Lectures: Condensed Matter II

1 – Electronic Transport in Quantum dots

2 – Kondo effect: Intro/theory.

3 – Kondo effect in nanostructures

Luis Dias – UT/ORNL

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➡ 3 – Kondo effect in nanostructures

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Lecture 3: Outline

- Quantum Dots: brief review.
- Kondo effect: Review.
- Kondo effect in quantum dots.
- Kondo effect in Single Molecule Transistors.
- Kondo effect in Surfaces (STM, “quantum mirage”).
- Kondo effect in carbon nanotubes.

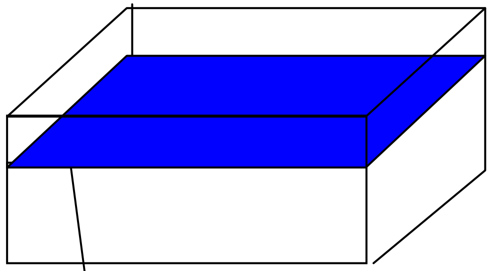
“More is Different”

“ The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of simple extrapolation of the properties of a few particles.

Instead, at each level of complexity entirely new properties appear and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other.“

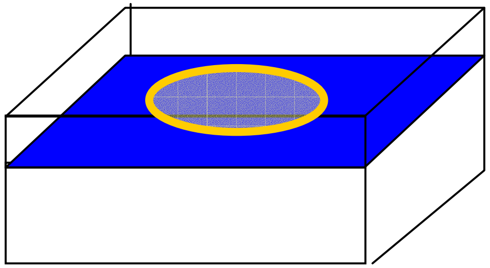
Phillip W. Anderson, “More is Different”,
Science **177** 393 (1972)

Can you make “atoms” out of atoms?

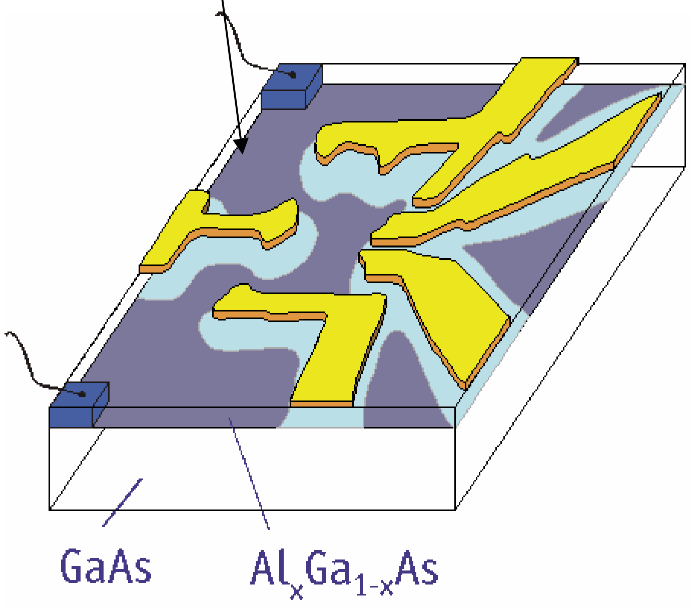


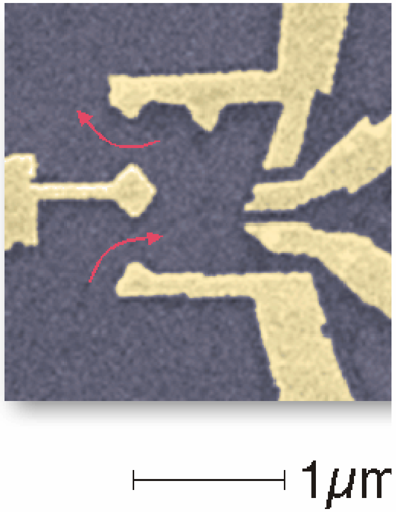
2D Electron gas

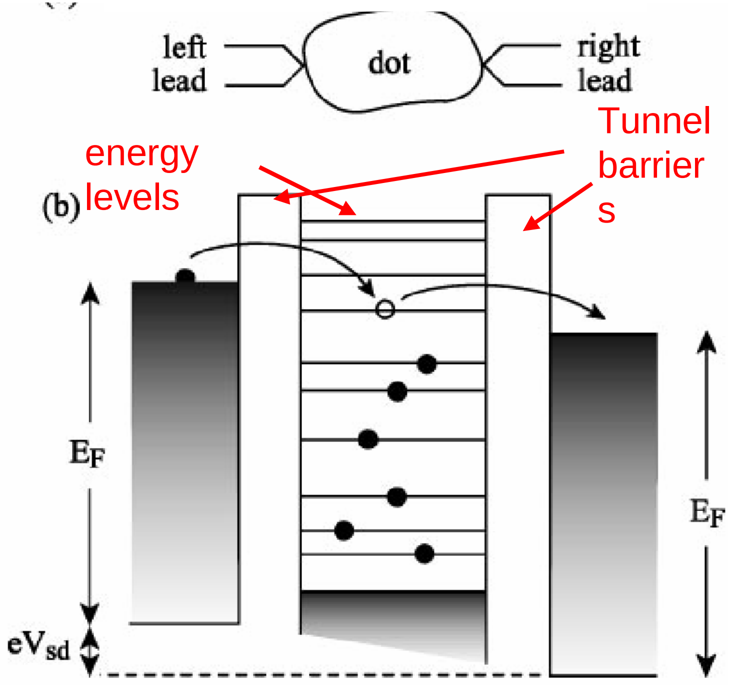
Electrostatically confine electrons within an small (nanometer-size) region.



“Quantum dot”

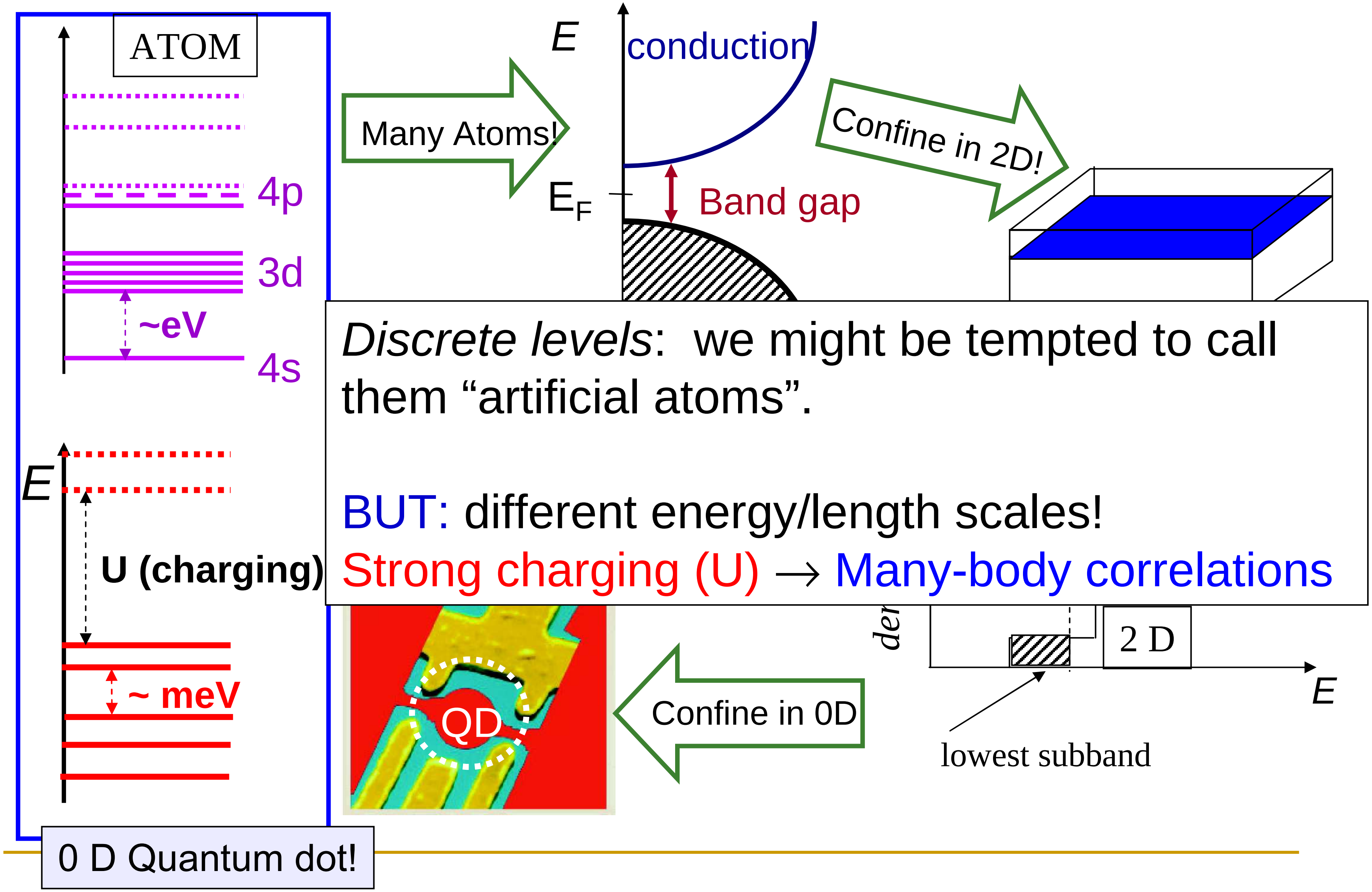






from Charlie Marcus’ Lab website (marcuslab.harvard.edu)

Making “artificial atoms”(?) out of atoms



Discrete levels: we might be tempted to call them “artificial atoms”.

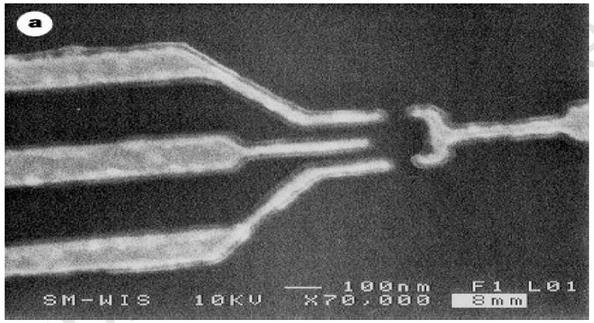
BUT: different energy/length scales!

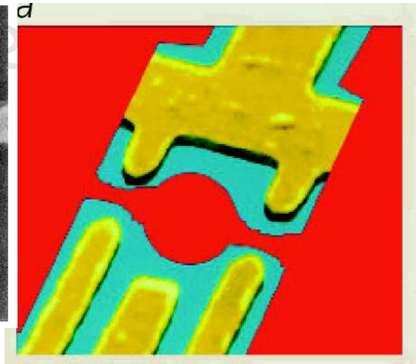
Strong charging (U) → Many-body correlations

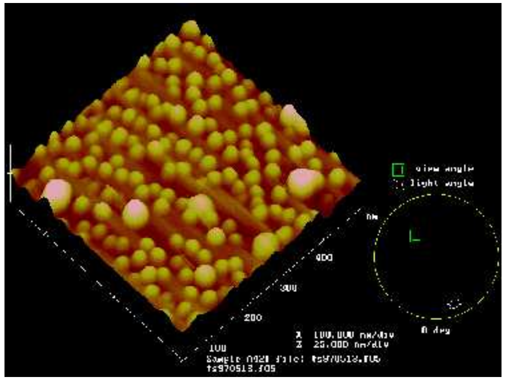
What are *Quantum Dots*?

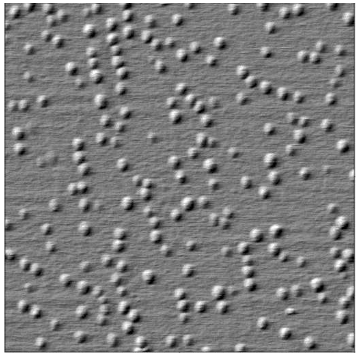
Semiconductor Quantum Dots:

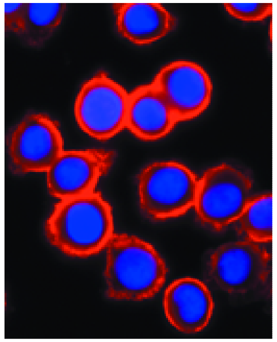
- Devices in which electrons are **confined** in nanometer size volumes.
- Sometimes referred to as “artificial atoms”.
- “Quantum dot” is a generic label: lithographic QDs, self-assembled QDs, colloidal QDs have different properties.



