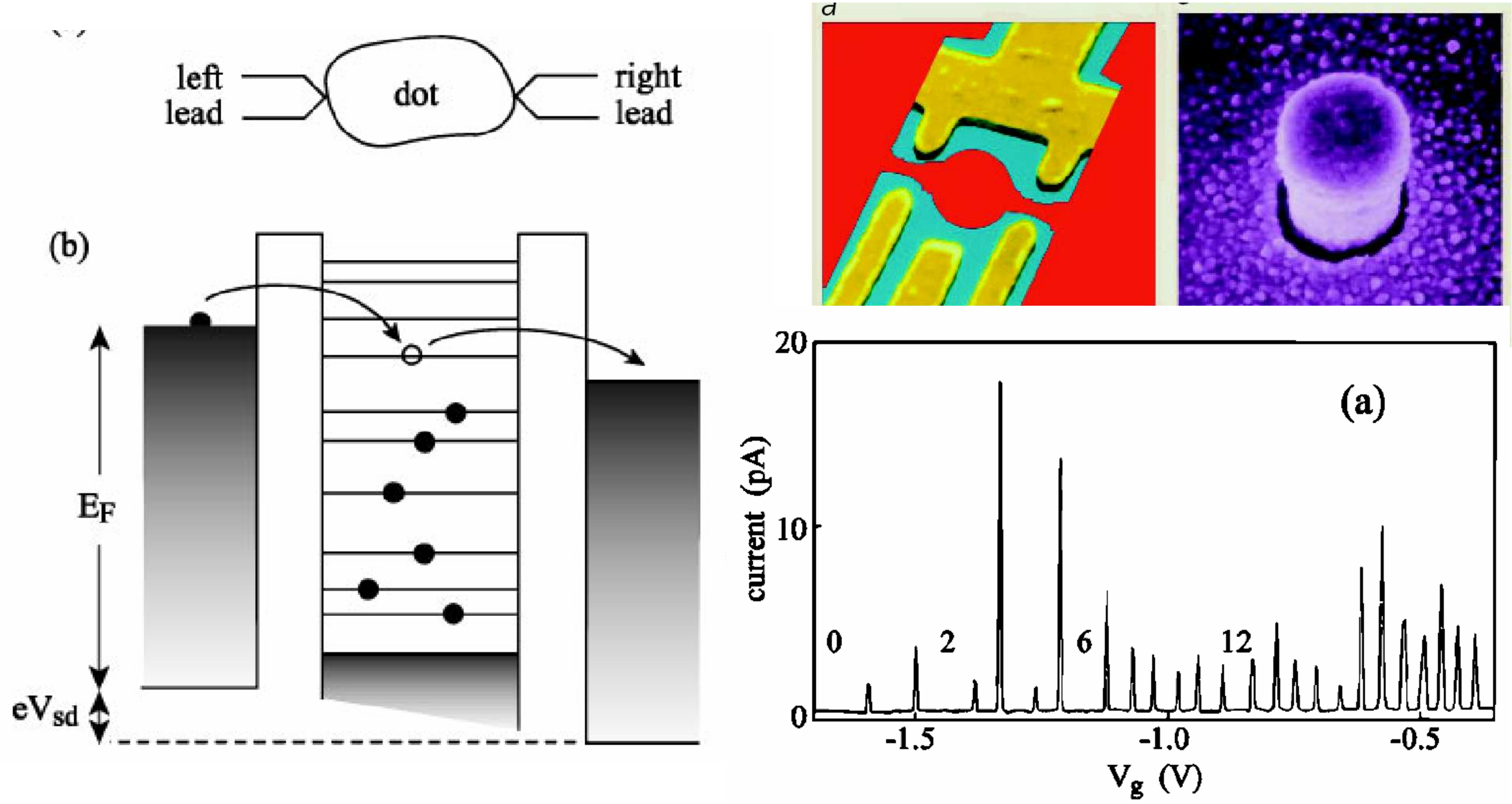


no conductance - Coulomb Blockade

The number of electrons on the QD is adjusted by the gate potential.

