Midterm Exam

P571

October 5, 2010

SHOW ALL WORK TO GET FULL CREDIT!

PART I:**ONLY TWO OF THE FOUR PROBLEMS WILL BE GRADED**. Take a look at the 4 problems. Each of them is worth 25 points. To make sure that you have enough time to do your work you will have to turn in only 2 of the 4 problems. **If you turn more than 2 problems only the two on top will be graded and 5 points will be deducted from your grade.**

PART II: Take the test home and bring **ALL** the problems solved on Tuesday October 12. Your grade for the test will be the **sum of the two** parts. A perfect score is worth 150 points.

Problem 1: In 3 dimensions you know that the electric field is a vector $E^j = (E_x, E_y, E_z)$ and, upon a transformation from a cartesian system S to another cartesian system S' that arises from a rotation about the z-axis by an angle ϕ the electric field in S' is given by

$$E'^i = a^i{}_j E^j, \tag{1}$$

with

$$a^i{}_j = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

In Minkowski space, on the other hand, the electric field is part of a tensor of rank 2, the strength tensor $\mathrm{F}^{\alpha\beta}$, where α and β take the values 0, 1, 2, and 3 with $x_0 = ct$, $x_1 = x$, $x_2 = y$, and $x_3 = z$ which in system S where we are measuring only an electric field $\mathbf{E}$ and there is no magnetic field,it has the form:

$$\mathrm{F}^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & 0 & 0 \\ E_y & 0 & 0 & 0 \\ E_z & 0 & 0 & 0 \end{pmatrix}. \tag{3}$$

In Minkowski space, this spacial rotation described above is given by

$$M^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

a) Obtain E'^i in S' in terms of E^j and the coefficients of the transformation matrix $a^i{}_j$ given in Eq.(2). (5 points)

b) Using the rules of tensor transformation write, in tensor notation, an expression for $\mathrm{F}'^{\gamma\delta}$, i.e., the stress tensor in the system S' in terms of $F^{\alpha\beta}$ and the coefficients of the transformation matrix $M^\mu{}_\nu$. (5 points)

c) Now, using the explicit values of the components of $\mathrm{F}^{\alpha\beta}$, provided above in Eq.(3), say what components of $\mathrm{F}'^{\gamma\delta}$ are non-zero.(5 points)

d) Provide the explicit value of the non-zero components of $\mathrm{F}'^{\gamma\delta}$. (5 points)

e) Compare your results in part (d) with the values for E'^i obtained in part (a). Comment on your result.(5 points)

Problem 2:

a) Write and prove in tensor notation (justifying all your steps) the identity given by

$$\nabla.(\mathbf{A} \times \mathbf{B}) = \mathbf{B}.(\nabla \times \mathbf{A}) - \mathbf{A}.(\nabla \times \mathbf{B})$$

(10 points)

b) Knowing that $\mathbf{A}$ and $\mathbf{B}$ are vectors show that $\nabla.(\mathbf{A} \times \mathbf{B})$ is a tensor and provide its rank.(5 points)

c) Is $\nabla.(\mathbf{A}\times\mathbf{B})$ a tensor or a pseudotensor? Why? (5 points)
d) Under what conditions is $\nabla.(\mathbf{A}\times\mathbf{B}) = 0$? (5 points)

Problem 3: Evaluate the following integral

$$\int_{-\infty}^{\infty}(x^2+18)\delta(3x^2+x-2)dx.$$

Hint: Find a simpler expression for the δ function. (25 points).

Problem 4: Consider the expression

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}). \tag{1}$$

a) Write Eq.(1) in tensor notation and indicate the rank of the tensor represented by it. (4 points)
b) Assume that **A**, **B**, and **C** are all vectors. Is the result of Eq.(1) a tensor or a pseudotensor? Justify your reply.(7 points)
c) Assume that **A**, **B**, and **C** are all pseudovectors. Is the result of Eq.(1) a tensor or a pseudotensor? Justify your reply.(7 points)
d) Assume that **A**, and **B** are pesudovectors and **C** is a vector. Is the result of Eq.(1) a tensor or a pseudotensor? Justify your reply.(7 points)