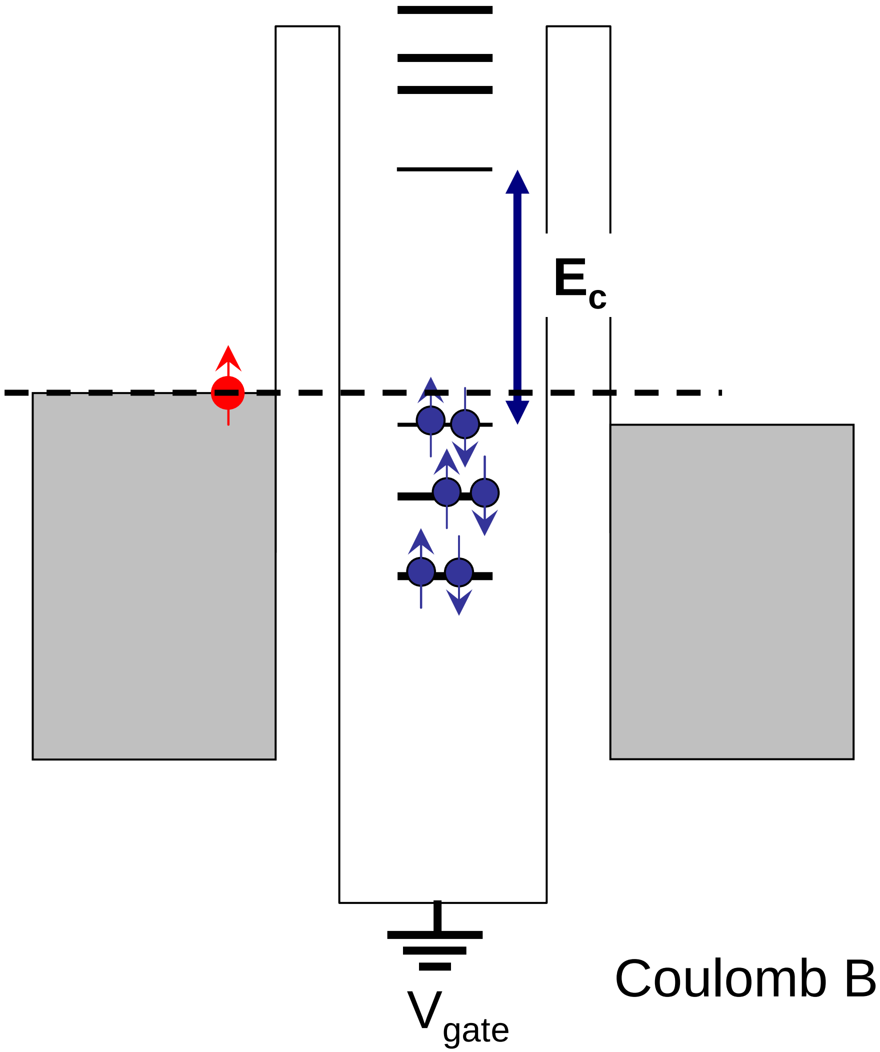


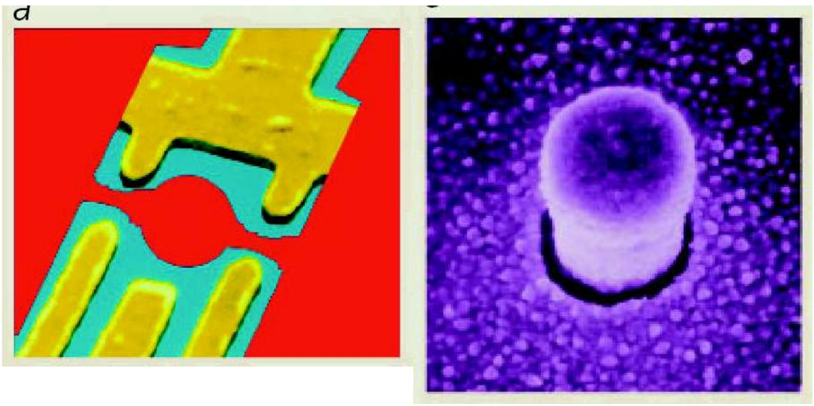


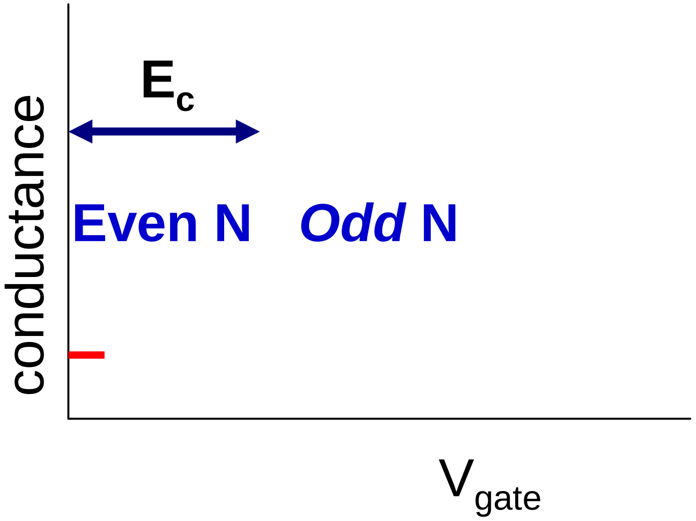




Coulomb Blockade in Quantum Dots

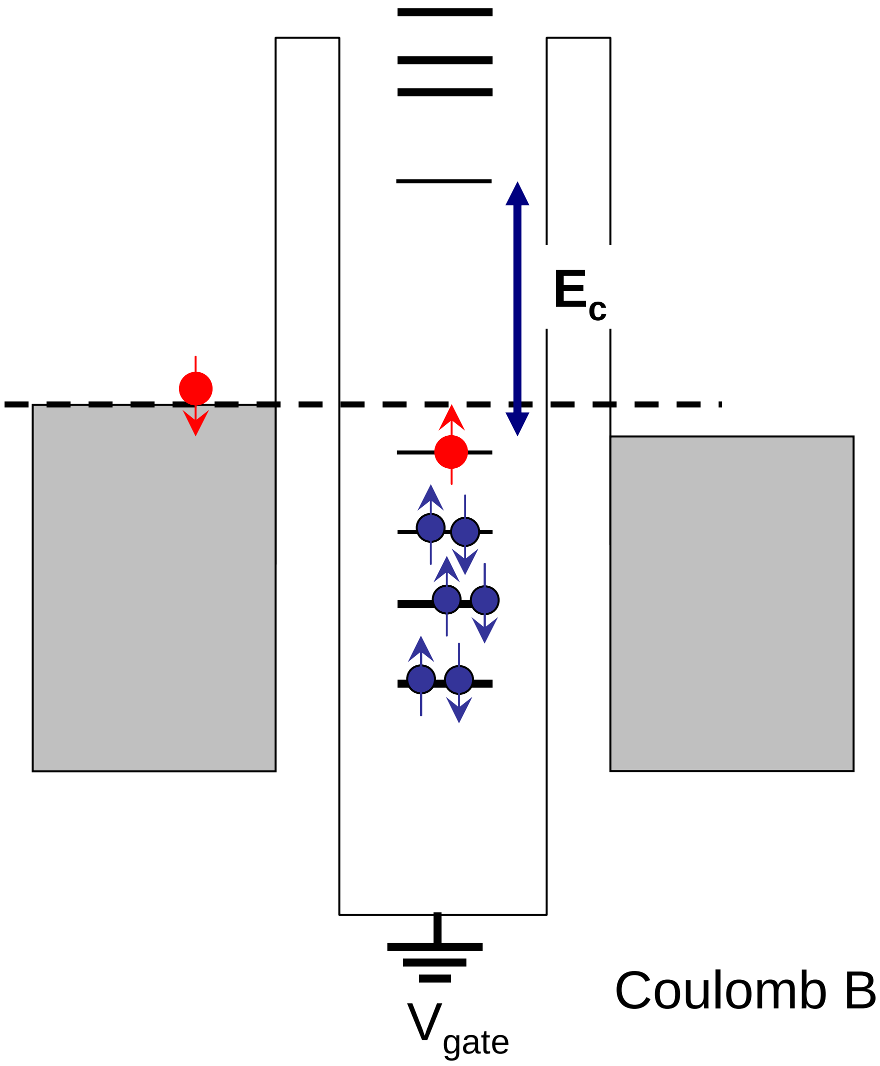


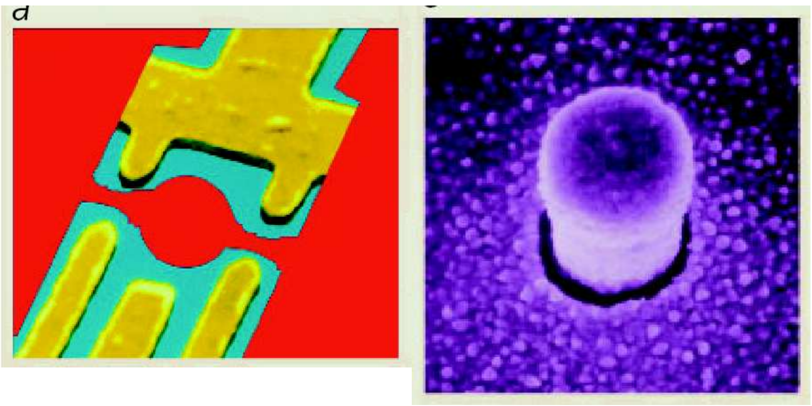


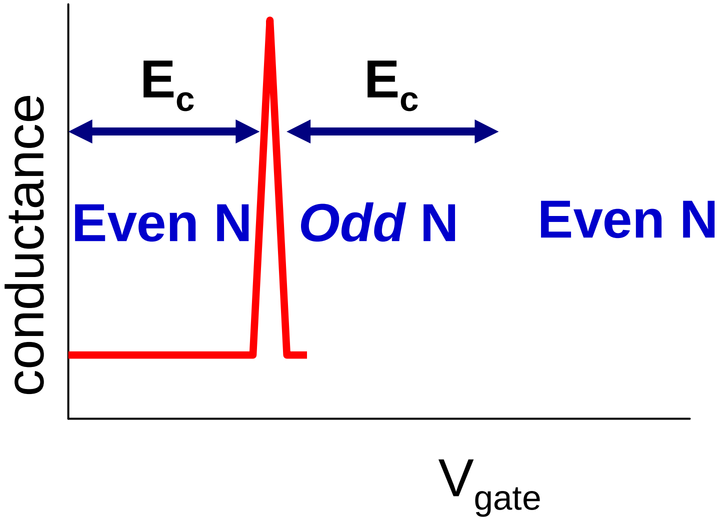


Coulomb Blockade in Quantum Dots

Coulomb Blockade in Quantum Dots

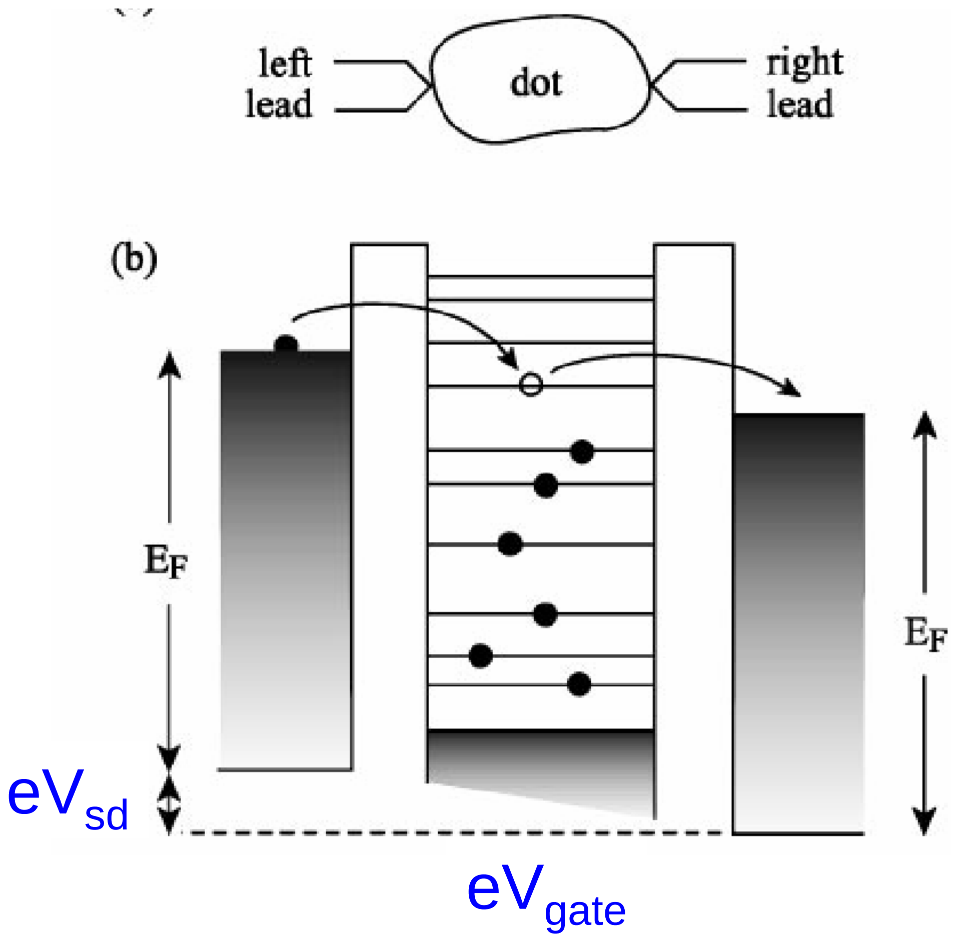




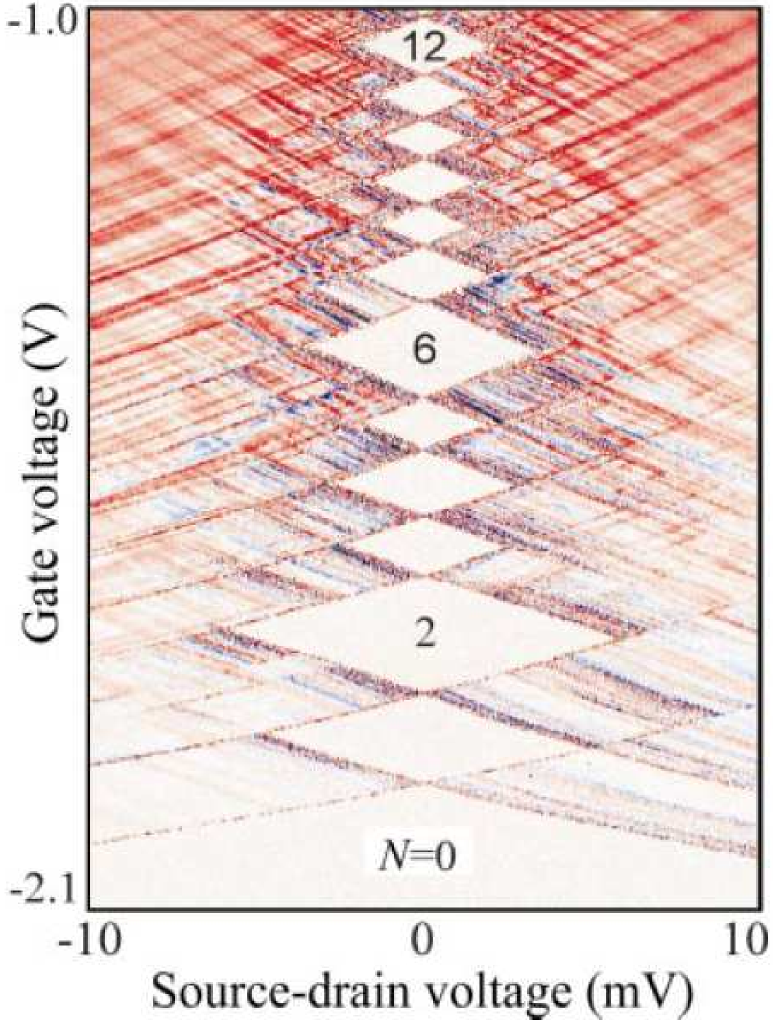


Coulomb Blockade in Quantum Dots

“Coulomb Diamonds” (Stability Diagram)

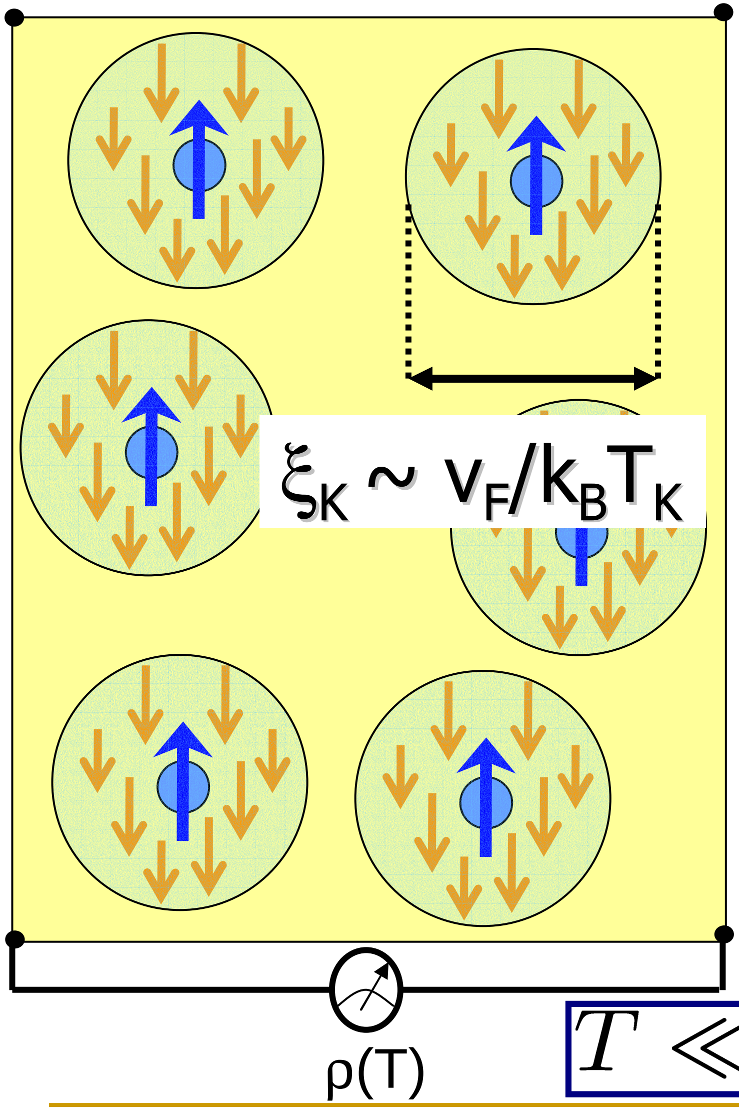


Coulomb Blockade in Quantum Dots



L. P. Kouwenhoven et al. *Science* **278** 1788 (1996).

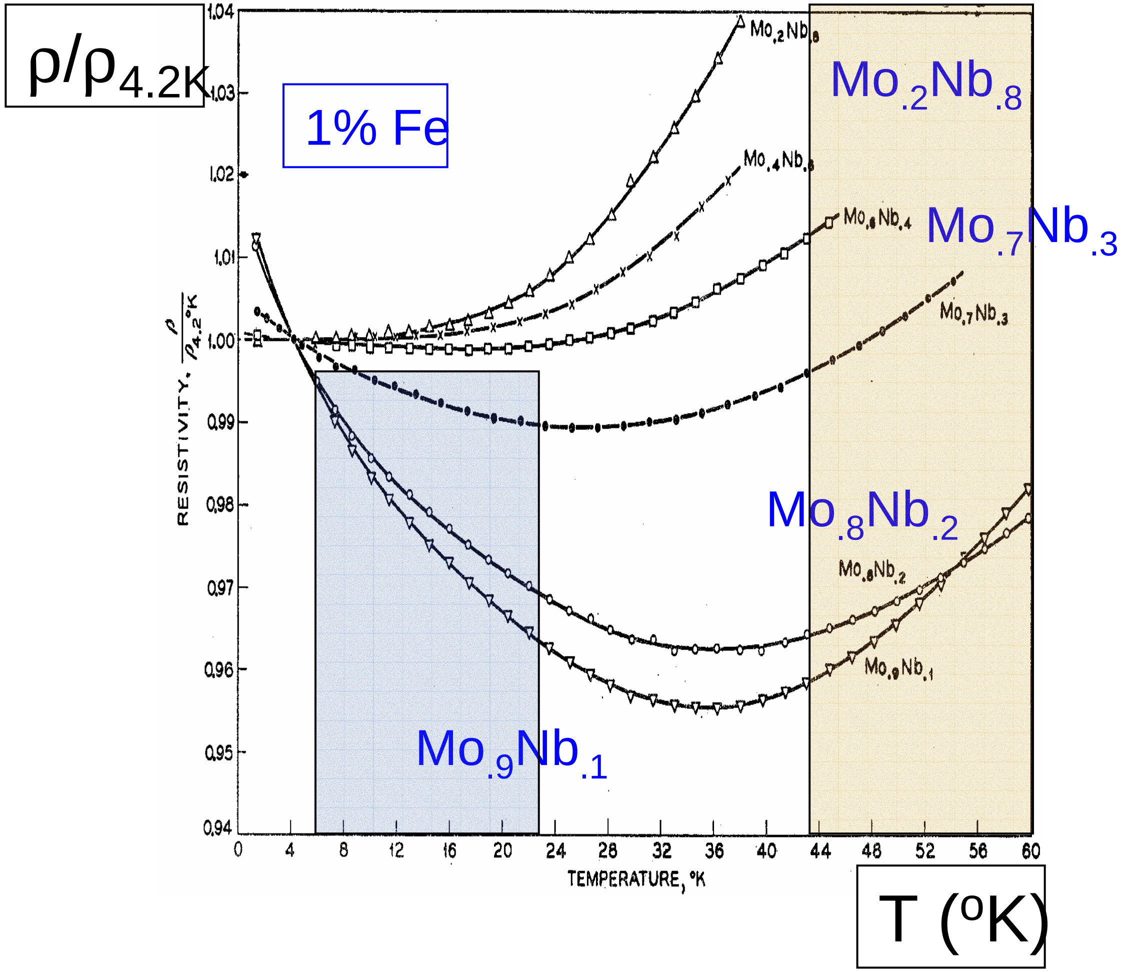
Kondo effect



$\xi_K \sim v_F/k_B T_K$

$T \ll T_K$

M.P. Sarachik *et al* Phys. Rev. **135** A1041 (1964).



Resistivity *decreases* with decreasing T (usual)

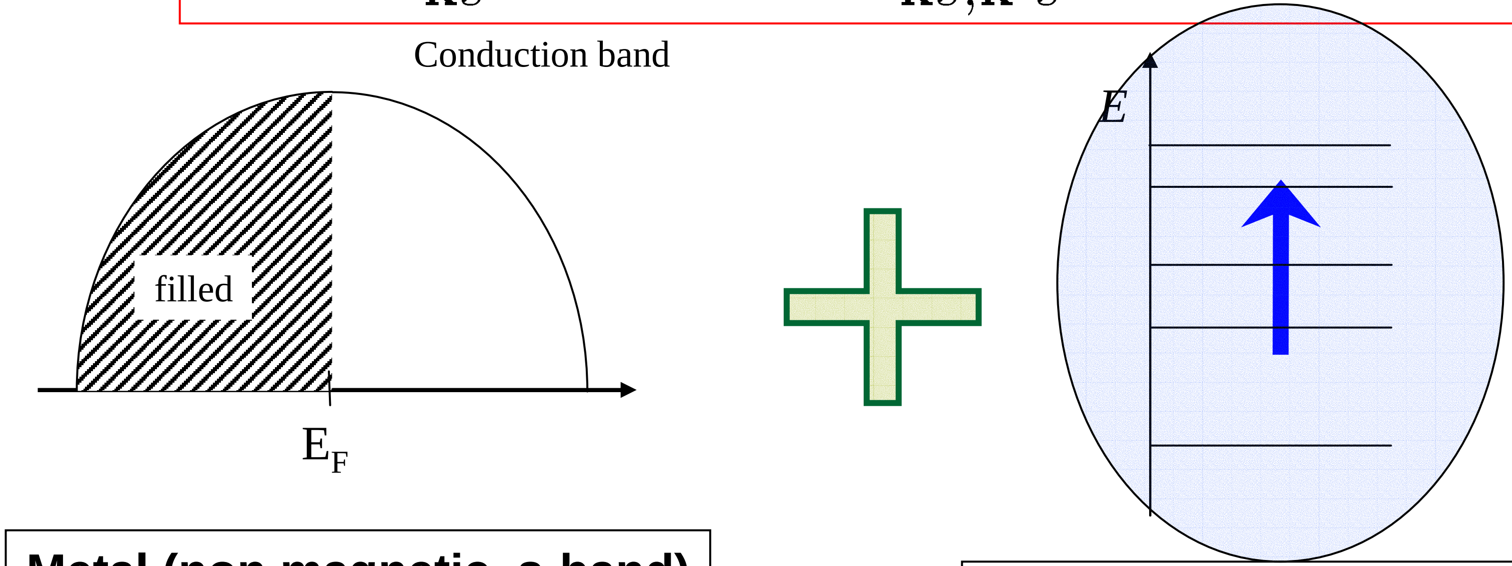
Resistivity *increases* with decreasing T (Kondo effect)

Characteristic energy scale: the Kondo temperature T_K

Kondo problem: s-d Hamiltonian

- Kondo problem: s-wave coupling with spin impurity (s-d model):

$$H_K = \sum_{\mathbf{k}s} \epsilon_{\mathbf{k}s}\hat{n}_{\mathbf{k}s} + J \sum_{\mathbf{k}s;\mathbf{k}'s'} c^{\dagger}_{\mathbf{k}s} \left(\mathbf{S}\cdot\vec{\sigma}\right)_{ss'} c_{\mathbf{k}'s'}$$



Metal (non magnetic, s-band)

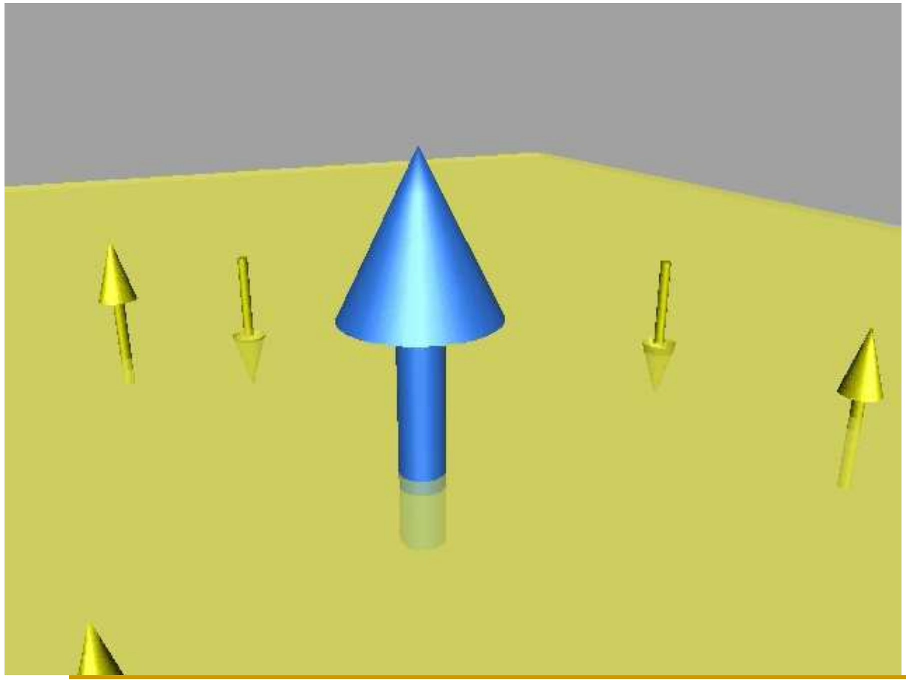
Magnetic impurity (unfilled d-level)