Conduction only occurs when $E_N = E_{N+1}$

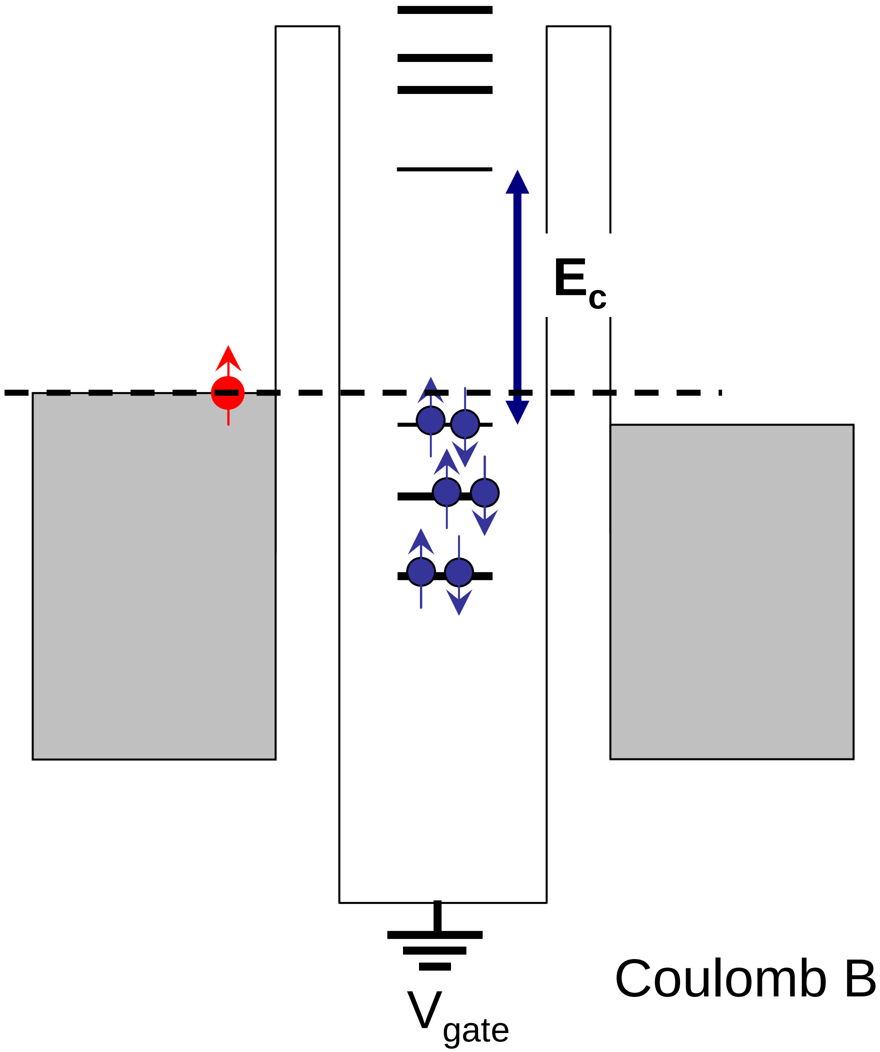
Coulomb Blockade in Quantum Dots

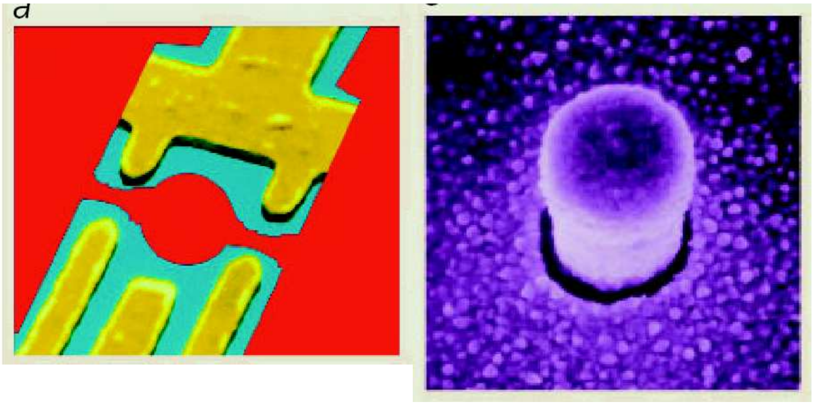


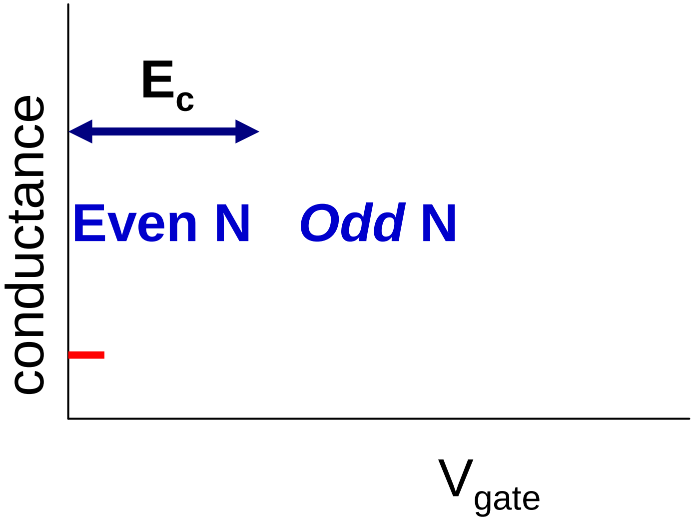
Coulomb Blockade in Quantum Dots

Y. Alhassid *Rev. Mod. Phys.* **72** 895 (2000).

Coulomb Blockade in Quantum Dots

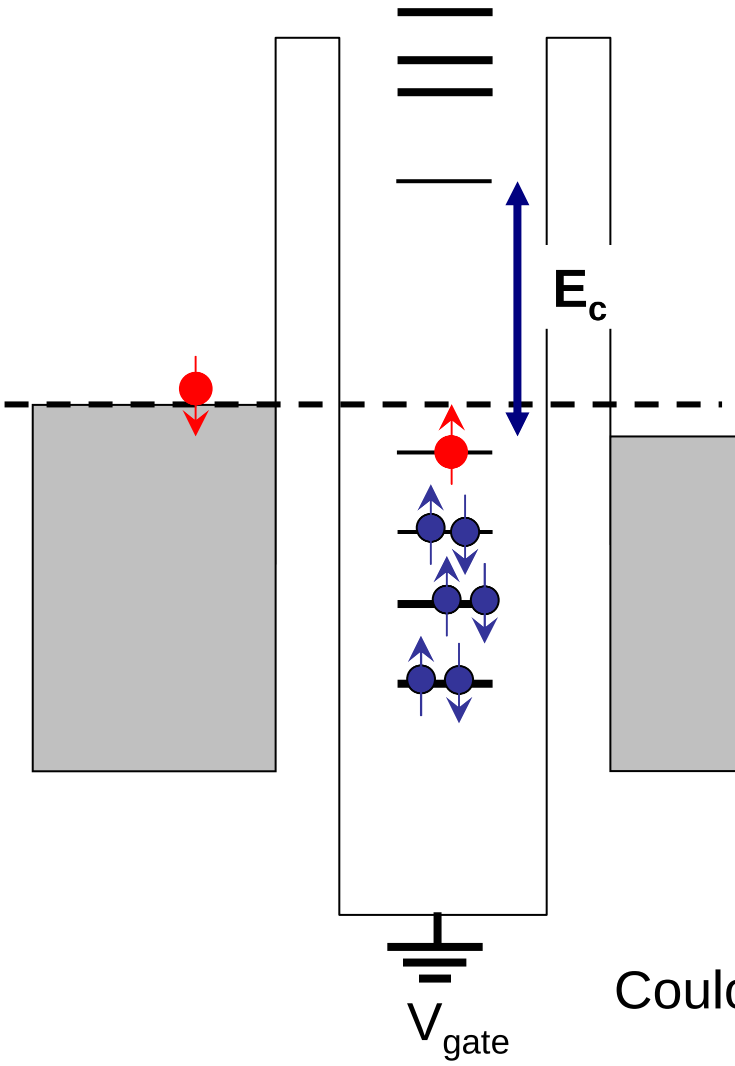


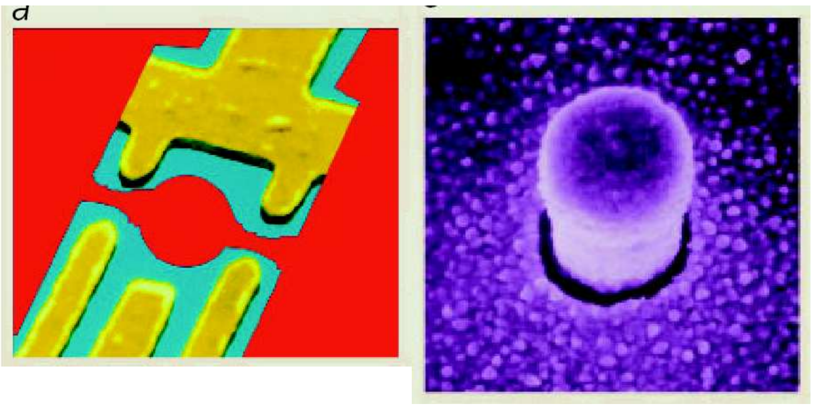


