Midterm Exam #2

P571

November 5, 2009

SHOW ALL WORK TO GET FULL CREDIT!

PART I:**ONLY TWO OF THE FOUR PROBLEMS WILL BE GRADED**. Take a look at the 4 problems. Each of them is worth 25 points. To make sure that you have enough time to do your work you will have to turn in only 2 of the 4 problems. **If you turn more than 2 problems only the two on top will be graded and 5 points will be deducted from your grade.**

PART II: Take the test home and bring **ALL** the problems solved on Tuesday October 6. Your grade for the test will be the **sum of the two** parts. A perfect score is worth 150 points.

Problem 1: Consider the region defined by $0 \leq x \leq a$ and $0 \leq y \leq b$ with $a > b$ and the following boundary conditions: $\phi(0,y) = \phi(x,0) = \phi(x,b) = 0$ and $\phi(a,y) = V \sin(\pi y/b)$.

a) In order to find the electrical potential $\phi(\mathbf{r})$ inside the region, what differential equation do you have to solve? (5 points)

b) Draw the region indicating the boundary conditions and write the differential equation explicitly in terms of the coordinates. (5 points)

c) Propose a general solution and adjust the coefficients using the boundary conditions.(10 points)

d) Provide the value of the potential at the center of the region. Your results should be given in terms of V, a, and b.(5 points)

Problem 2: A spherical shell of radius a has a surface charge given by $\sigma(\theta) = \sigma_0 \cos\theta$.

a) In order to find the electrical potential produced by the charged shell in all space, what differential equation do you need to solve? Why? (5 points)

b) In terms of what functions do you expect to obtain the solution? Why? (5 points)

c) Write an expression for the solution with as few undetermined coefficients as possible. (5 points)

d) Write the boundary conditions that you will use to obtain the undetermined coefficients.(5 points)

e) Find the coefficients and provide an explicit expression for the electrical potential $\Phi(\mathbf{r})$ produced everywhere by the spherical shell. (5 points)

Problem 3: Consider the function $f(x)$ defined in the interval $-1 \leq x \leq 1$ given by $f(x) = 0$ if $-1 \leq x < a$ and $f(x) = 2$ if $a \leq x \leq 1$. Consider $a > 0$.

a) Draw the function $f(x)$ and write a formal expression for its expansion in terms of Legendre polynomials. Why is it possible to expand $f(x)$ in terms of $P_l(x)$? (4 points)

b) Find explicitly the coefficients of the first 3 terms in the expansion. (7 points)

c) Now provide an expression for the coefficient of $P_l(x)$ for $l > 0$. To confirm that your expression is correct compare with the results obtained in part b). Hint: This expression, valid for all $l > 0$, may be useful: $P_l(x) = \frac{1}{2l+1}[P'_{l+1}(x) - P'_{l-1}(x)]$. (7 points)

d) Calculate $\int_{-1}^{1}[f(x)]^2 dx$ in terms of the coefficients of the expansion that you found in c).(7 points) If you did not find the coefficients in c) you can give the expression in terms of undetermined coefficients a_l and you'll get 6 points.

Problem 4: Consider a charge q at $(a,0,0)$, another charge q at $(0,a,0)$, a charge $-q$ at $(-a,0,0)$ and another charge $-q$ at $(0,-a,0)$ where the positions are given in Cartesian coordinates (x,y,z).

a) Draw the array of charges and write an expression for the electrical potential generated by all of them at a generic point $\mathbf{r}$ in space. (10 points)

b) Now write the total potential $\phi(r,\theta,\phi)$ for $r > a$ in terms of spherical harmonics.(15 points)