Kondo’s explanation for T_{min} (1964)

- Perturbation theory in J^3:
  - Kondo calculated the conductivity in the linear response regime

$$R_{\text{imp}}^{\text{spin}} \propto J^2\left[1-4J\rho_0 \log\left(\frac{k_B T}{D}\right)\right]$$

$$R_{\text{tot}}(T) = aT^5 - c_{\text{imp}} R_{\text{imp}} \log\left(\frac{k_B T}{D}\right)$$

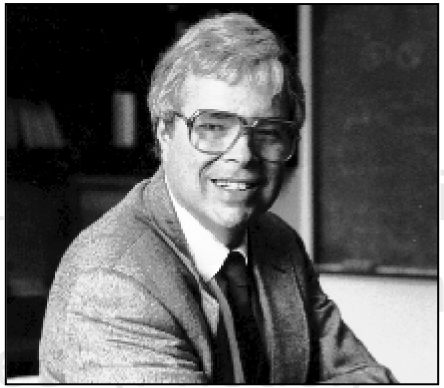


$$T_{\min} = \left(\frac{R_{\text{imp}} D}{5ak_B}\right)^{1/5} c_{\text{imp}}^{1/5}$$

- Only *one* free paramenter: the Kondo temperature T_K
  - Temperature at which the perturbative expansion ***diverges.***

$$k_B T_K \sim De^{-1/2J\rho_0}$$

A little bit of Kondo history:

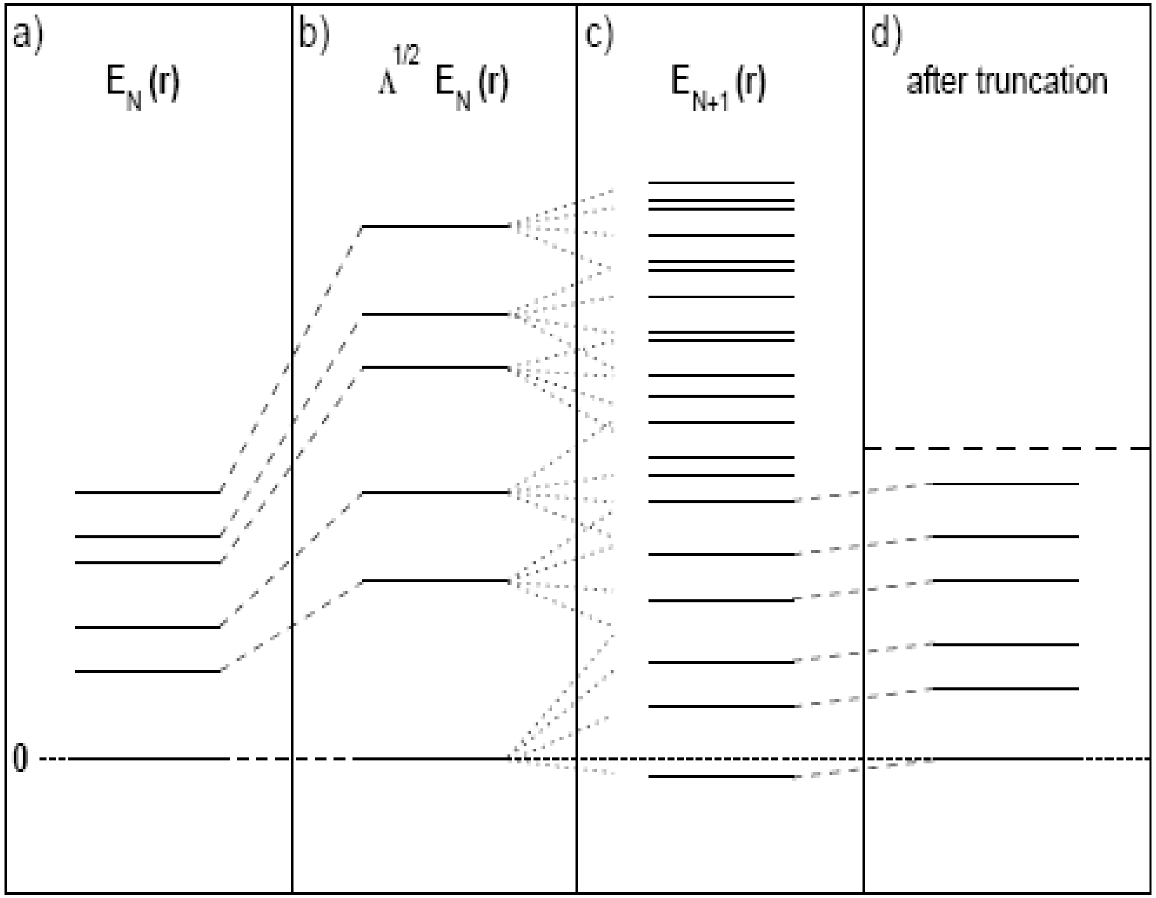


Kenneth G. Wilson – Physics Nobel Prize in 1982
"for his theory for critical phenomena in connection with phase transitions"

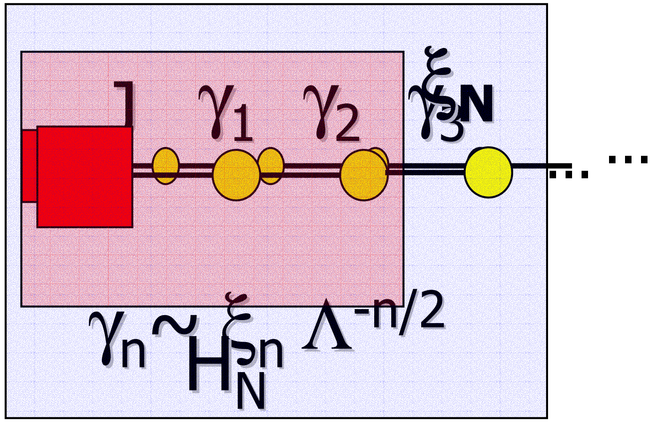
- Early '30s : Resista in some metals
- Early '50s : theoreti purities in metals
  "Virtual Bound State
- 1961: An in
  metals
- 1964: s-d model and Kon solution (PT)
- 1970: Anderson "Poor's m n scaling"
- 1974-75: Wilson's Numerical Renormalization Group (non PT)
- 1980 : Andrei and Wiegmann's exact solution

Renormalization Procedure

$$H_{N+1} = \sqrt{\Lambda} H_N + \xi_N \sum_{\sigma} f^{\dagger}_{N+1\sigma} f_{N\sigma} + f^{\dagger}_{N\sigma} f_{N+1\sigma}$$

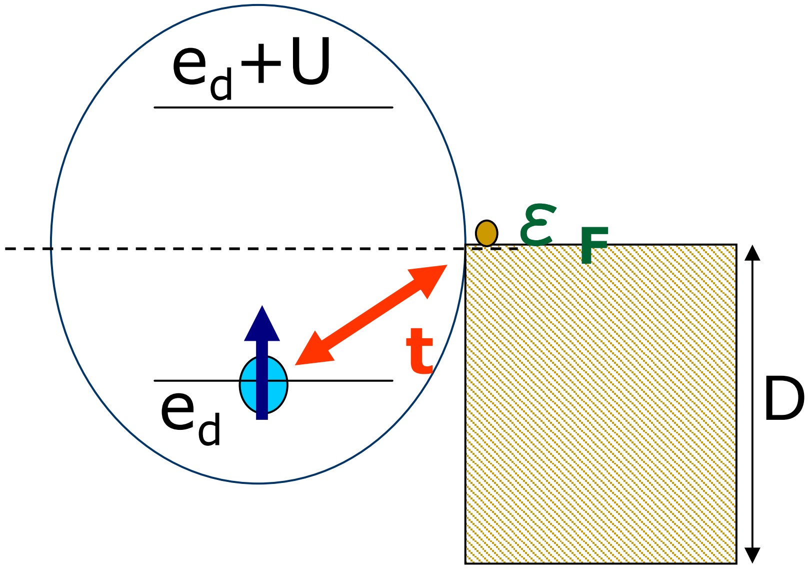


- Iterative numerical solution.
- Renormalize by $\Lambda^{1/2}$.
- Keep low energy states.



H_{N+1}

Anderson Model



$$\begin{aligned} H &= \epsilon_d \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow}\hat{n}_{d\downarrow} \\ &\quad + \sum_k \epsilon_k \hat{n}_{k\sigma} \\ &\quad + t \sum_k c^\dagger_{d\sigma} c_{k\sigma} + \mathrm{h.c.} \end{aligned}$$

with

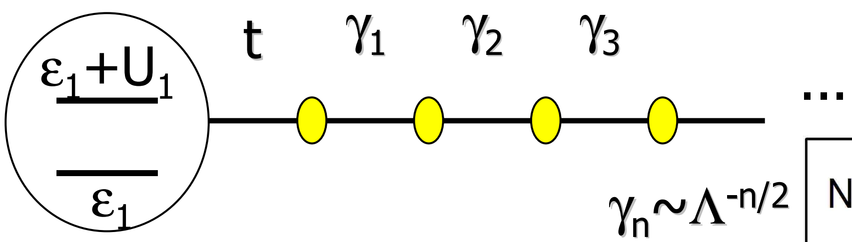
$$\begin{aligned} \hat{n}_{d\sigma} &= c^\dagger_{d\sigma} c_{d\sigma} \\ \hat{n}_{k\sigma} &= c^\dagger_{k\sigma} c_{k\sigma} \end{aligned}$$

- e_d: energy level
- U: Coulomb repulsion
- e_F: Fermi energy in the metal
- t: Hybridization
- D: bandwidth

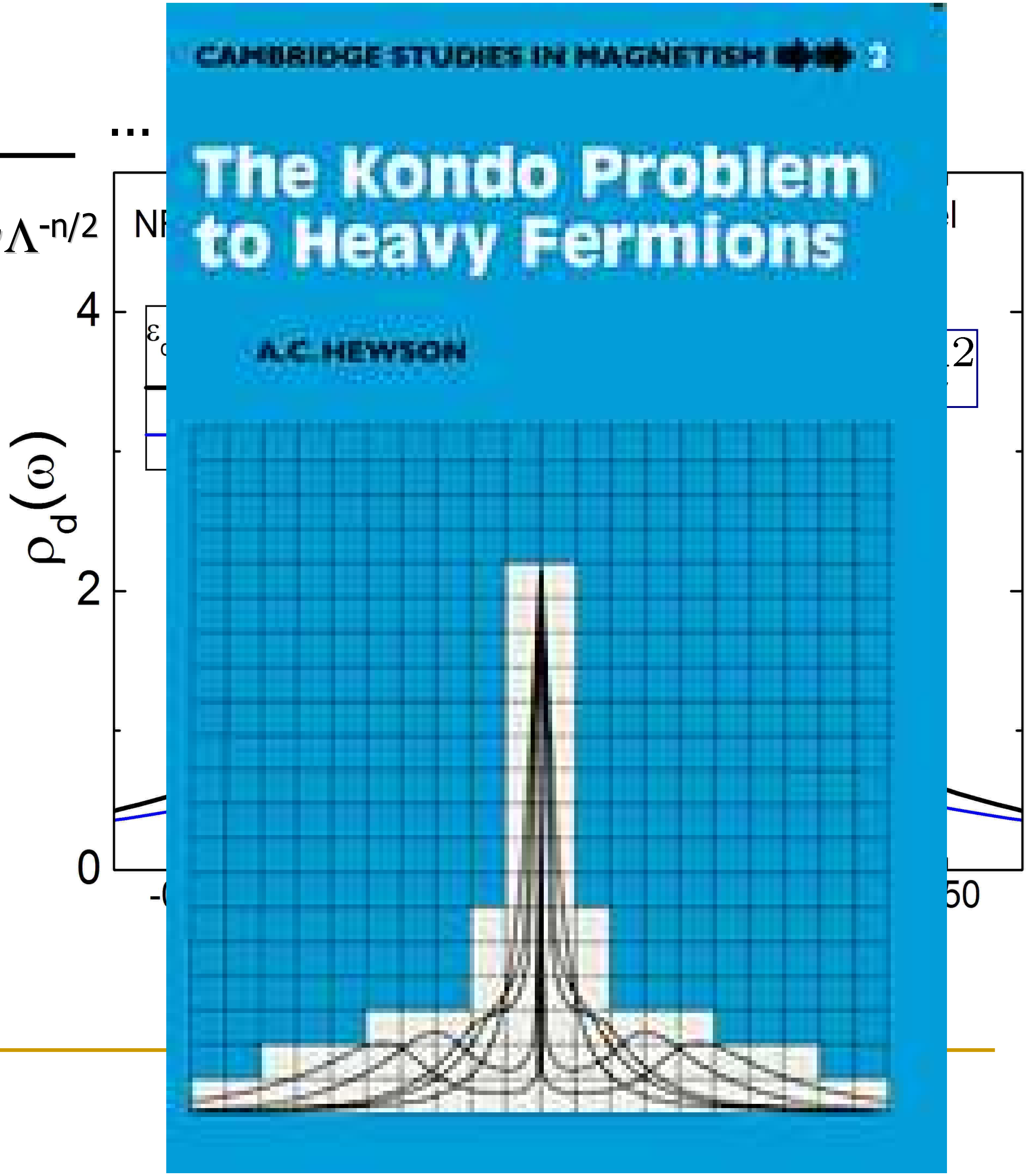
“Quantum dot language”

- e_d: position of the level (V_g)
- U: Charging energy
- e_F: Fermi energy in the leads
- t: dot-lead tunneling
- D: bandwidth

NRG on Anderson model: LDOS



- Single-particle peaks at ε_d and ε_d+U.
- *Many-body* peak at the Fermi energy: ***Kondo resonance*** (width ~T_K).
- NRG: good resolution at low ω (log discretization).



History of Kondo Phenomena

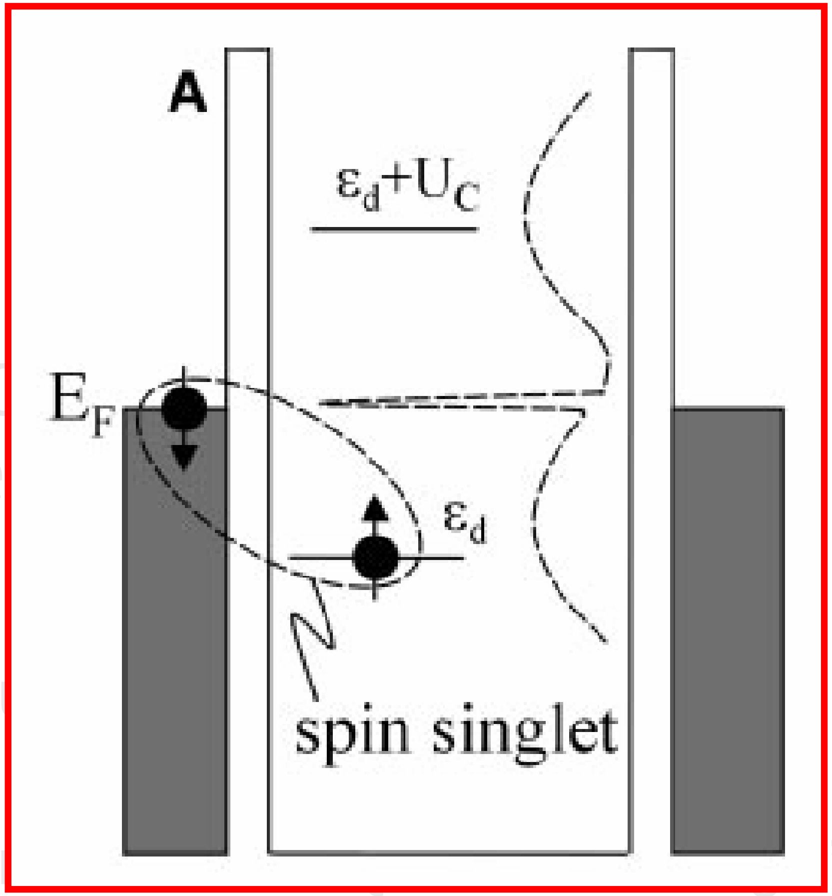
- Observed in the ‘30s
- Explained in the ‘60s
- Numerically Calculated in the ‘70s (NRG)
- Exactly solved in the ‘80s (Bethe-Ansatz)

So, what’s new about it?

Kondo correlations observed in many different set ups:

- Transport in quantum dots, quantum wires, etc
- STM measurements of magnetic structures on metallic surfaces (e.g., single atoms, molecules. “Quantum mirage”)
- ...

History of Kondo Phenomena



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So, what’s ne

Kondo correlations observed

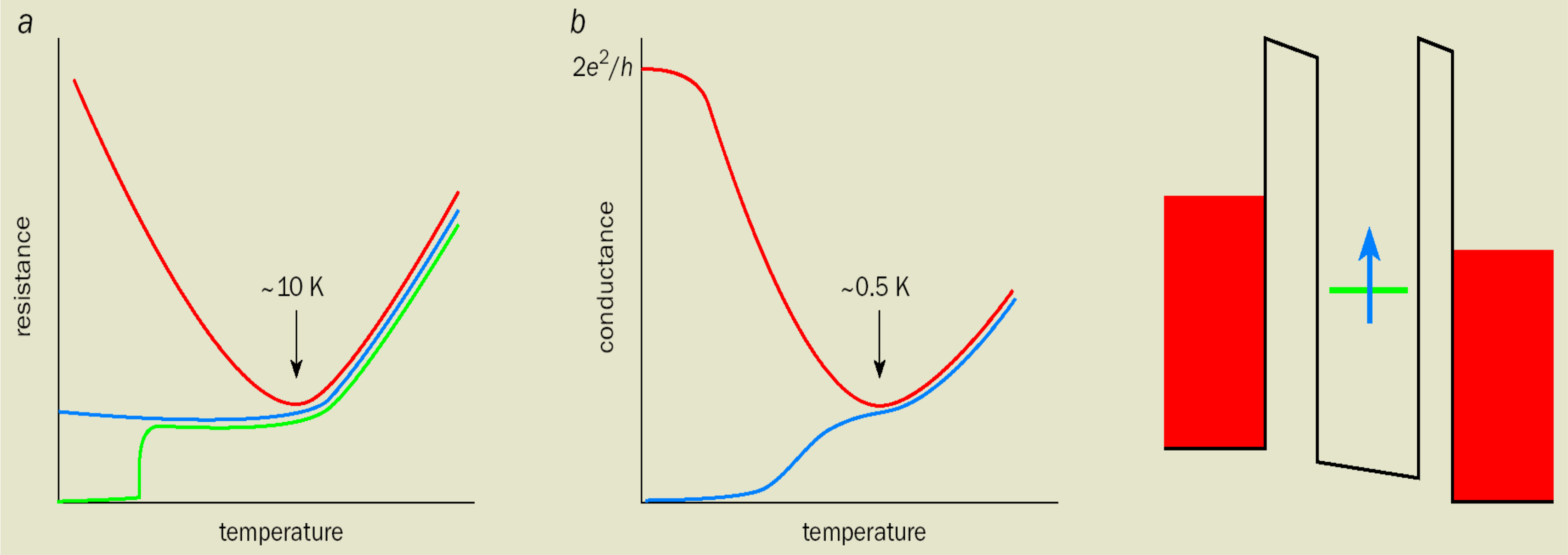
- <u>Transport in quantum dots</u>, q
- STM measurements of magn
  surfaces (e.g., single atoms,
- ...