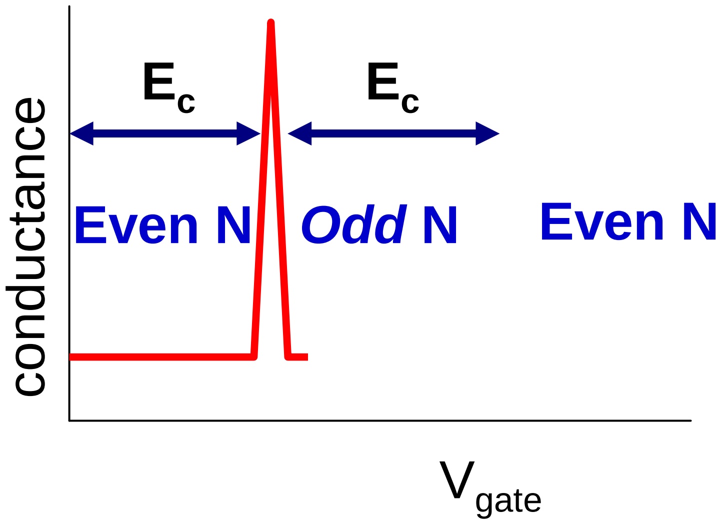


Coulomb Blockade in Quantum Dots

Coulomb Blockade in Quantum Dots

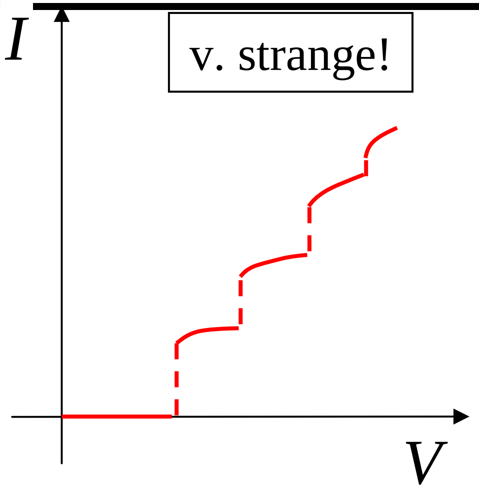




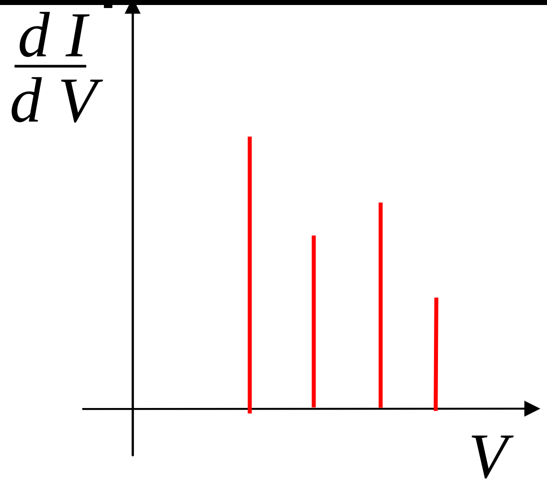


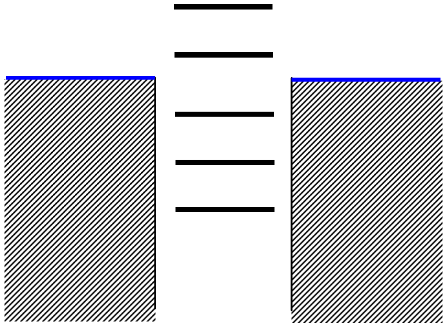
Coulomb Blockade in Quantum Dots

Electrical Transport: Coulomb staircase

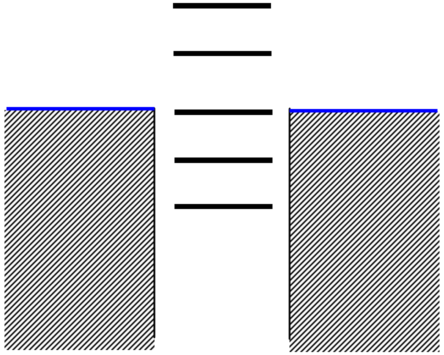


v. strange!

