

Midterm Exam

P571

October 7, 2008

SHOW ALL WORK TO GET FULL CREDIT!

IMPORTANT!!! ONLY TWO OF THE FOUR PROBLEMS WILL BE GRADED. Take a look at the 4 problems. Each of them is worth 25 points. To make sure that you have enough time to do your work you will have to turn in only 2 of the 4 problems. **If you turn more than 2 problems only the two on top will be graded and 5 points will be deducted from your grade.**

Problem 1: If $\mathbf{F} = F_x(x, y)\hat{\mathbf{x}} + F_y(x, y)\hat{\mathbf{y}}$ and $\nabla \times \mathbf{F} \neq 0$

- show that $\nabla \times \mathbf{F}$ is perpendicular to \mathbf{F} . (10 points)
- In what plane is the vector $\mathbf{F} \times (\nabla \times \mathbf{F})$? Why? (10 points)
- Find the angle between $\mathbf{F} \times (\nabla \times \mathbf{F})$ and \mathbf{F} . (5 points)

Problem 2: If T_{iklm} is a tensor antisymmetric with respect to all pairs of indices, answer the following questions:

- How many components does the tensor have if it is defined in 3-dimensional space? (6.5 points)
- How many independent components does it have in 3-dimensional space? Why? (6 points)
- How many components does the tensor have if it is defined in 4-dimensional space? (6.5 points)
- How many independent components does it have in 4-dimensional space? Why? (6 points)

Problem 3: A charge q is placed inside a grounded sphere of radius R at a distance a from the center of the sphere.

- In order to find the electrostatic potential inside the sphere, should you solve Laplace or Poisson equation? Explain. (8 points)
- Write a general expression for the potential inside the sphere and explain what boundary conditions you will use to determine the potential. (8 points)
- Find the electrostatic potential inside the sphere. (9 points)

Problem 4: An atomic electron with angular momentum L and magnetic quantum number M has a wave function

$$\psi(r, \theta, \phi) = f(r)Y_L^M(\theta, \phi).$$

- Calculate (15 points)

$$\sum_{M=-L}^L \psi(r, \theta, \phi)^* \psi(r, \theta, \phi).$$

- The expression calculated in a) represents the sum of the electron densities in a complete shell. On what variables r , θ or ϕ does this quantity depend? (10 points)