Kondo Effect in Quantum Dots

Revival of the Kondo effect

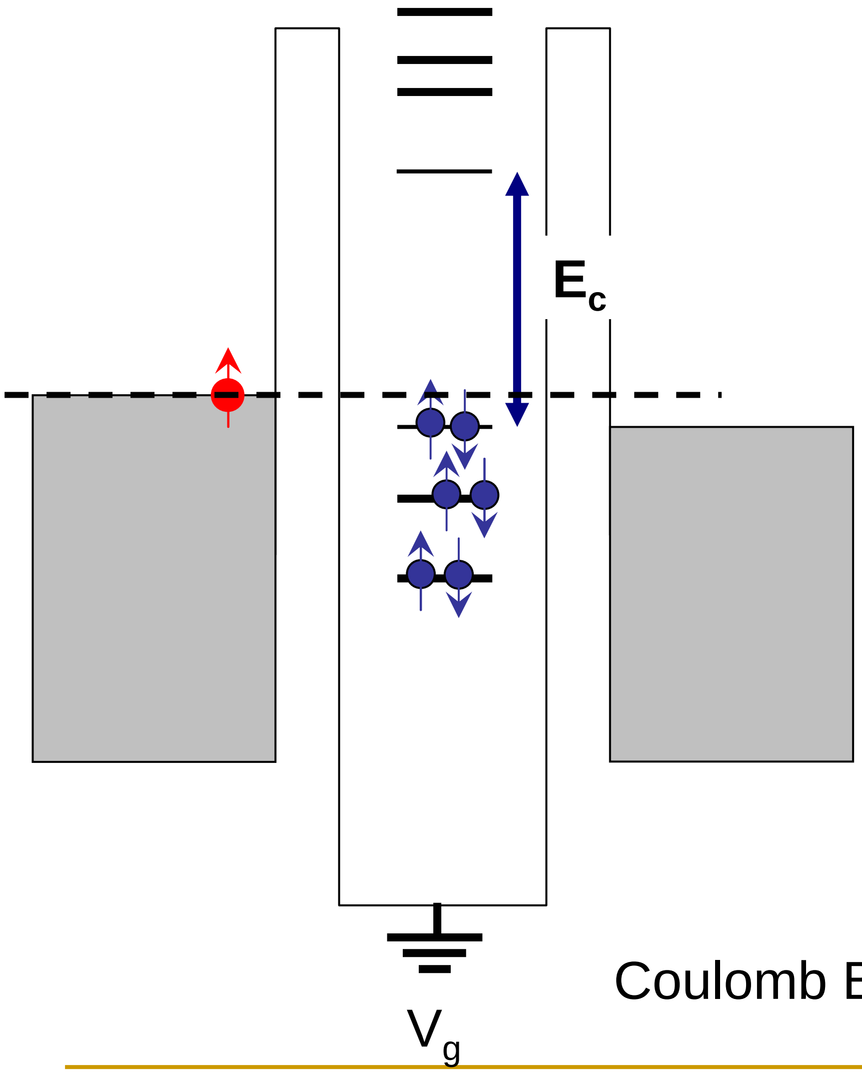
Leo Kouwenhoven and Leonid Glazman

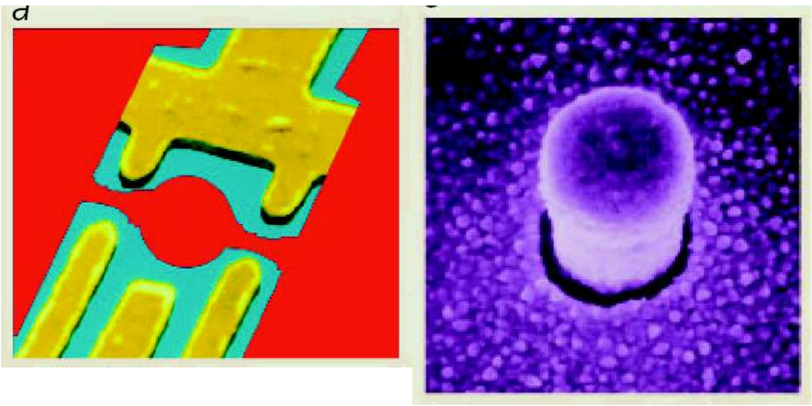
1 The Kondo effect in metals and in quantum dots

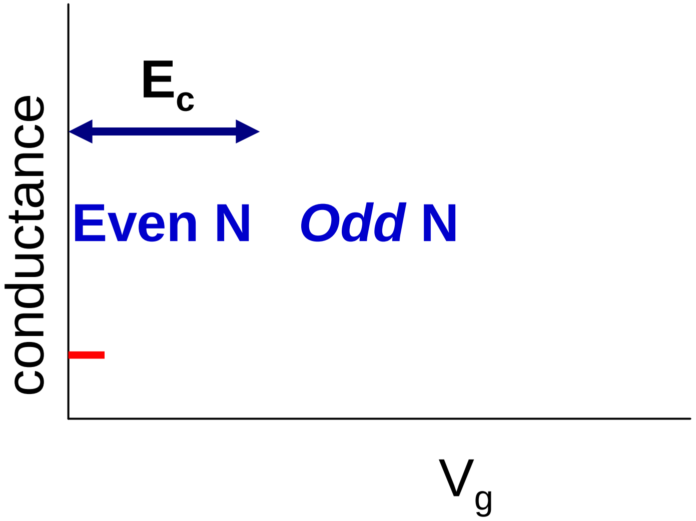


Kowenhoven and Glazman *Physics World* – Jan. 2001.

Coulomb Blockade in Quantum Dots







Coulomb Blockade in Quantum Dots

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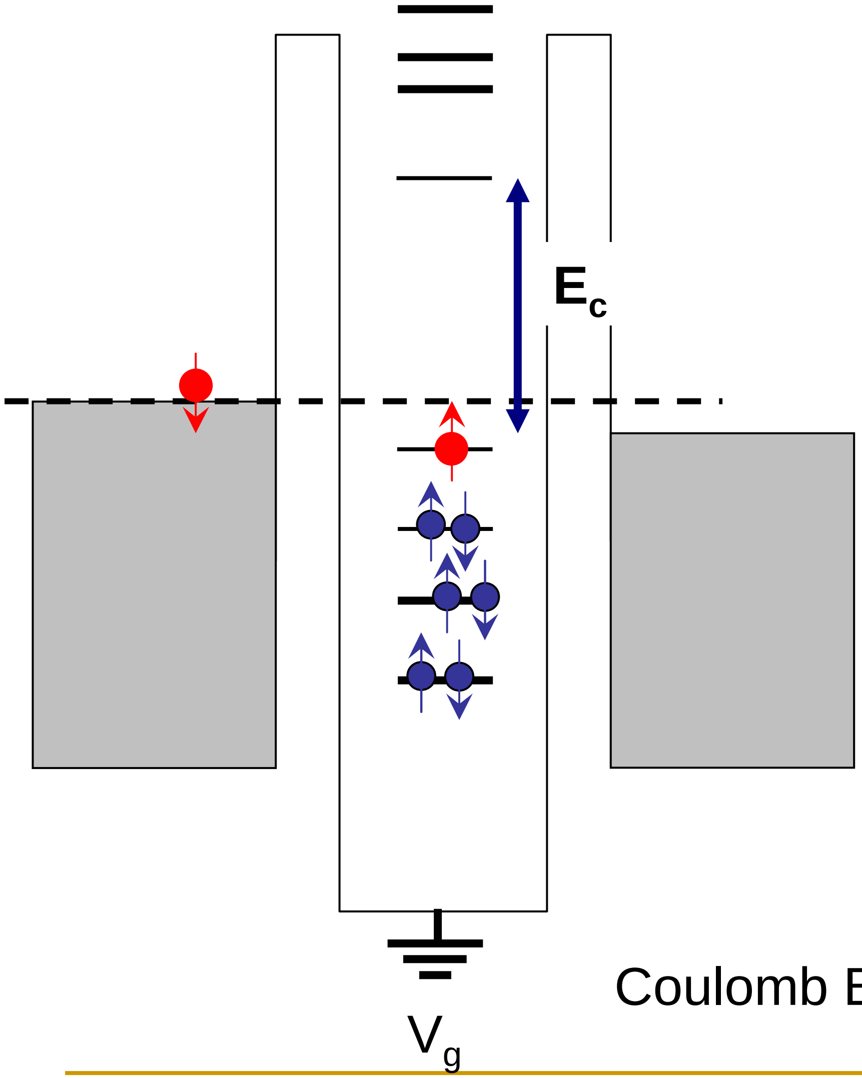
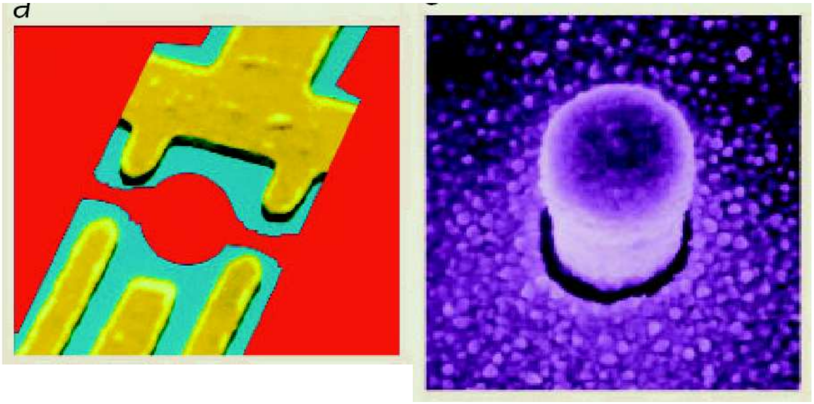
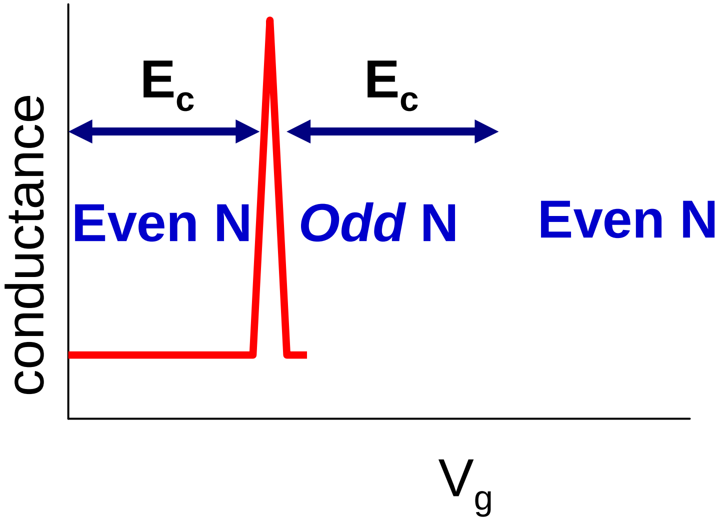


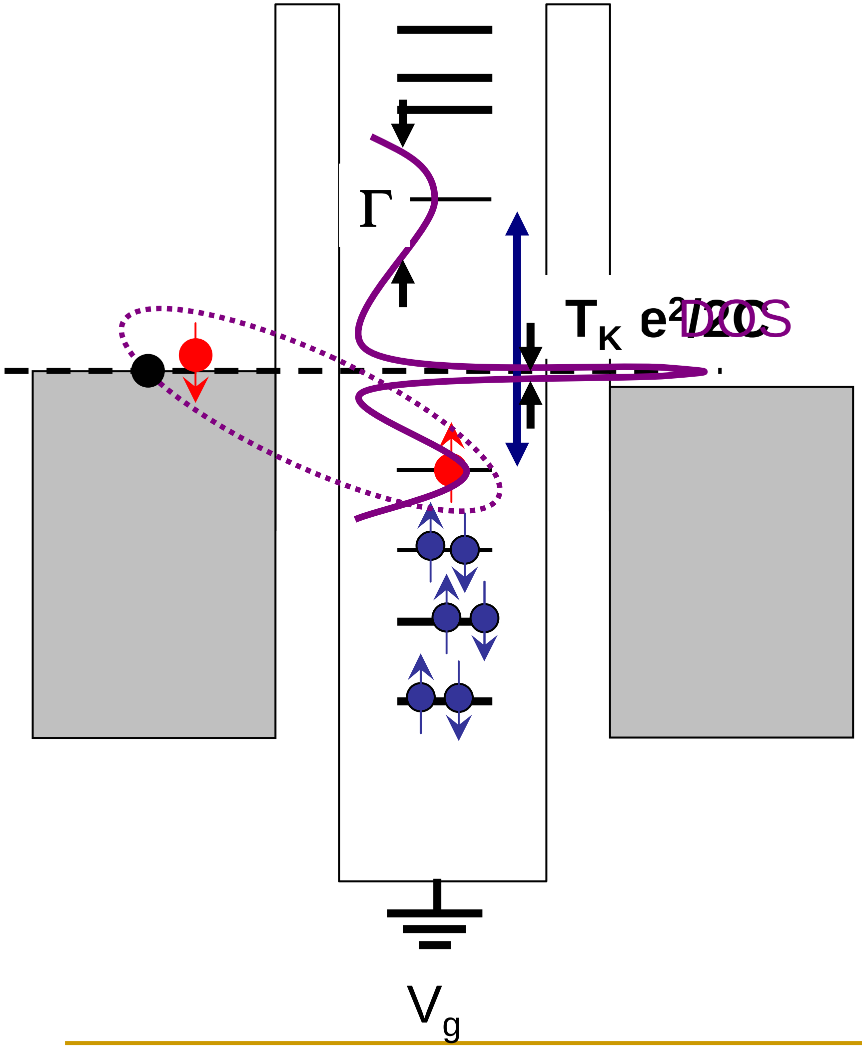


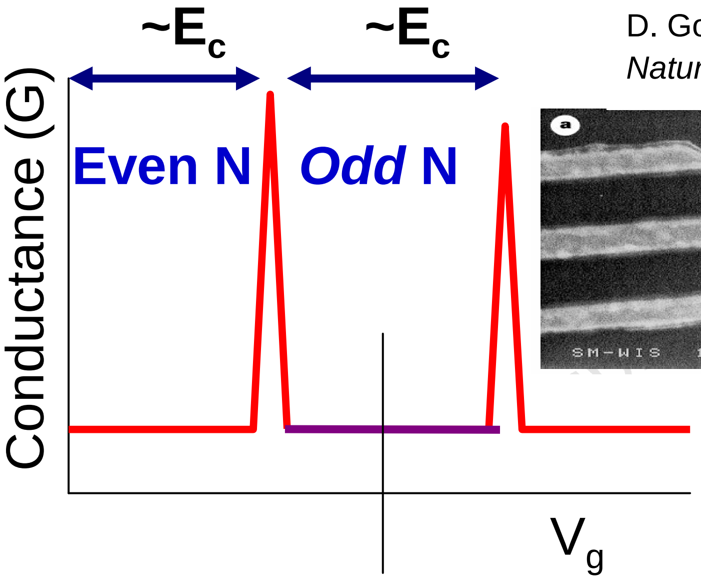


Coulomb Blockade in Quantum Dots

Y. Alhassid *Rev. Mod. Phys.* **72** 895 (2000).

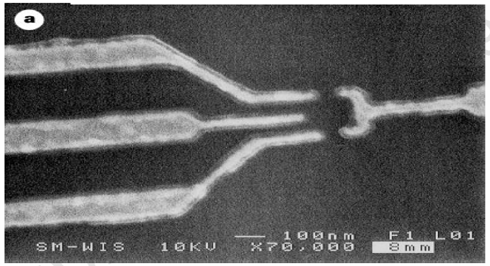
Kondo Effect in Quantum Dots





D. Goldhaber-Gordon et al

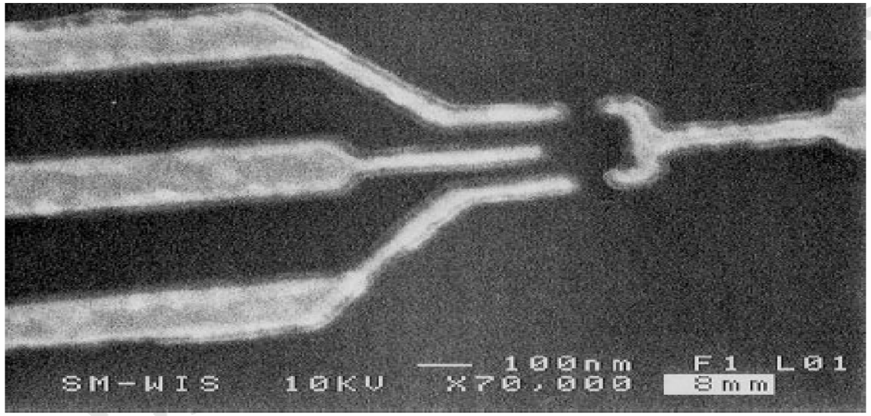
Nature **391** 156 (1998)

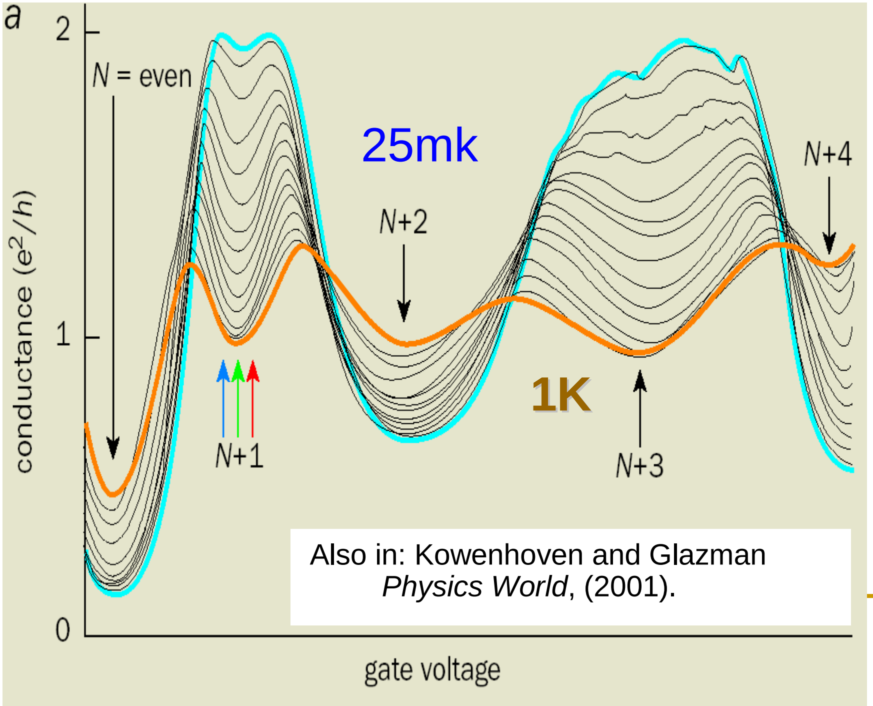


- $T>T_K$: Coulomb blockade (low G)
- $T<T_K$: Kondo singlet formation
- Kondo resonance at E_F (width T_K).
- New conduction channel at E_F: Zero-bias enhancement of G

Kondo effect in Quantum Dots

D. Goldhaber-Gordon et al. Nature **391** 156 (1998)



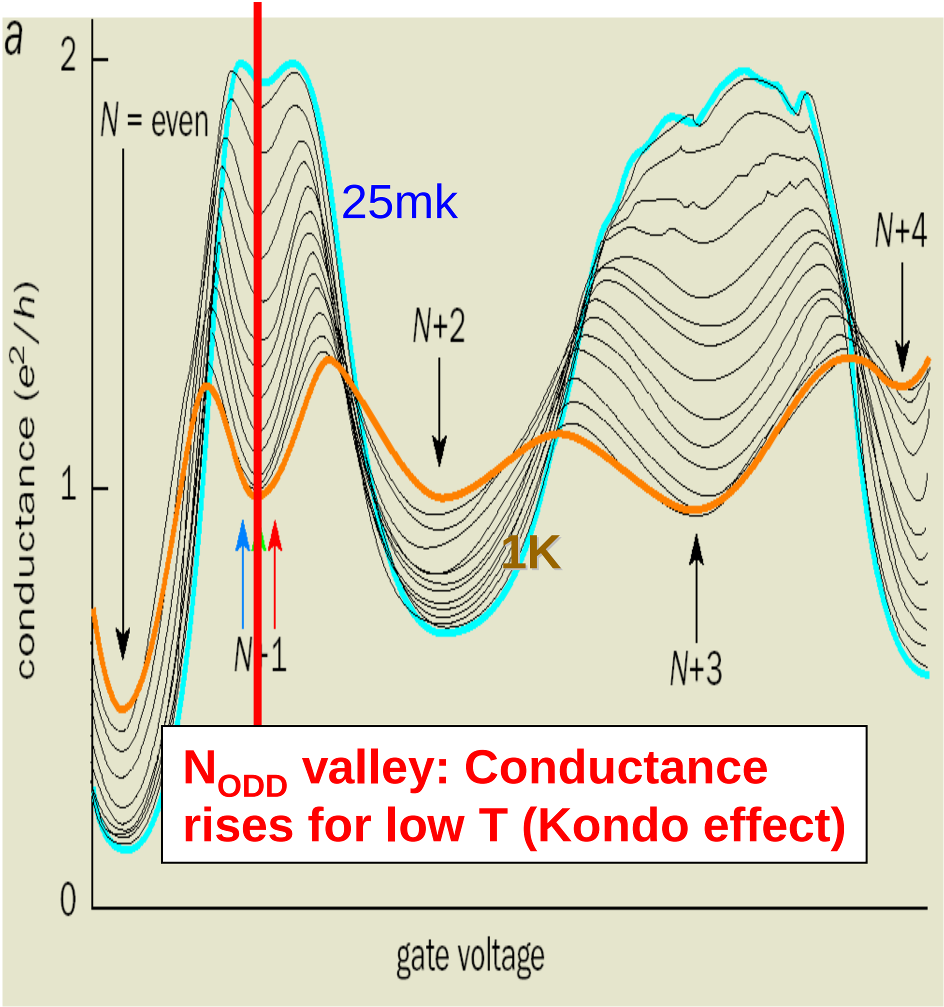


Also in: Kowenhoven and Glazman
Physics World, (2001).