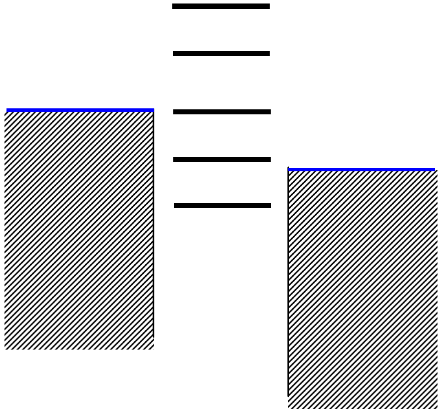




no conductance



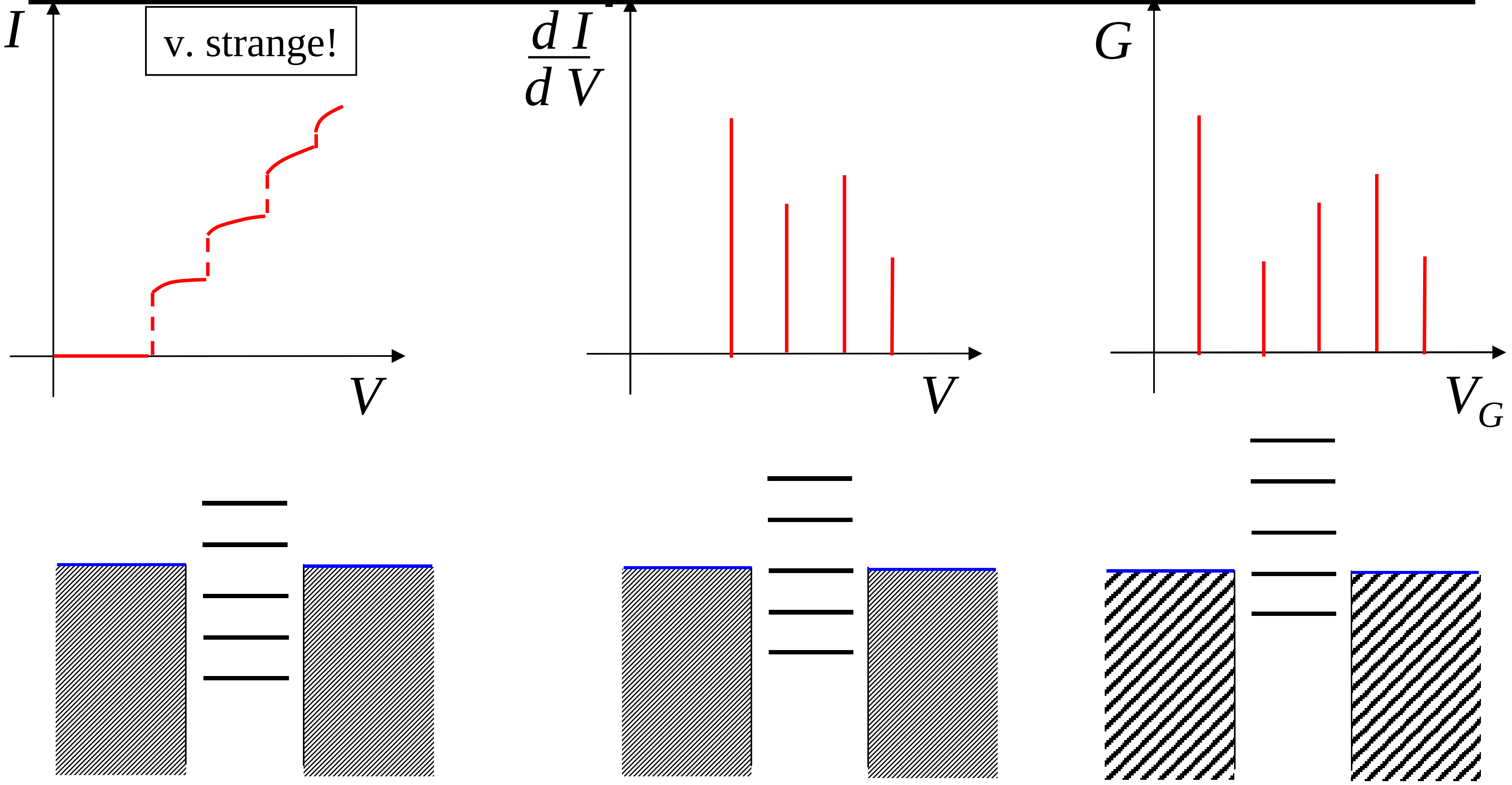
conductance



increased conductance

Probing the energy levels in the ‘artificial atom’

Electrical Transport: Coulomb staircase



v. strange!

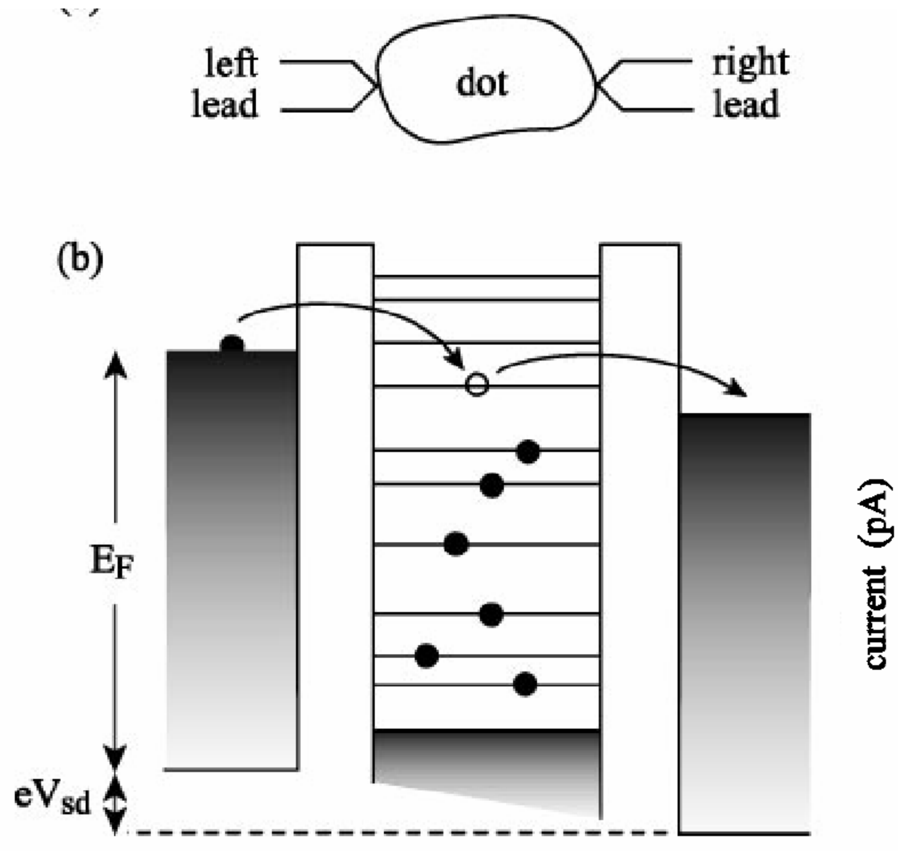
no conductance

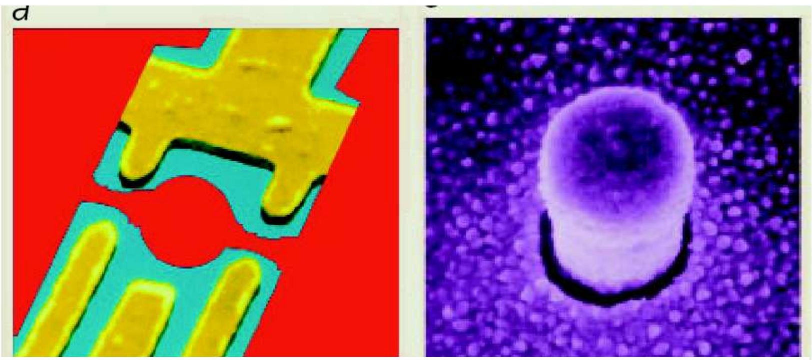
conductance

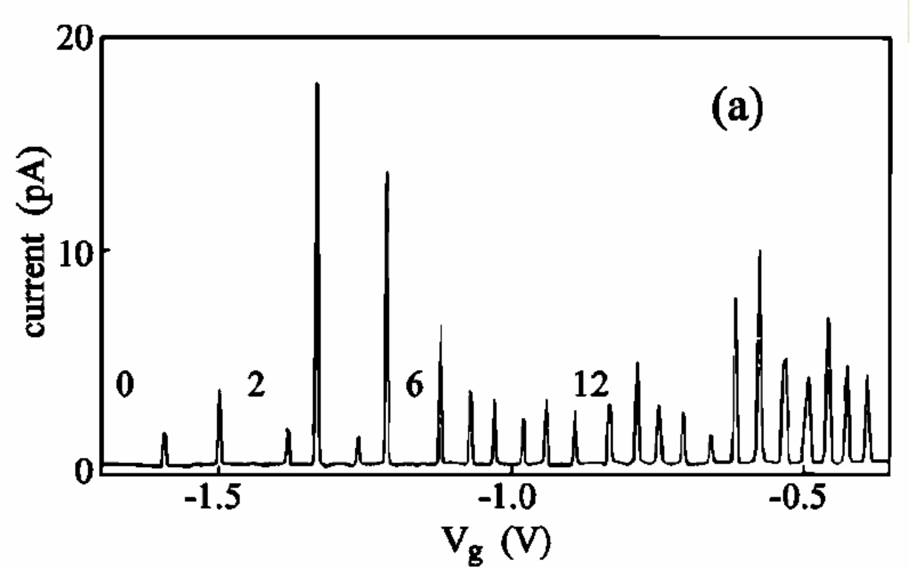
conductance

Probing the energy levels in the ‘artificial atom’

Coulomb Blockade in Quantum Dots



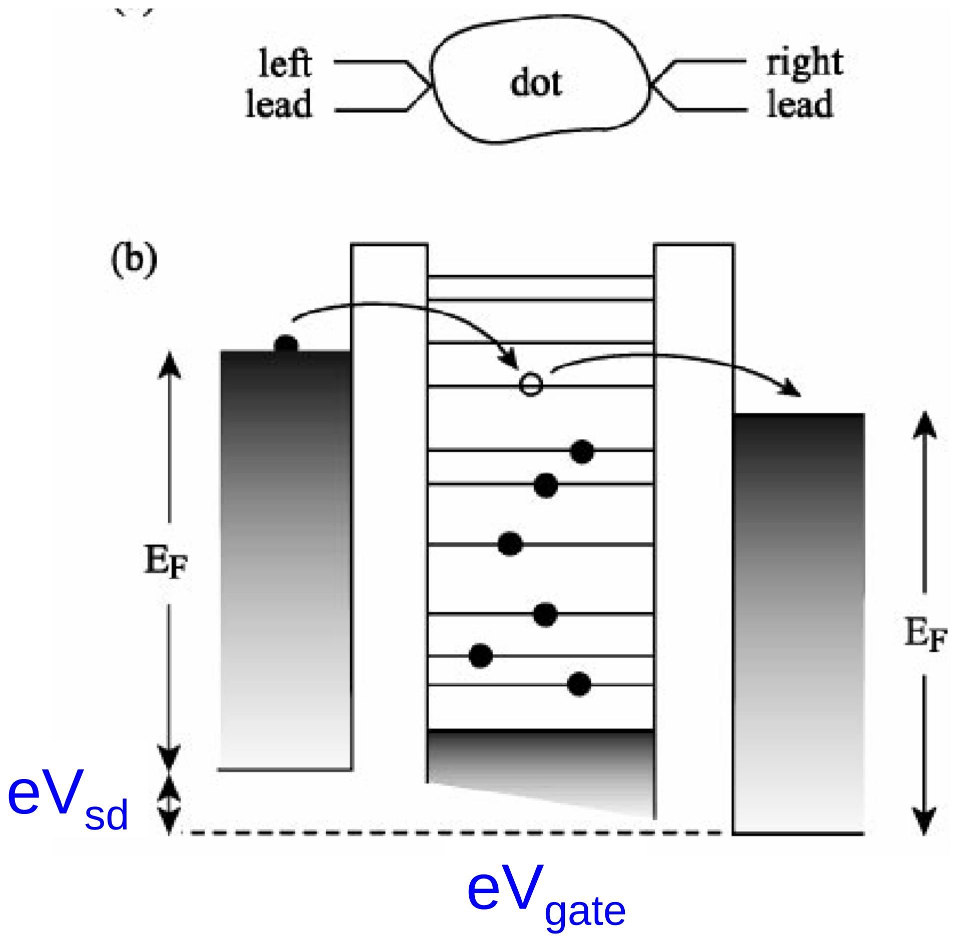




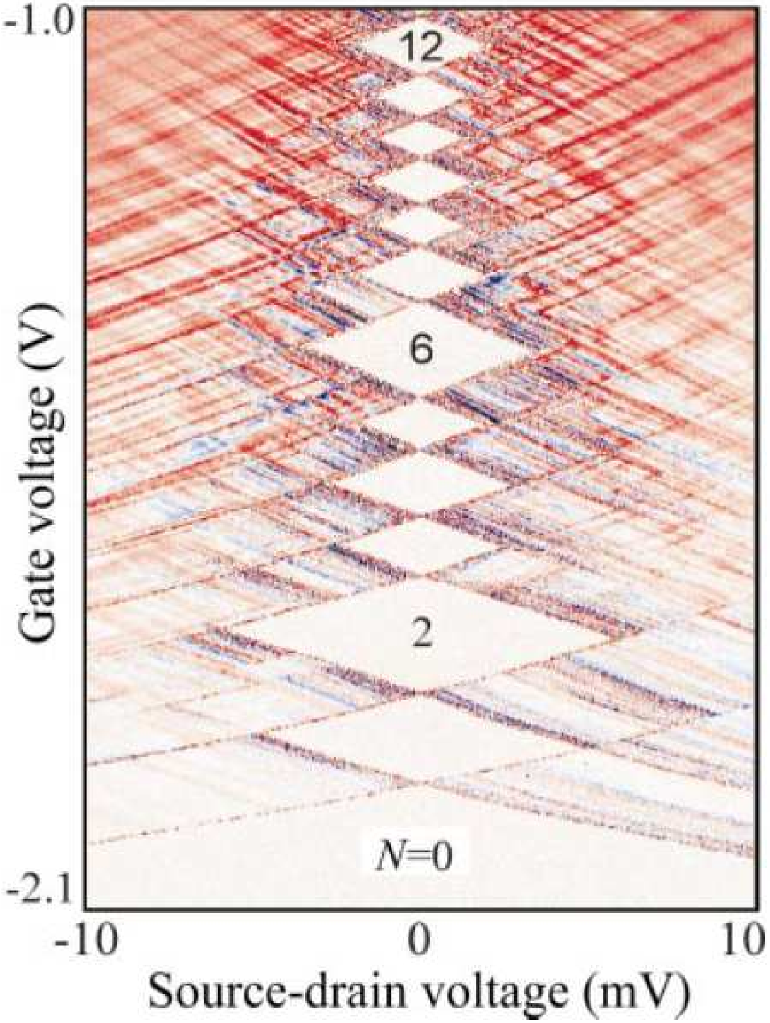
Coulomb Blockade in Quantum Dots: “dot spectroscopy”