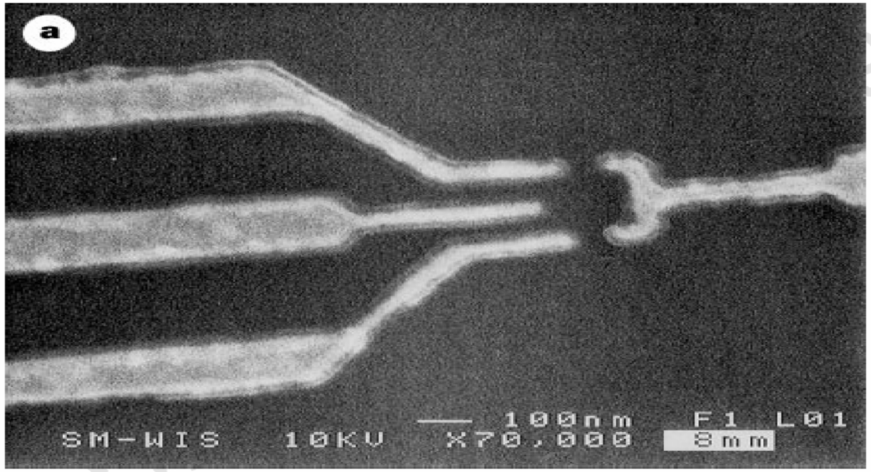
Semiconductor Quantum Dots:

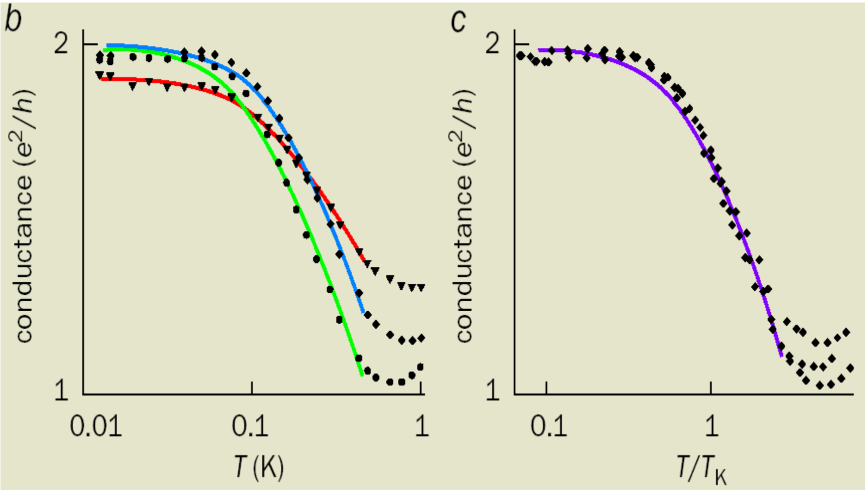
- Allow for systematic and *controllable* investigations of the Kondo effect.
- QD in N_{odd} Coulomb Blockade valley: realization of the Kondo regime of the Anderson impurity problem.

Kondo Effect in CB-QDs



N_{ODD} valley: Conductance rises for low T (Kondo effect)





Kondo Temperature T_k : only scaling parameter (~0.5K, depends on V_g)

Kowenhoven and Glazman *Physics World* – Jan. 2001.

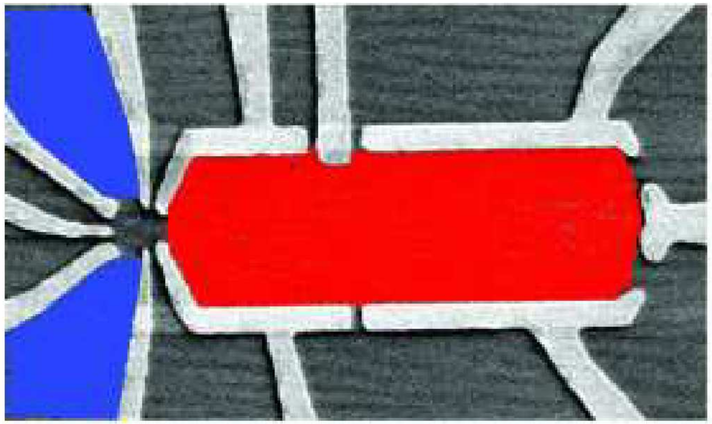
From: Goldhaber-Gordon *et al. Nature* **391** 156 (1998)

Examples of “fundamental” physics (in the “More is Different” sense) we can learn from quantum dots:

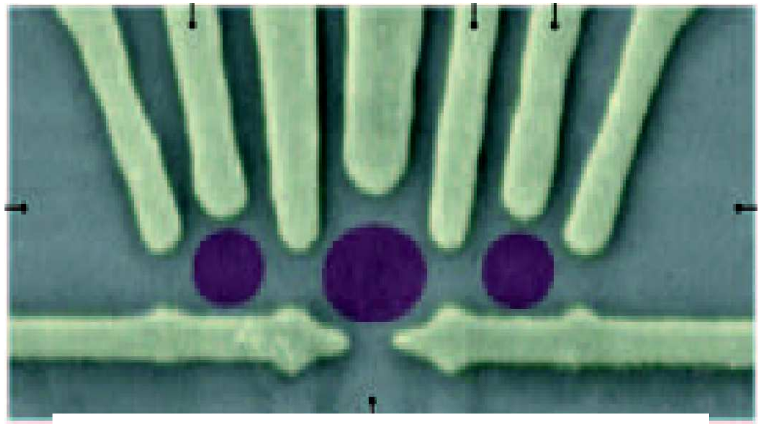
- *Strongly correlated effects*: Charging effects due to electron-electron interactions are dominant in QDs.
- *Quantum effects* (spin, tunneling, discreet energy levels, interference) probed in a very controllable way.
- Quantum Many-body physics:
  - Kondo effect, Quantum phase transitions, …

Kondo Effect in *Double* QDs

“Side dot” configuration

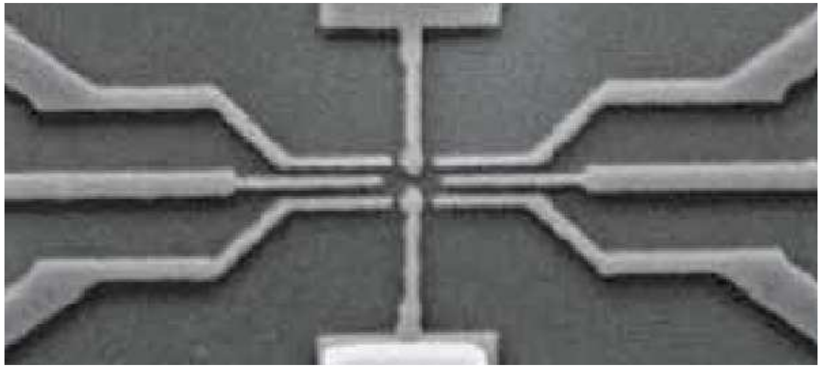


R. Potok *et al. Nature* 446 167 (2007).



Craig et al., *Science* **304** 565 (2004)

“Parallel” configuration



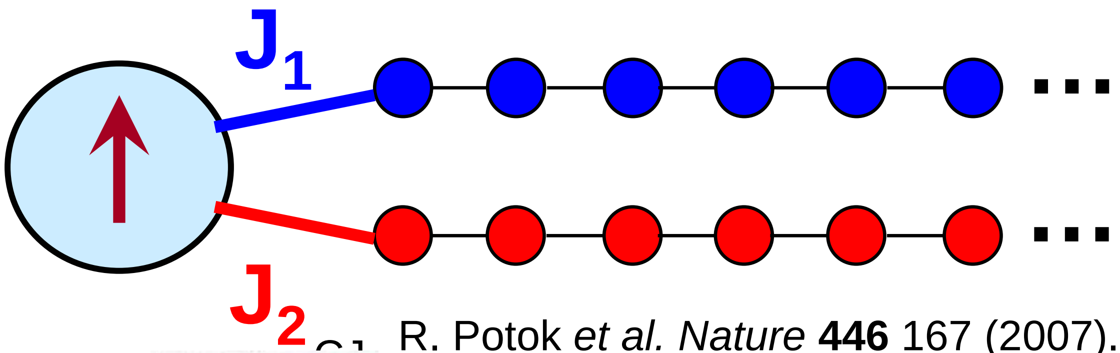
Chen, Chang, Melloch, *PRL* **92** 176801 (2004)

- Tunability of intradot and interdot parameters (couplings, gate voltage).

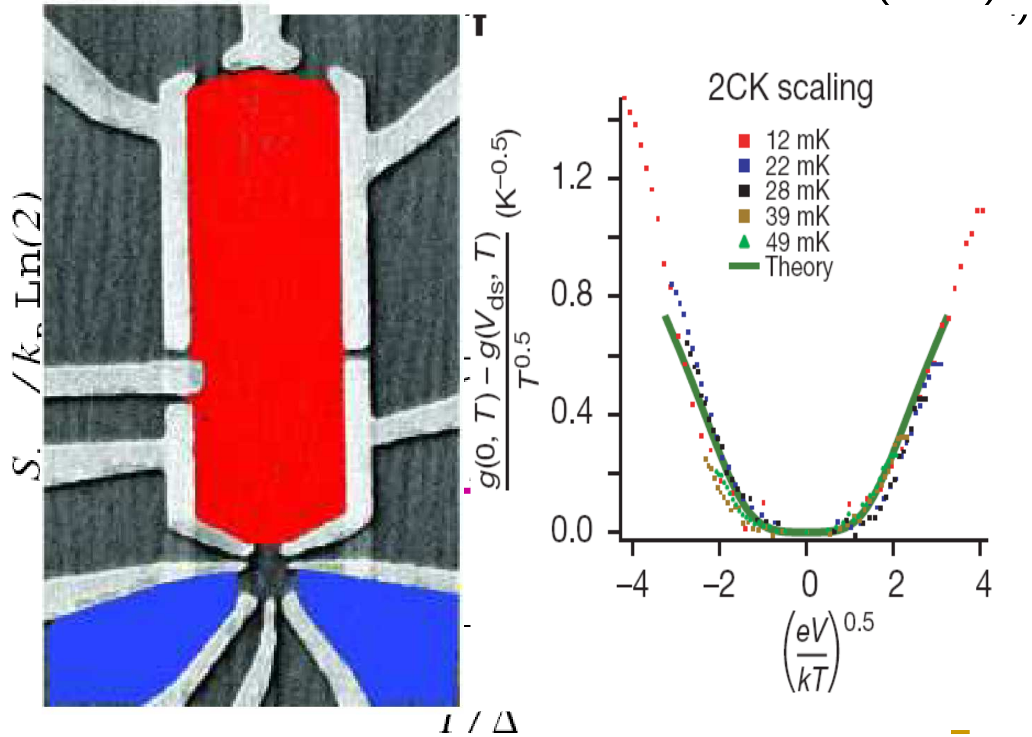
- Prospects for experimental probe of many-body phenomena, e.g:
- - SU(4) Kondo, RKKY interactions,...
- - Non-Fermi liquid physics (2-ch Kondo)
- **- Quantum phase transitions.**

Two-channel Kondo effect.

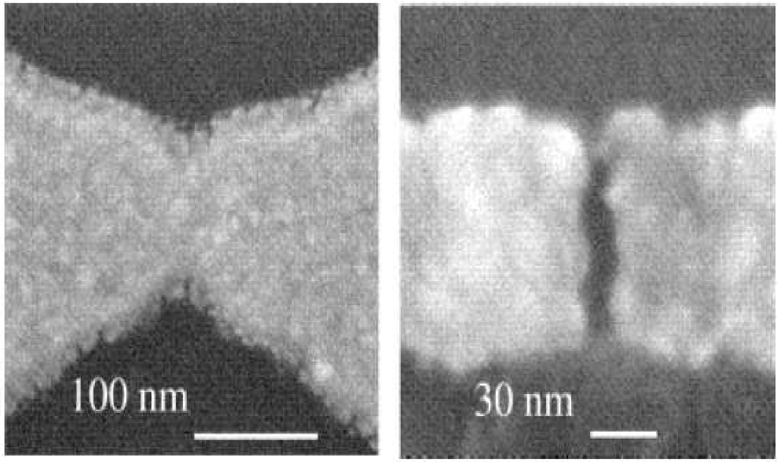
- Spin 1/2 coupled to two ***independent*** bands: 2-channel Kondo model ("overscreened").
- Non-Fermi-liquid (NFL) behavior for $J_1=J_2$.
- Impurity entropy (NFL): $S_{imp}= k_B \log(\sqrt{2})$ (NRG, Bethe ansatz).
- Recent expts in q. dots.

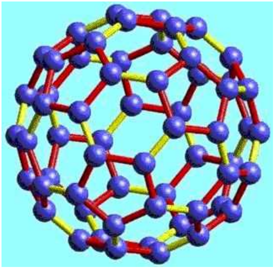


R. Potok *et al. Nature* **446** 167 (2007).

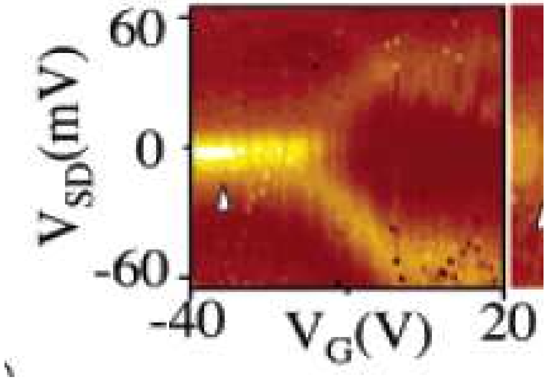


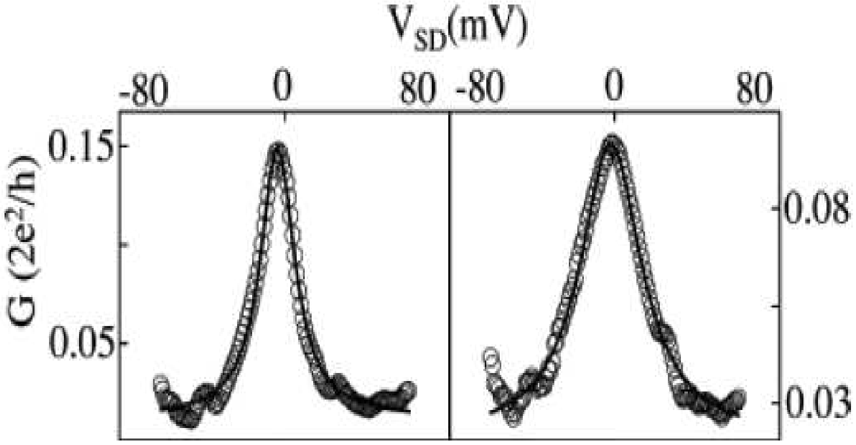
Kondo effect in Single Molecule Transistors





C_{60}





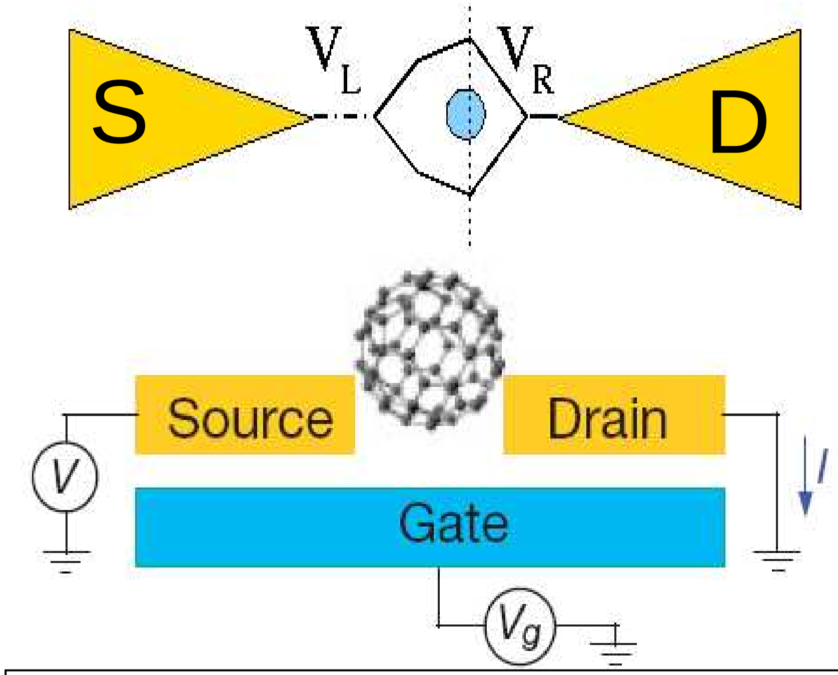
T=18K T=50K

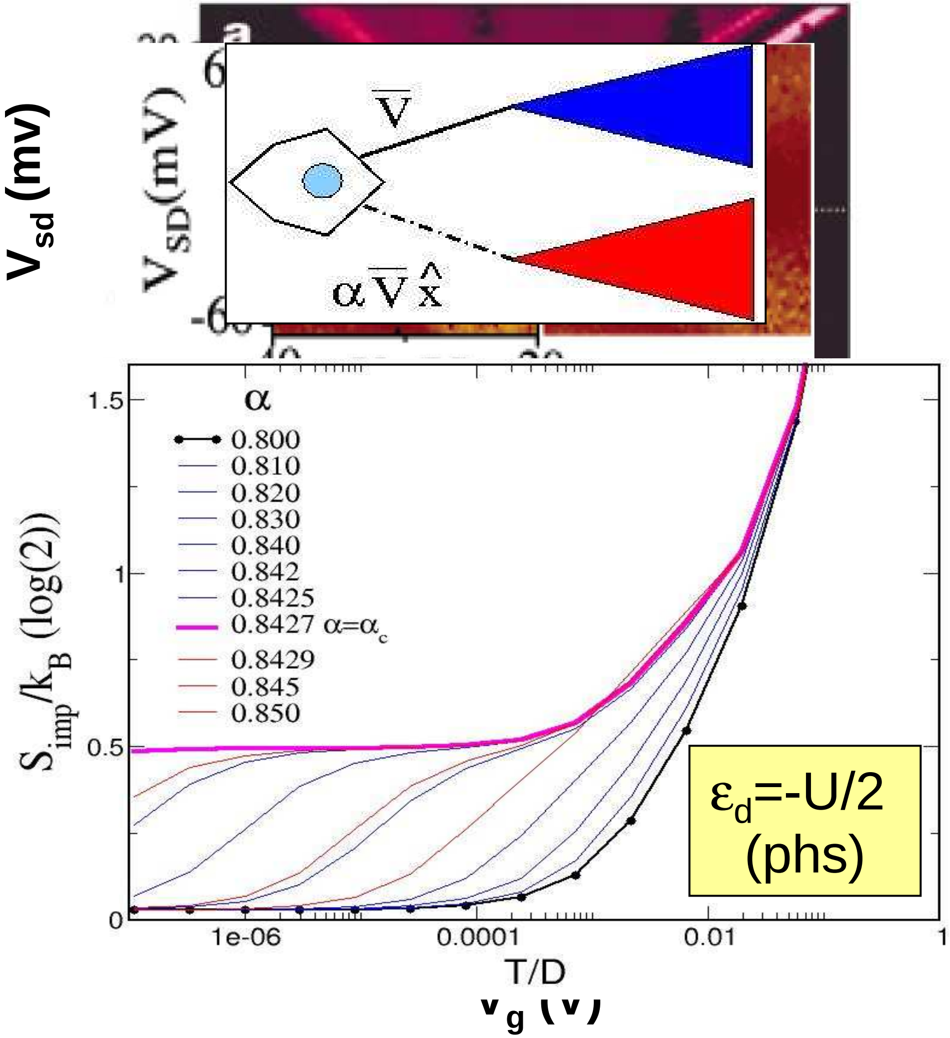
Yu, Natelson, *NanoLett.* **4** 79 (2004).