Y. Alhassid *Rev. Mod. Phys.* **72** 895 (2000).

"Coulomb Diamonds" (Stability Diagram)

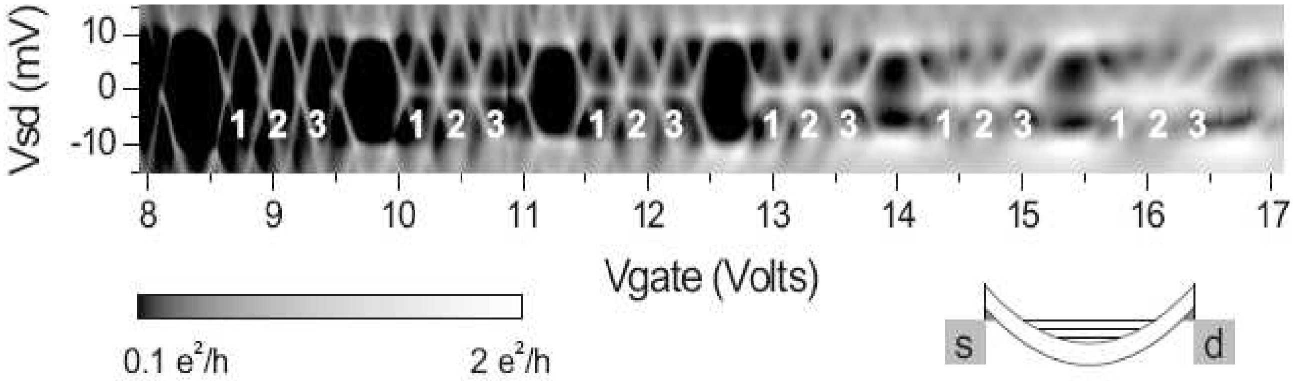


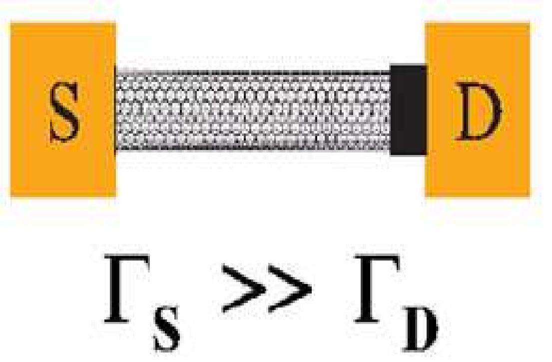
Coulomb Blockade in Quantum Dots



L. P. Kouwenhoven et al. *Science* **278** 1788 (1996).

“Carbon nanotube Quantum dots”.

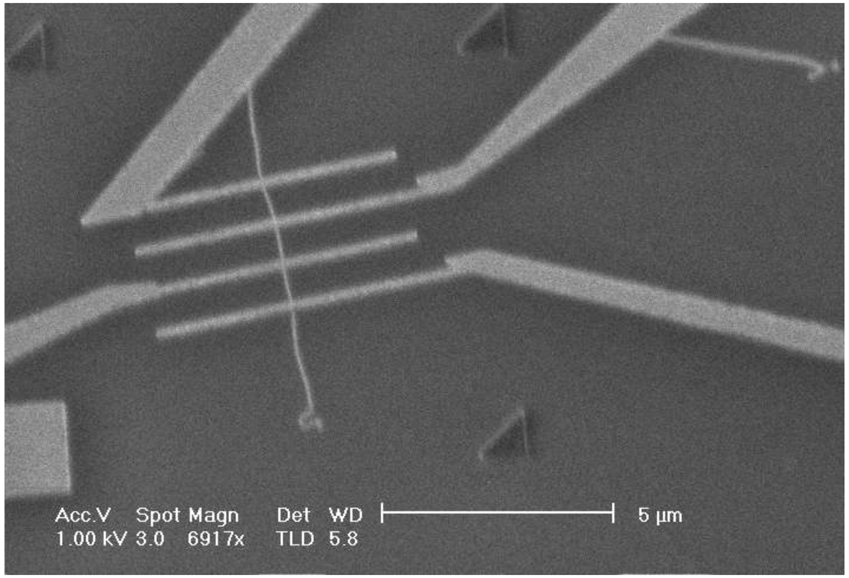




$\Gamma_S >> \Gamma_D$

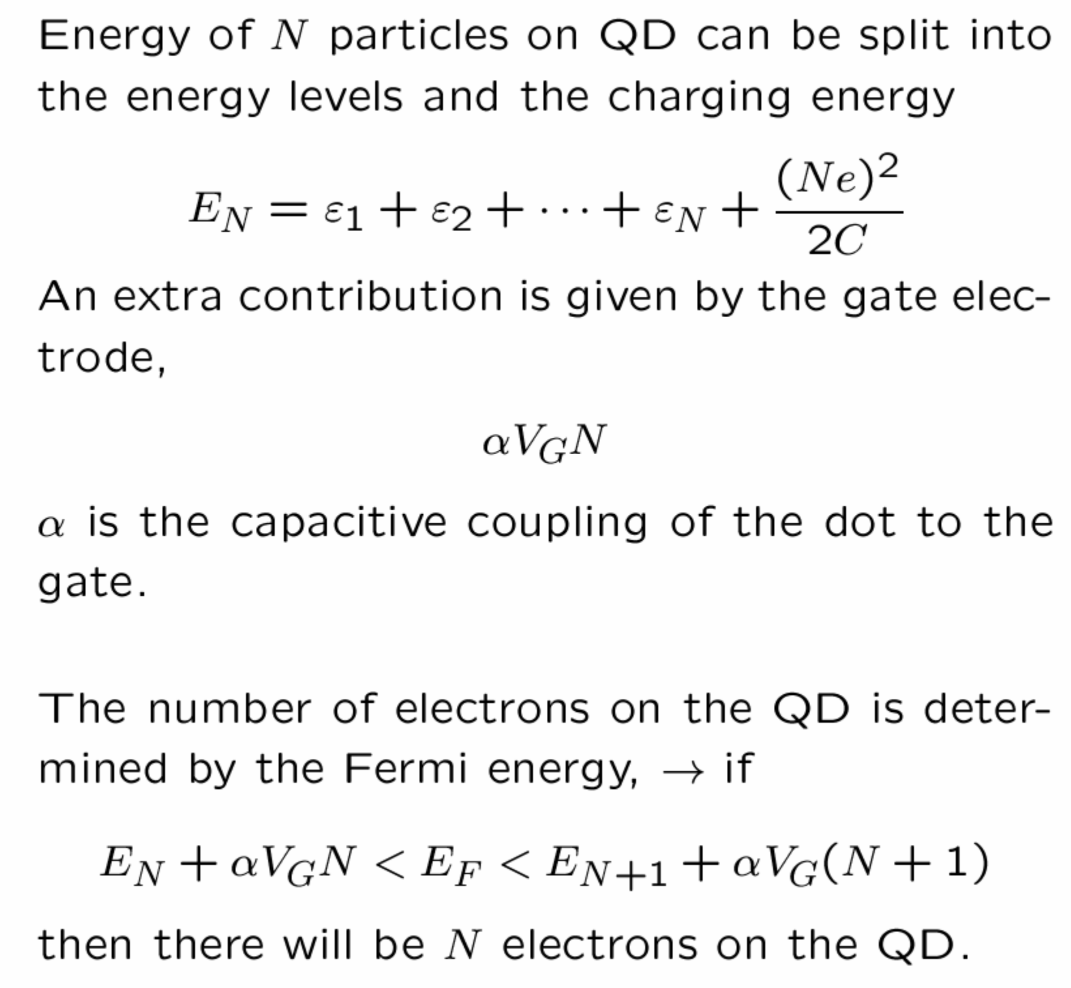
Makarovski, Zhukov, Liu, Filkenstein *PRB* **75** 241407R (2007).