- Single molecule transistors: C_{60} molecules “caught” between electrodes (break junction).
- Zero-bias peak as a function of gate voltage: correct Kondo scaling.
- Correct behavior vs. Bias.
- T_K>50K .

Transport in molecular junctions.

- Coulomb blockade effects .
- Features consistent with vibrational modes in dI/dV.
- Kondo signatures.

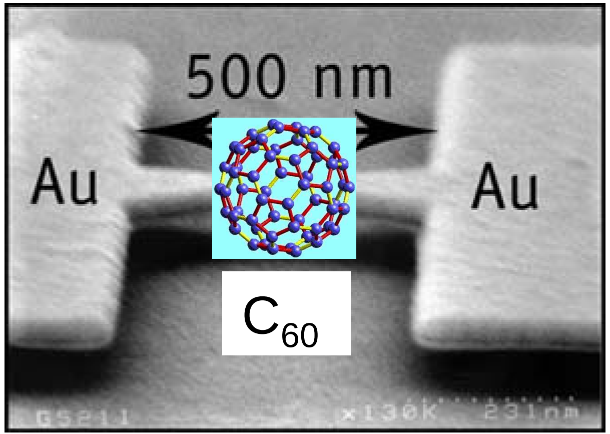


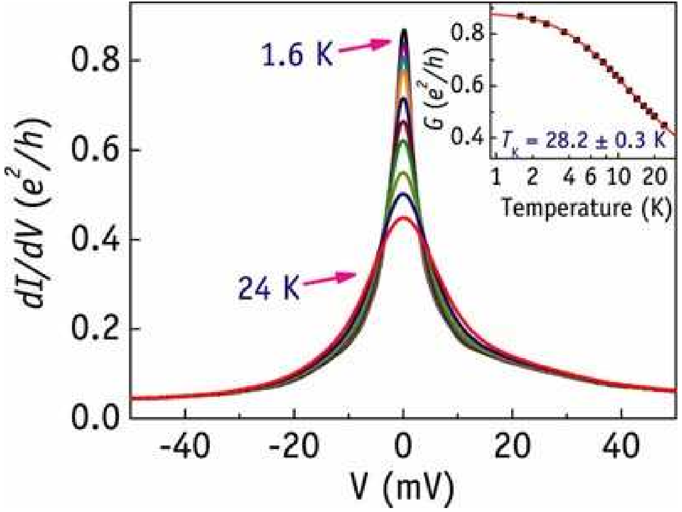


Park et al. *Nature* **407** 57 (2000)

Connection of vibrational modes with 2-channel Kondo:
LDS, E. Dagotto – PRB **79** 155302 (2009); arXiv:0902.3225.

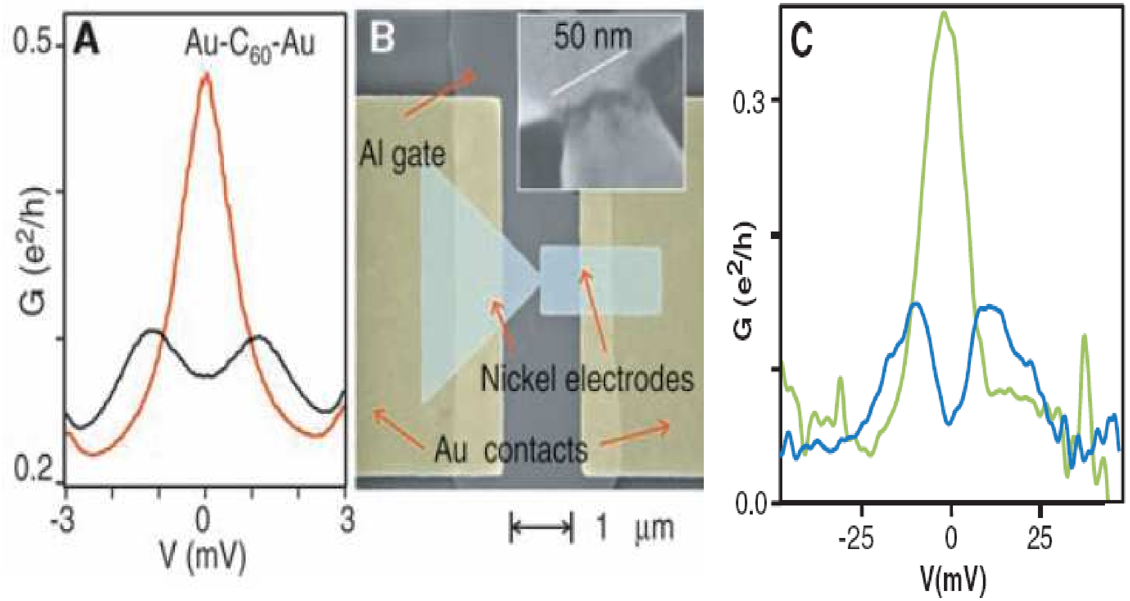
Kondo effect in Single Molecule Transistors





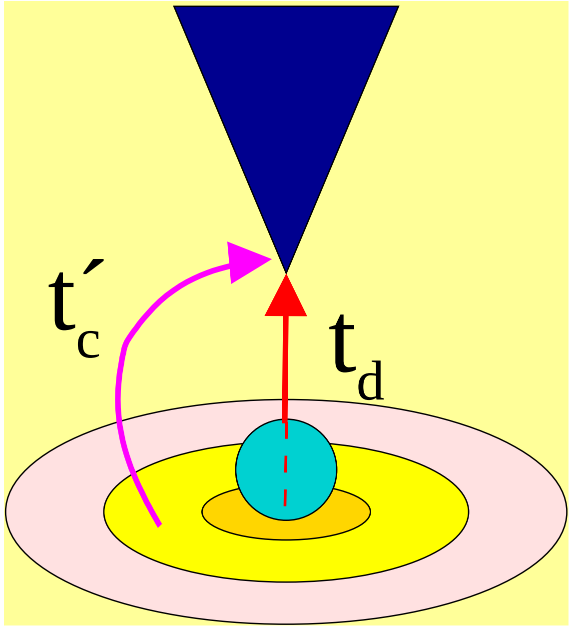
From Dan Ralph's webpage:
http://people.ccmr.cornell.edu/~ralph/

Pasupathy et al., *Science* **306** 86 (2004)

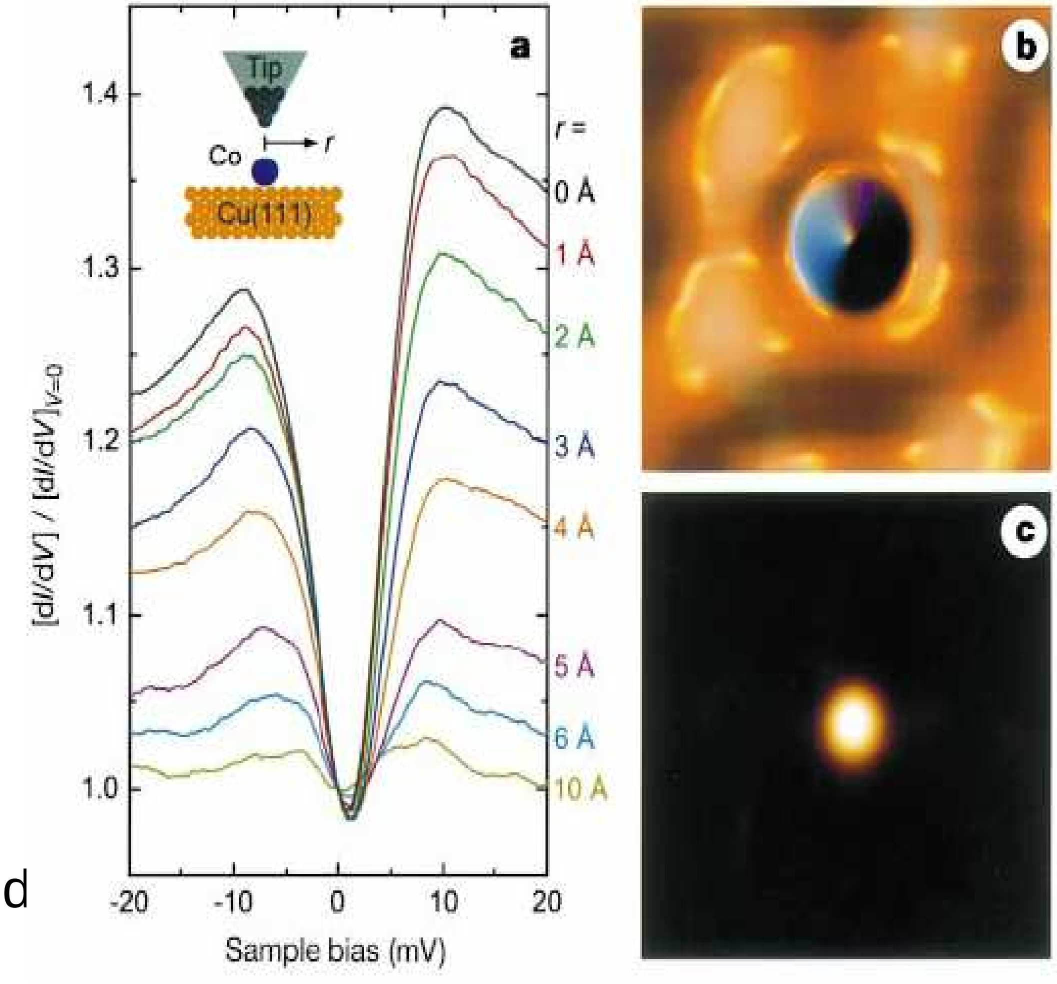


- Similar expts (D. Ralph's group).
- Supression of the Kondo resonance in the presence of a magnetic field (top left, black curve, B=10T) and *magnetic* leads (top right, parallel [green] and antiparallel [blue] magnetizations).

Kondo effect in surfaces (STM images).

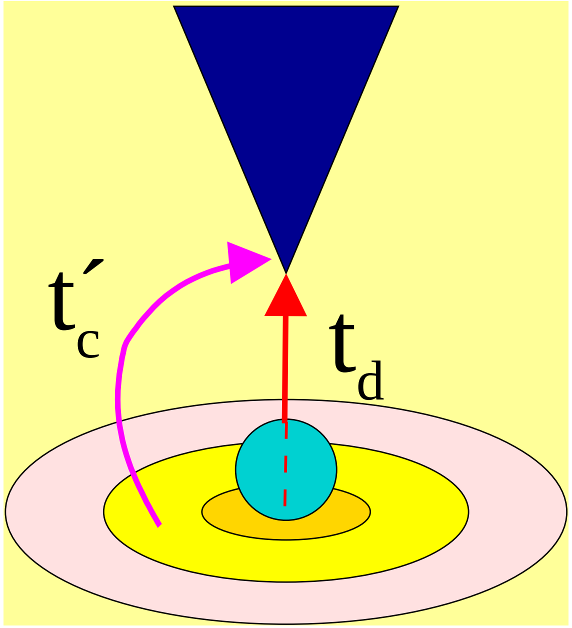


Manoharan et al., *Nature* **403** 512 (2000).

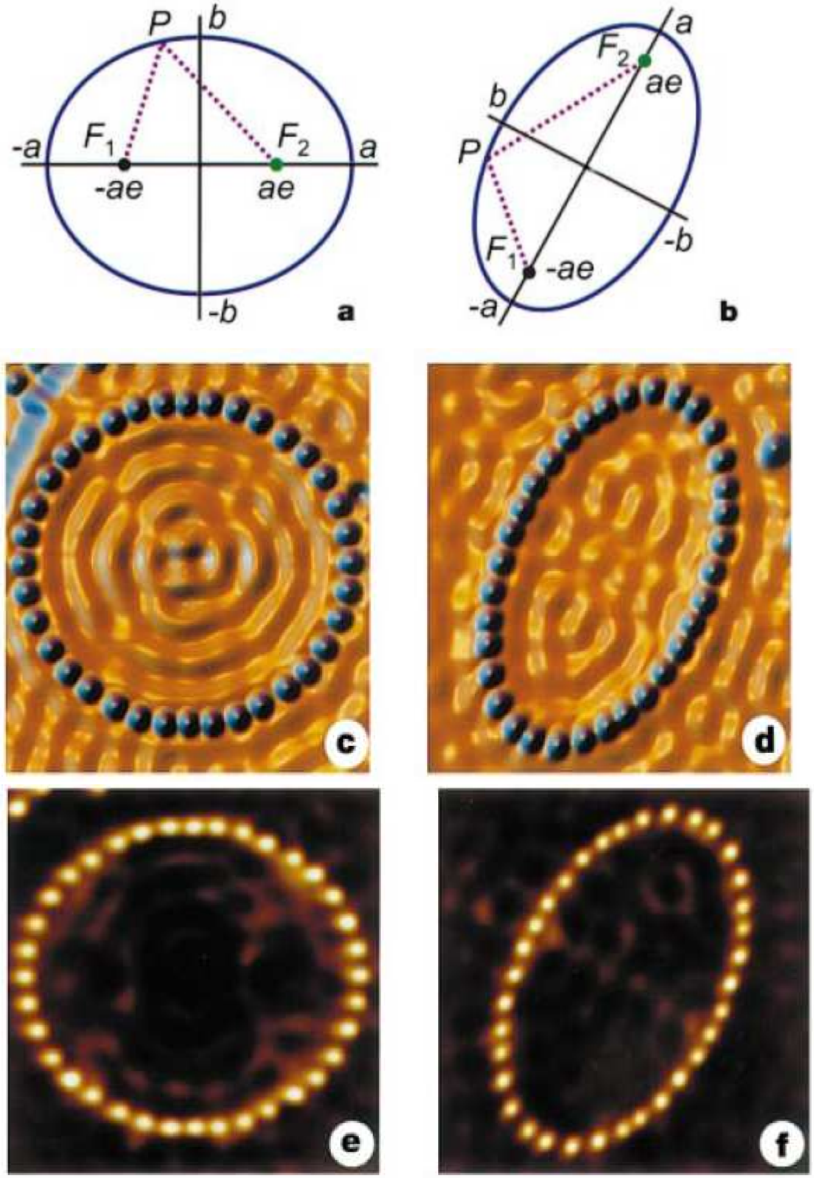


- Magnetic (Co, Fe) atoms on metallic *surfaces!* Right ingredients for Kondo.
- In this case, Kondo is marked by a *dip* at zero-bias conductance (dI/dV at V=0).

Kondo effect surfaces: STM measurements.

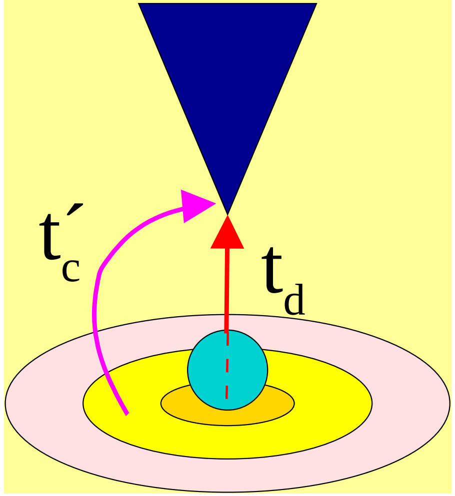


- STM atomic manipulation: can build local structures ("quantum corrals").
- Elliptical shape: imaging (top) and dI/dV measurements (bottom).
- Cobalt atoms on Cu(111) shown.

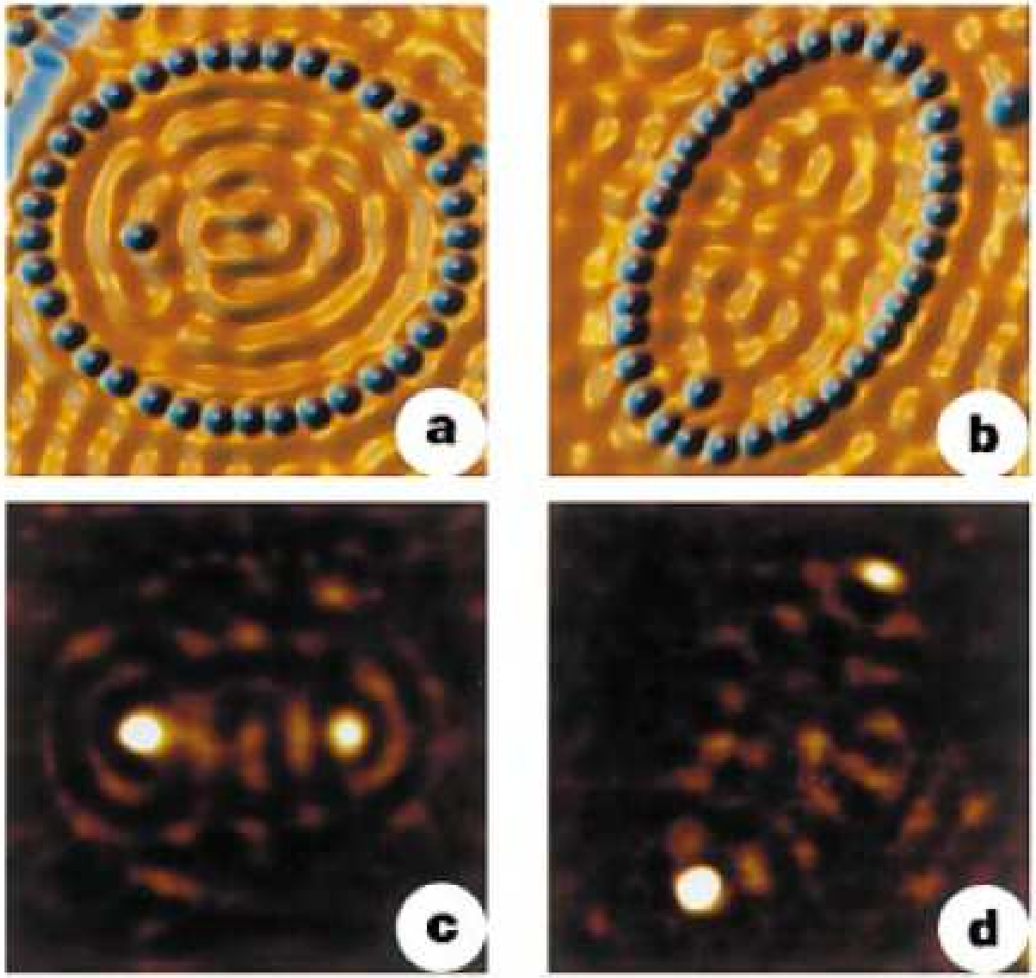


Manoharan et al., *Nature* **403** 512 (2000).