- Carbon nanotubes depsited on top of mettalic electrodes.
- Quantum dots defined *within* the carbon nanotubes.
- More structure than in quantum dots: “shell structure” due to *orbital* degeneracy.



Gleb Filkenstein’s webpage: http://www.phy.duke.edu/~gleb/

Charging Energy Model



Energy of N particles on QD can be split into the energy levels and the charging energy

$$E_N = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_N + \frac{(Ne)^2}{2C}$$

An extra contribution is given by the gate electrode,

$$\alpha V_G N$$

α is the capacitive coupling of the dot to the gate.

The number of electrons on the QD is determined by the Fermi energy, → if

$$E_N + \alpha V_G N < E_F < E_{N+1} + \alpha V_G (N+1)$$

then there will be N electrons on the QD.

The no. of electrons on the QD is adjusted by the gate potential.

Conductance peak spacing

The energy of $N+1$ electrons is

$$E_{N+1} = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_{N+1} + \frac{(N+1)^2 e^2}{2C}$$

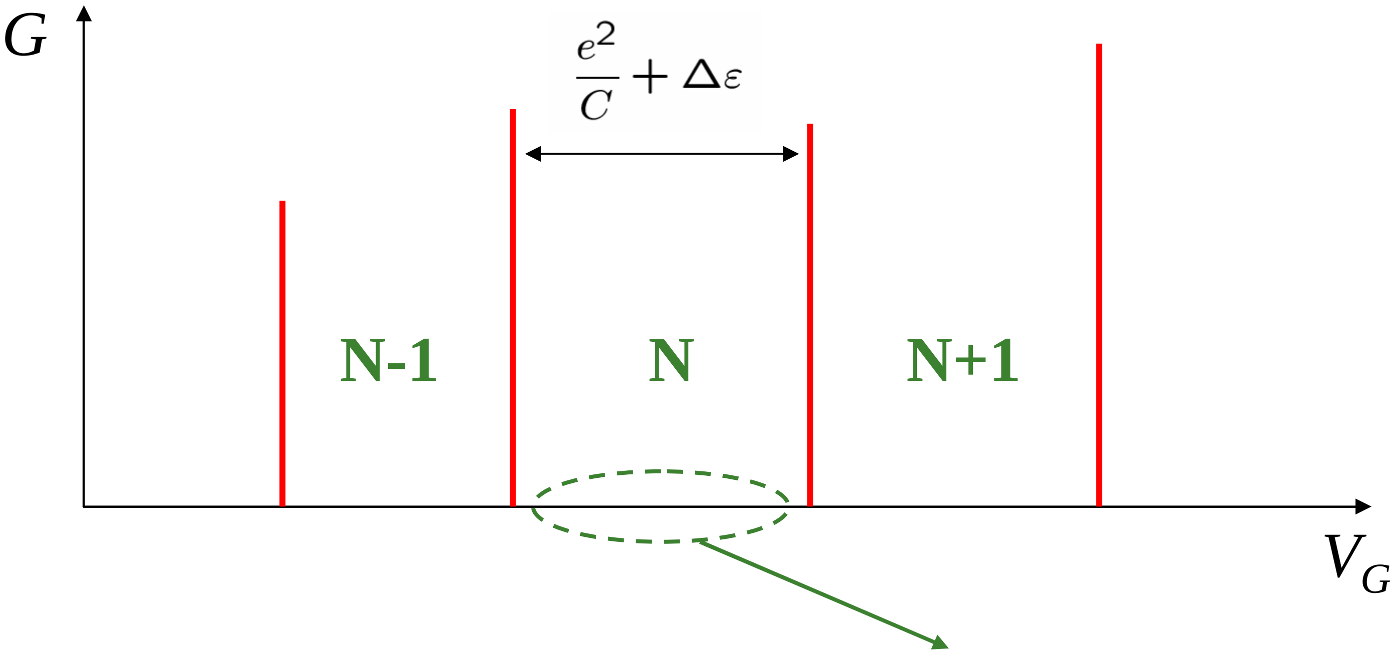
The difference in energy between N and $N+1$ electrons is

$$\Delta E_{N\to N+1} = E_{N+1} - E_N = \varepsilon_{N+1} + (2N+1)\frac{e^2}{2C}$$

The separation between conductance peaks in this model is given by

$$\alpha \Delta V_G = \Delta E_{N\to N+1} - \Delta E_{N-1\to N} = \underbrace{\varepsilon_{N+1} - \varepsilon_N}_{\text{level spacing } \Delta\varepsilon} + \underbrace{\frac{e^2}{C}}_{\text{charging energy}}$$