Kondo effect surfaces: STM measurements.



Manoharan et al., *Nature* **403** 512 (2000).



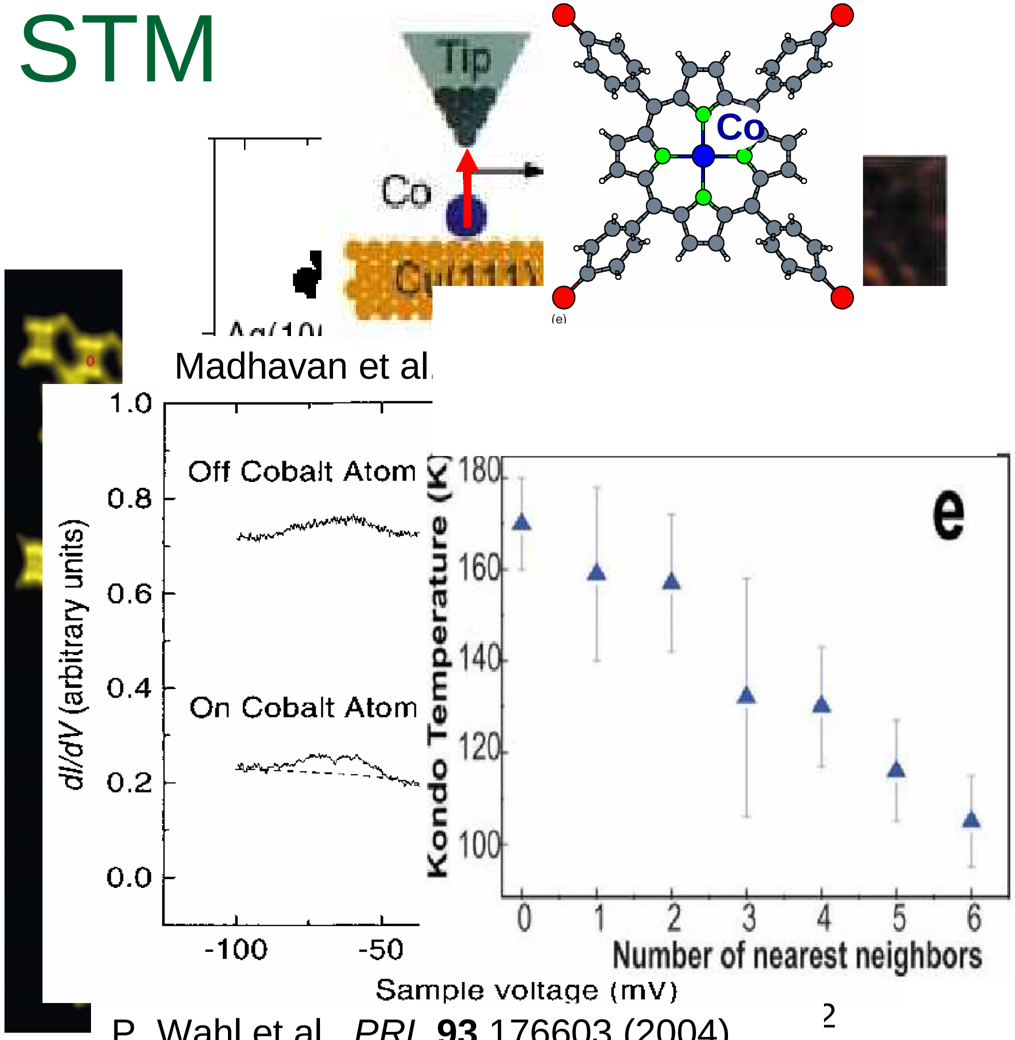
- One extra atom placed in one foci: a peak in the dIdV appears in the other focus athough NO ATOM is there! (“quantum mirage”).
- Theory: “focusing” of Kondo-scattered surface electrons*.

*Schiller and Agam, *PRL* **86** 484 (2001)..

Magnetic impurities on metallic surfaces: Kondo + STM

Kondo/Fano phenomenology quite generic:

- Early experiments: Co on Au(111)
- Co on Cu(111): “Quantum mirage”
- Co on different surfaces: Cu(100), Ag(111), Ag(100),...
- Magnetic *molecules* on Cu(111) Control of the Kondo temperature by atomic manipulation.



Madhavan et al.

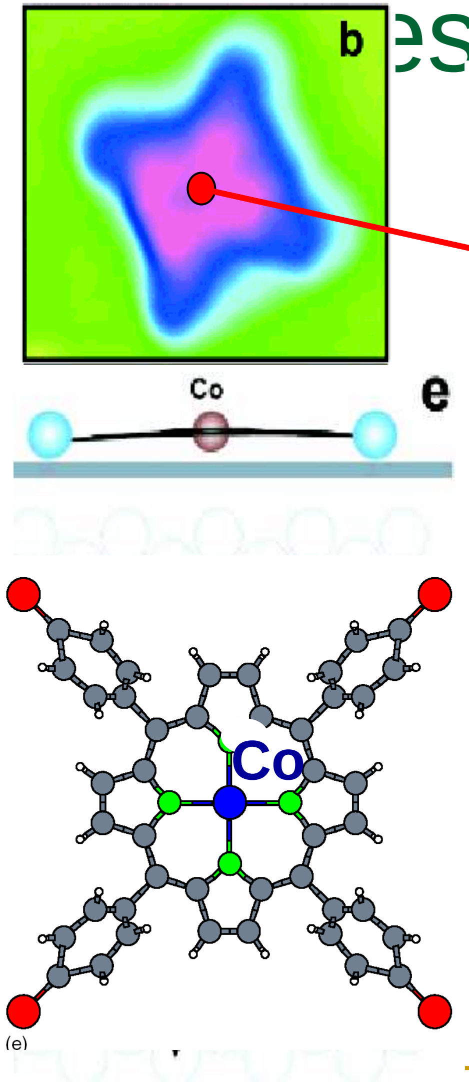
P. Wahl et al., *PRL* **93** 176603 (2004).

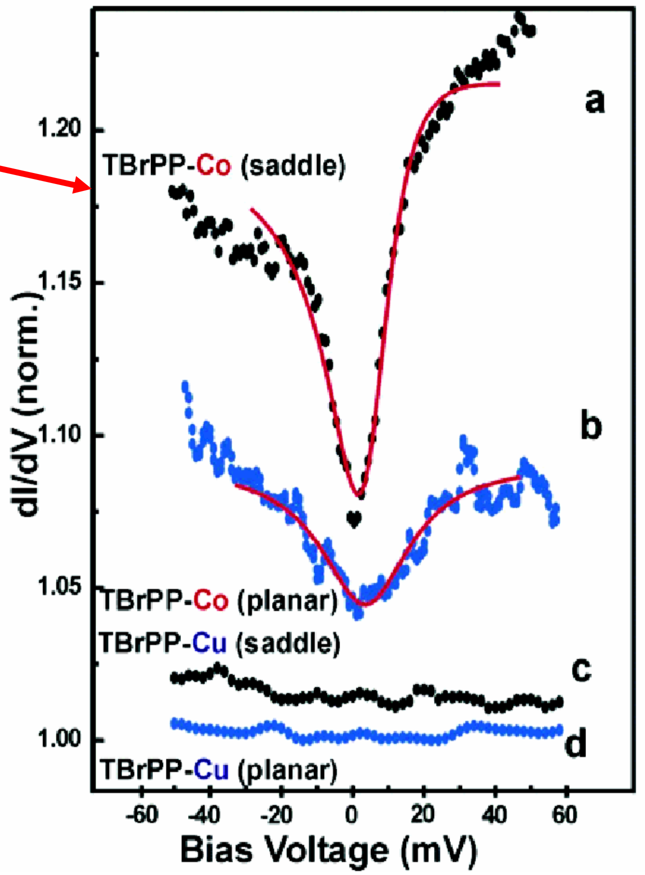
V. Iancu, A. Deshpande, Saw W. Hla *PRL* **97** 266603 (2006).

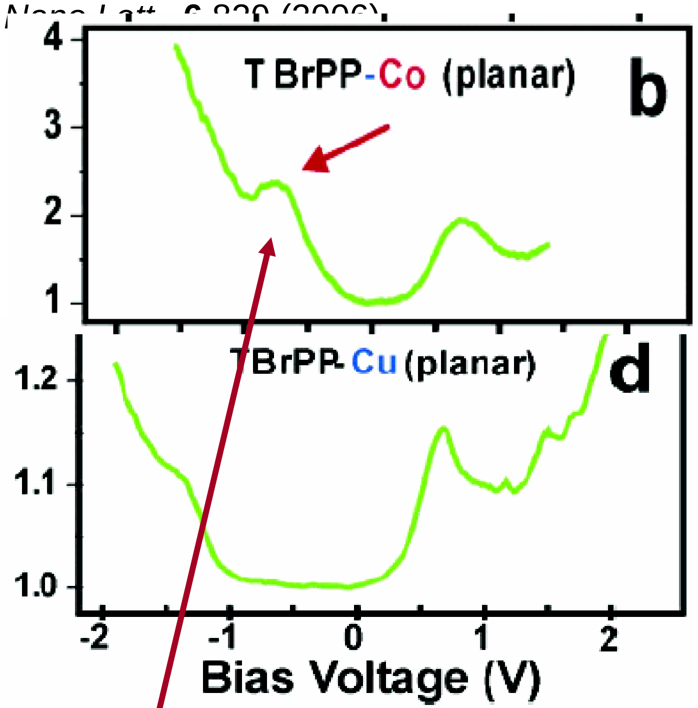
Kondo: Magnetic molecules on es

STM measurements

V. Iancu, A. Deshpande, Saw W. Hla
Nano Lett. **6** 820 (2006)







Co d_{3z2-1} level (at ~-0.7 eV

Zero-bias dip: Kondo effect. T_K~130-170K

First-principles calculations (GW): hints for a microscopic model.

Model: Anderson-like Hamiltonian

$$H = H_{\text{Molecule}} + H_{\text{Mol-Surface}}$$

$$H_{\text{Molecule}} = \sum_{\sigma} E_d \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \sum_{\sigma} E_M \hat{n}_{M\sigma}$$

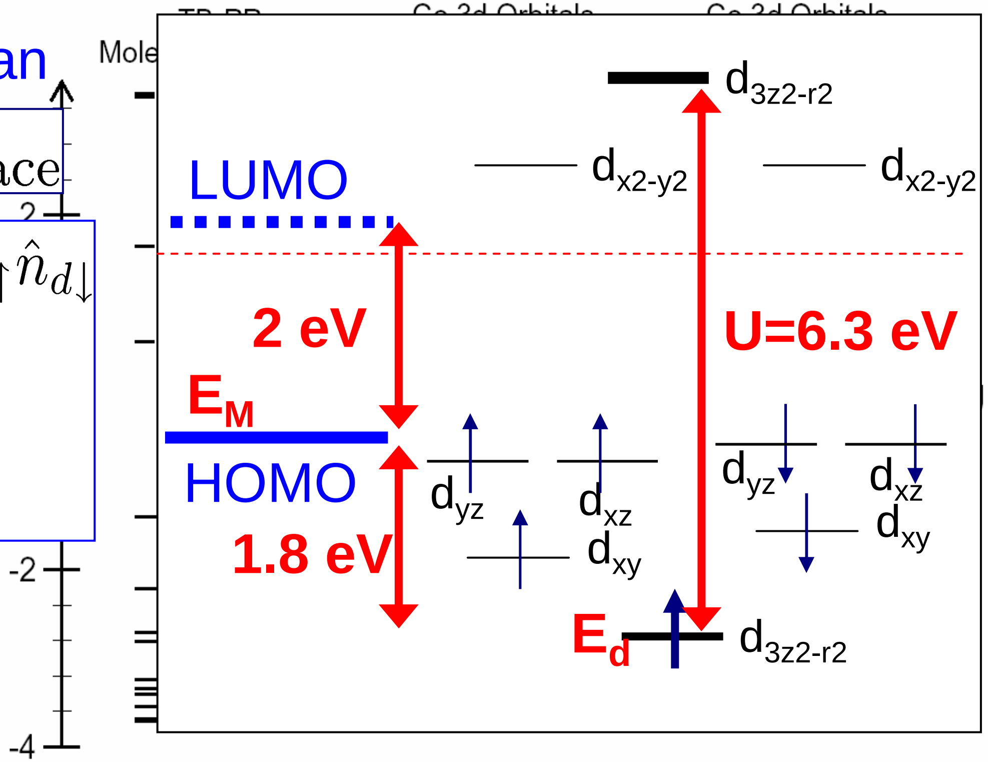
OK!

$$H_{\text{Mol-Surf}} = \sum\nolimits_{\mathbf{k},\sigma} V_{d\mathbf{k}} c^{\dagger}_{d\sigma} c_{\mathbf{k}\sigma} + \sum\nolimits_{M\mathbf{k},\sigma} V_{M\mathbf{k}} c^{\dagger}_{M\sigma} c_{\mathbf{k}\sigma} + \text{ h.c.}$$

$$V_{d(M)\mathbf{k}} = \langle \phi_{d(M)} | \hat{H} | \psi_{\mathbf{k}} \rangle$$

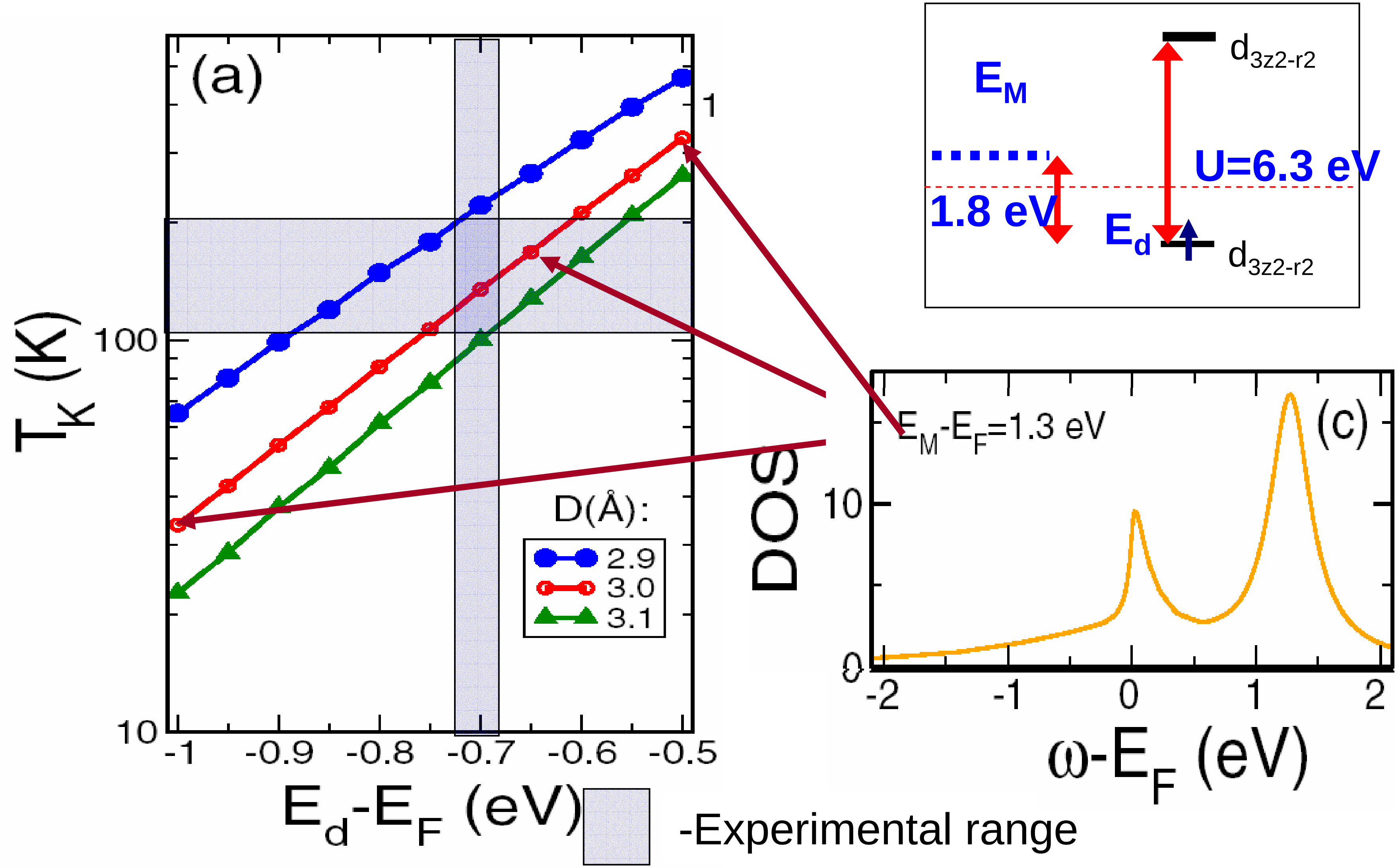
Not easy to calculate with GW!

S=1/2 model



GW: Molecular levels“Co-like” levels

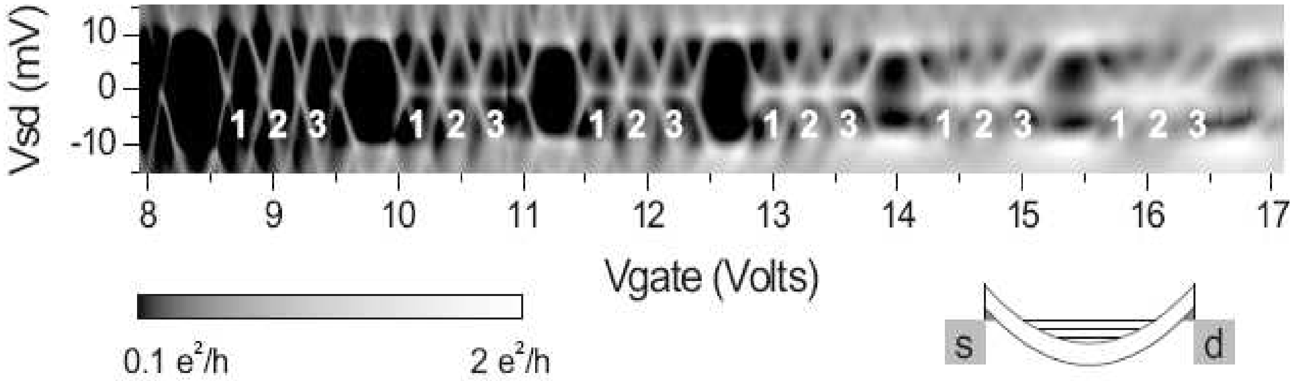
NRG calculations (Kondo temperature).



▨ -Experimental range

LDS, M Tiago, S Ulloa, F Reboredo E.
Dagotto *PRB* **80** 155443 (2009).