Conductance peak spacing II

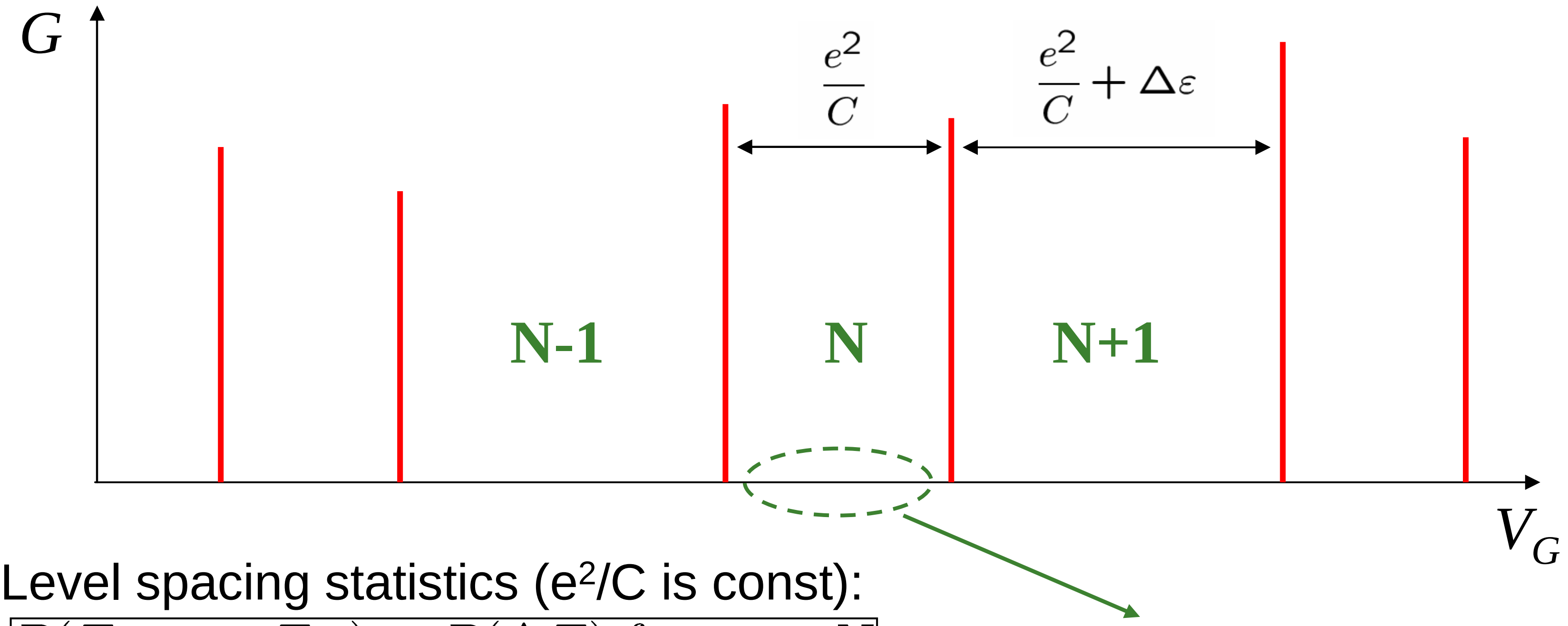


no conductance - Coulomb Blockade

Conductance peak spacing III: spin degeneracy

In the event of degeneragy, e.g. spin degeneracy

$$\varepsilon_N = \varepsilon_{N+1}$$

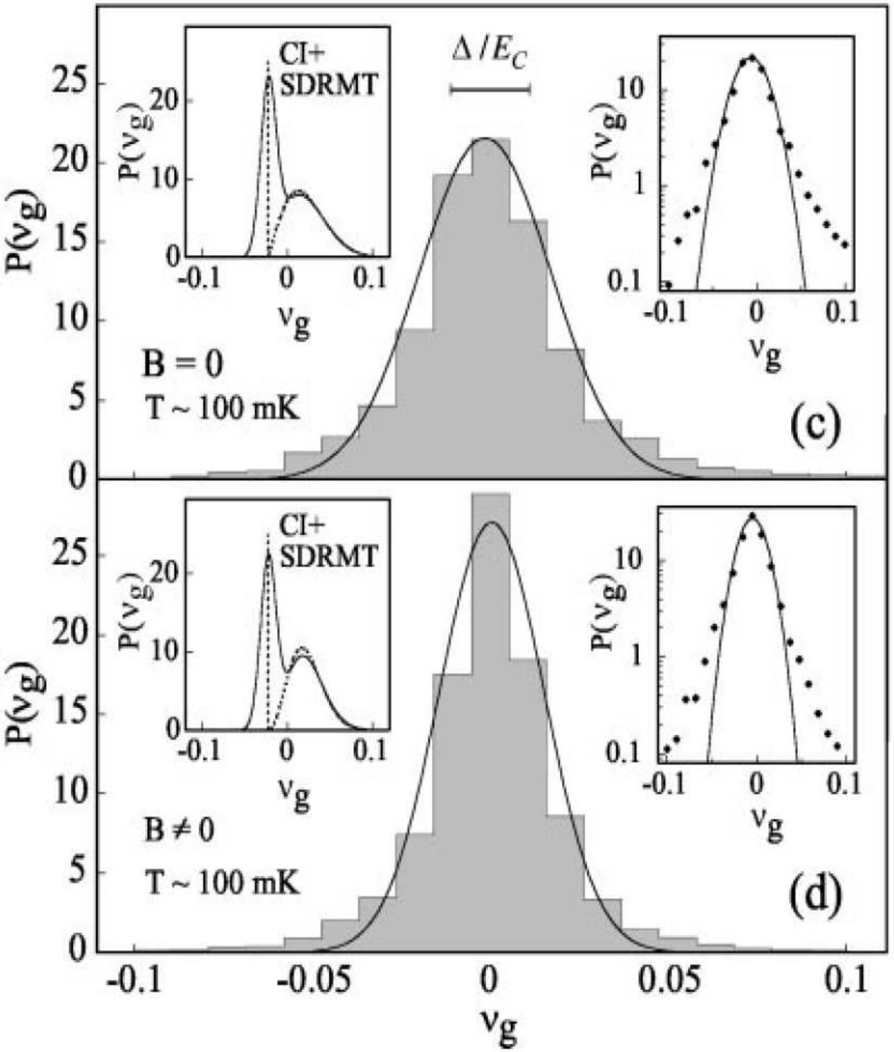


Level spacing statistics (e^2/C is const):

$$\boxed{\begin{array}{l} P(E_{N+1} - E_N) = P(\Delta E) \text{ for even } N \\ P(E_{N+1} - E_N) = \delta(E) \text{ for odd } N \end{array}}$$

no conductance - Coulomb Blockade

Peak spacing statistics: e-e interactions



Sivan et al. *PRL* **77** 1123 (1996).

$$P(E_{N+1}-E_N)=P(\Delta E)\text{ for even }N$$
$$P(E_{N+1}-E_N)=\delta(E)\text{ for odd }N$$

Level spacing distribution P(ΔE):
Theory: CI+RMT prediction
Superposition of two distributions:
- even N: large, chaotic dots:
P(ΔE) obeys a well known
RMT distribution (*not* Gaussian)
- odd N: Dirac delta function.

Experiment:
P(ΔE) gives a Gaussian.

Possible explanations:
Spin exchange, residual e-e interaction effects*.

* Jiang, Baranger, Yang *PRL* **90** 026806 (2003).