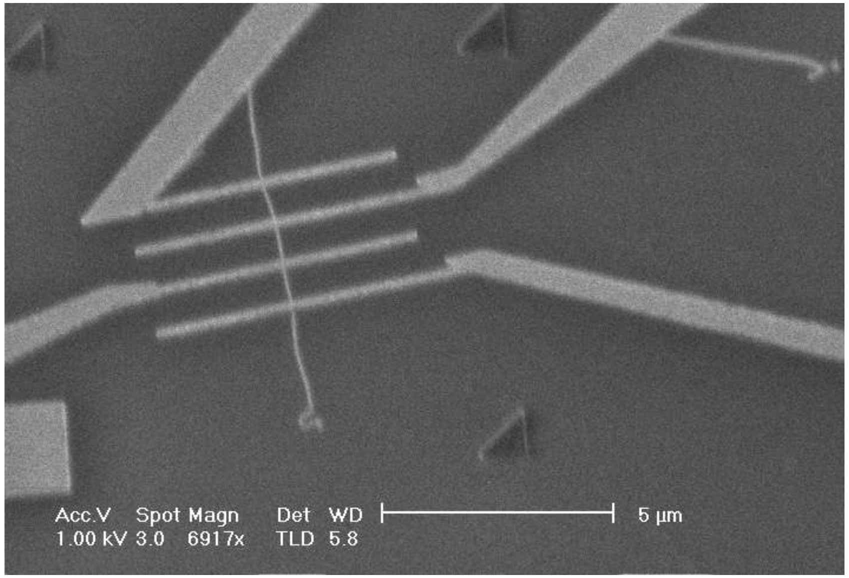
Kondo effect In Carbon nanotubes.



$\Gamma_S >> \Gamma_D$

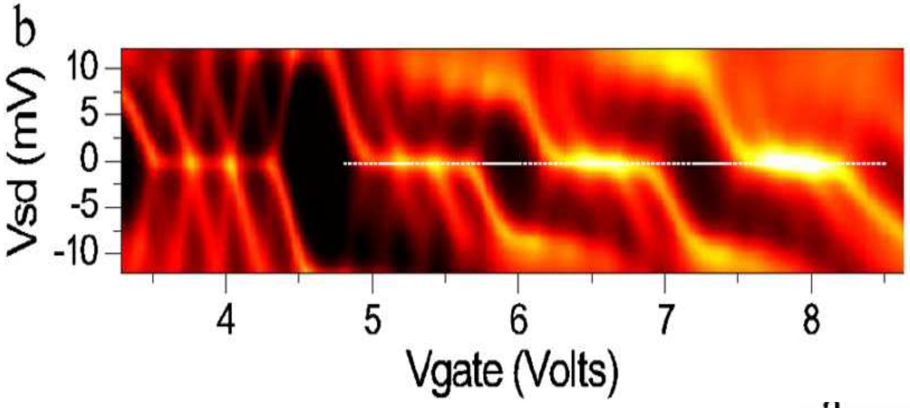
Makarovski, Zhukov, Liu, Filkenstein *PRB* **75** 241407R (2007).

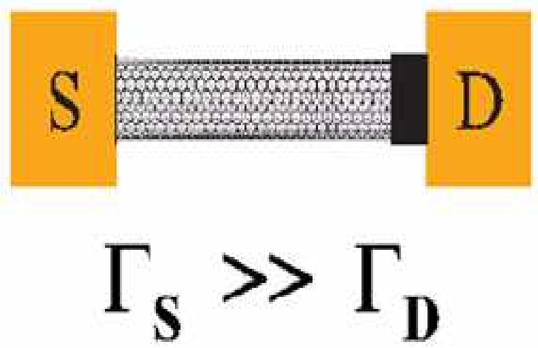
- Carbon nanotubes depsited on top of mettalic electrodes.
- Quantum dots defined *within* the carbon nanotubes.
- More structure than in quantum dots: “shell structure” due to *orbital* degeneracy.



Gleb Filkenstein’s webpage: http://www.phy.duke.edu/~gleb/

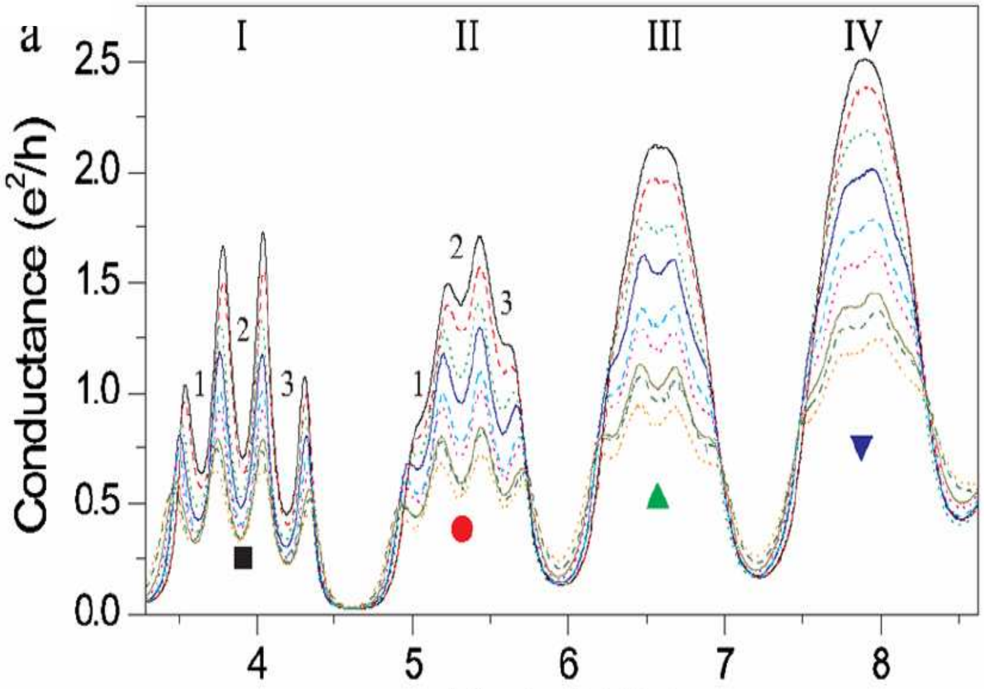
Kondo effect In Carbon nanotubes.





$\Gamma_S >> \Gamma_D$

- Temperature behavior is Kondo-like.
- Interesting *merging of the four shells* at high V_g ("SU(4)" Kondo instead of the usual SU(2) Kondo).
- NRG calculations* support that picture.



Makarovski, Liu, Filkenstein *PRL* **99** 066801 (2007).

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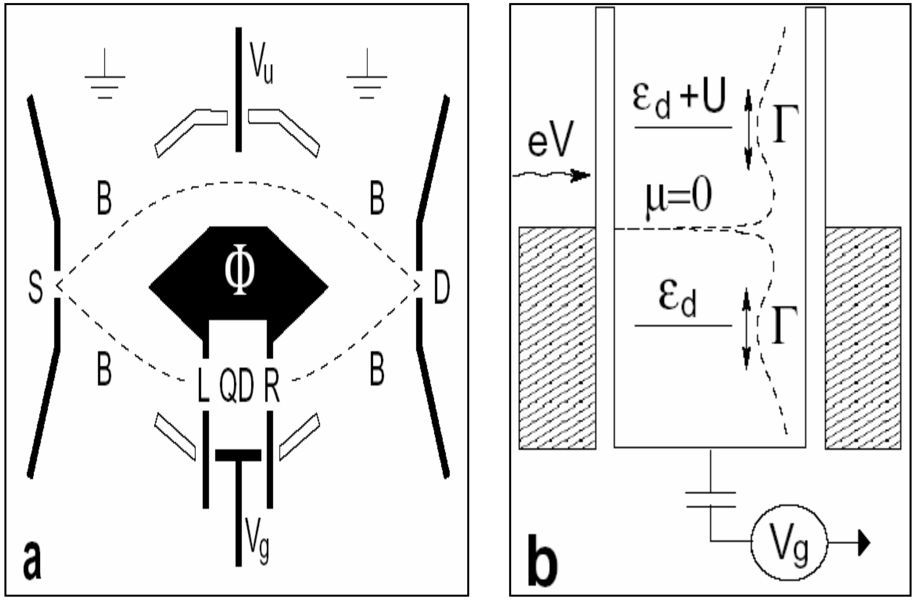
* Anders, Logan, Galpin, Filkenstein *PRL* **100** 086809 (2008).

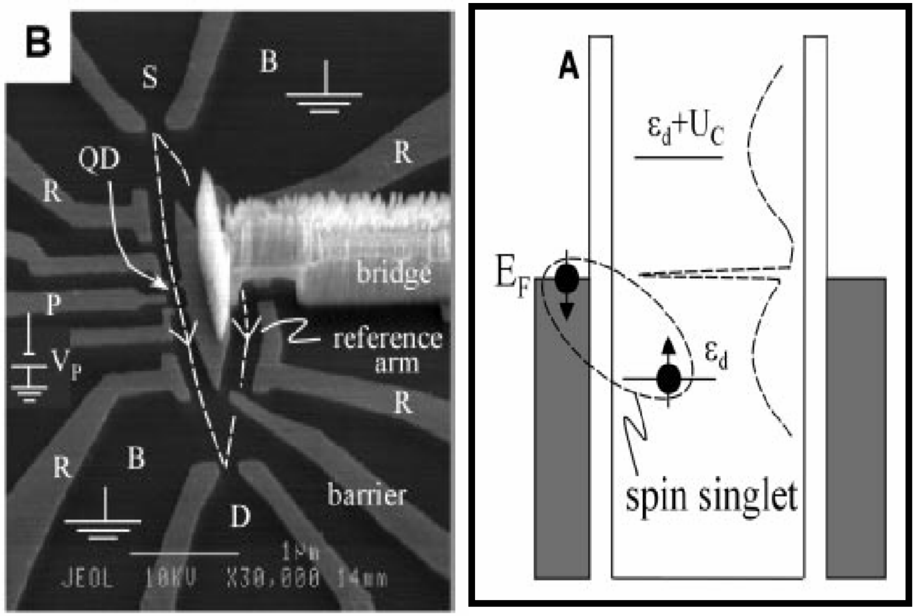
More “Theory-Experiment ballgame”

Transmission Phase Shift of a Quantum Dot with Kondo Correlations



Theory
(Gerland et al.
PRL **84** 3710 (2000))

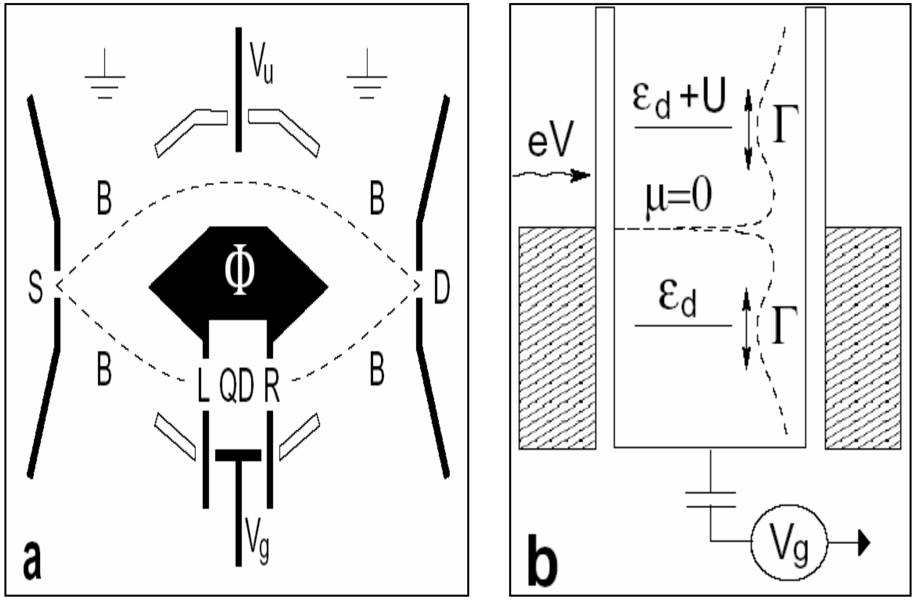
Phase Evolution in a Kondo-Correlated System



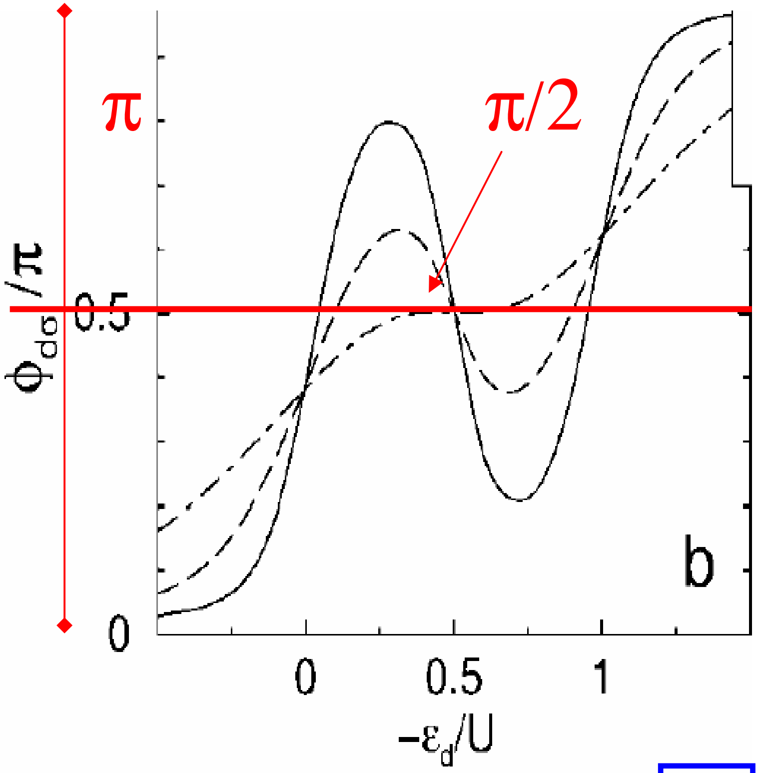
Experiment
(Ji, Heiblum et al.
Science **290** 779 (2000))

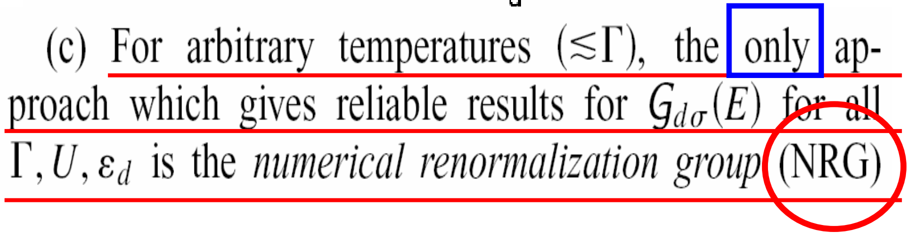
More “Theory-Experiment ballgame”

Transmission Phase Shift of a Quantum Dot with Kondo Correlations



Theory
(Gerland et al.
PRL **84** 3710 (2000))

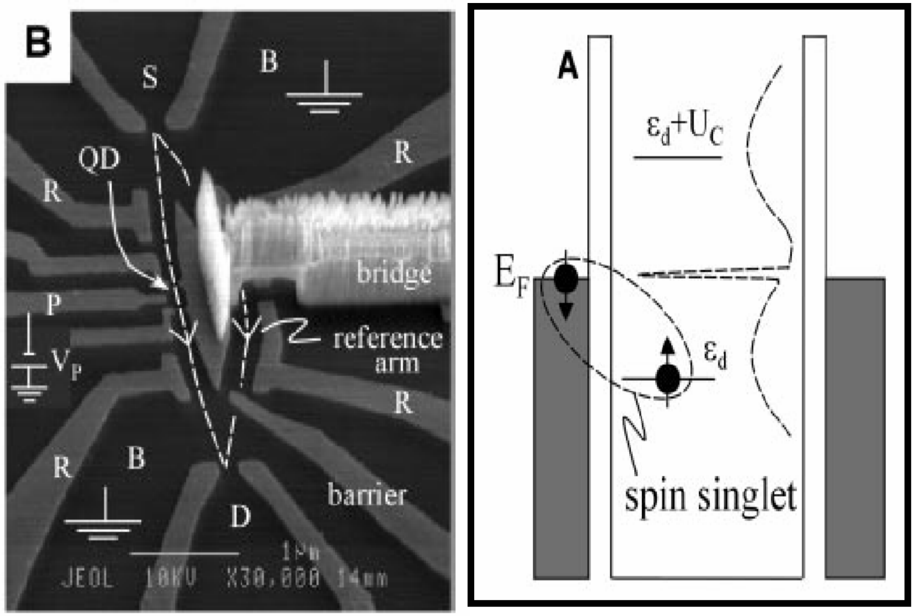



(c) For arbitrary temperatures ($\lesssim \Gamma$), the only approach which gives reliable results for $\mathcal{G}_{d\sigma}(E)$ for all Γ, U, ε_d is the *numerical renormalization group* (NRG)

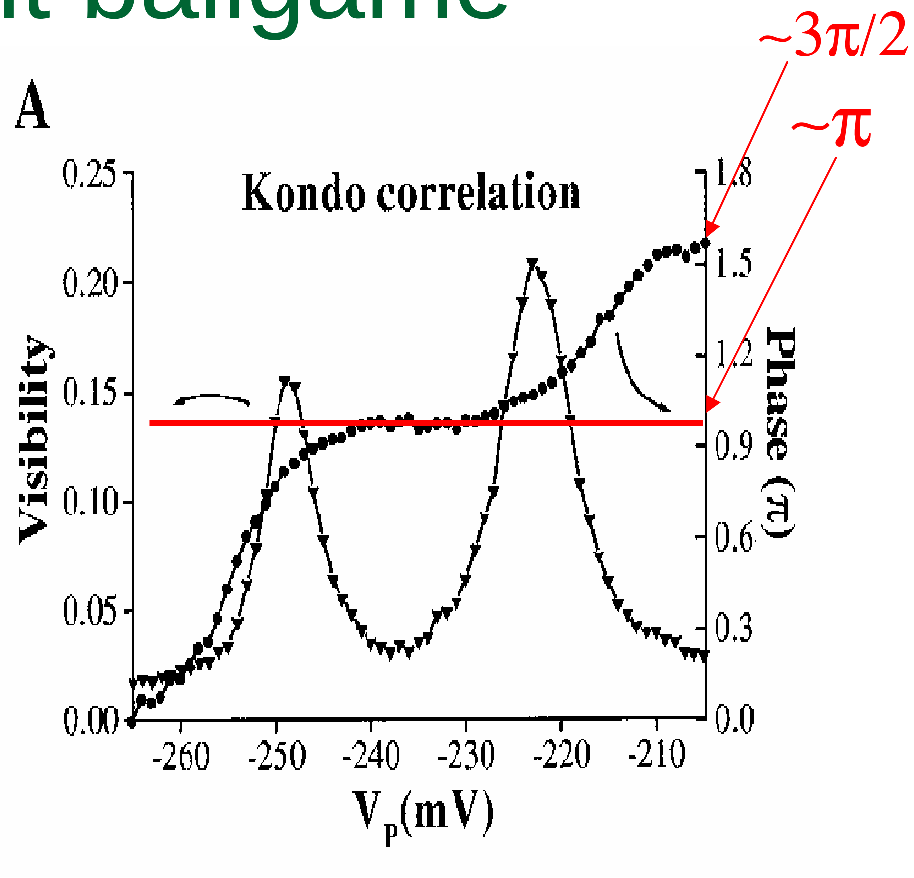
Theory (Gerland et al. PRL **84** 3710 (2000))

Theory-Experiment ballgame

Phase Evolution in a Kondo-Correlated System



Experiment
(Ji, Heiblum et al.
Science **290** 779 (2000))



Experiment (Ji et al. *Science* **290** 779 (2000))

Summary: Lectures on strongly correlated phenomena in nanostructures.

- *Lecture 1*: Quantum Dots.
- *Lecture 2*: Kondo effect/NRG.
- *Lecture 3*: Kondo effect in nanostructures.
- **Nanostructures display an array of strongly correlated phenomena:** (Kondo and 2ch Kondo effects (= non-Fermi-liquid behavior), interplay of spin and vibrational effects… quantum phase transitions, SU(4) Kondo effect).
- Opportunity: *controlled* studies of all these features.
  - *Thanks for your attention.*