Homework #12

Problem 2 - 20.2.2:

a) Since the function is symmetric we calculate the cosine Fourier transform

$$g_c(\omega) = \sqrt{\frac{2}{\pi}} \int_0^1 cos(\omega x) dx = \sqrt{\frac{2}{\pi}} \frac{sin\omega}{\omega}. \tag{1}$$

b) The inverse cosine transform is given by

$$f_c(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty g_c(\omega) cos(\omega x) d\omega. \tag{2}$$

Plugging (2) in (3) we obtain:

$$f_c(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{sin(\omega) cos(\omega x)}{\omega} d\omega. \tag{3}$$

c) In an integral table we find that

$$\int_0^\infty \frac{sin(p\omega) cos(q\omega)}{\omega} d\omega = \begin{cases} \frac{\pi}{2} & \text{if } p > q > 0, \\ 0 & \text{if } 0 < p < q, \\ \frac{\pi}{4} & \text{if } p = q > 0. \end{cases} \tag{4}$$

In our case $p = 1$ and $q = x$ then

$$\int_0^\infty \frac{sin(\omega) cos(\omega x)}{\omega} d\omega = \begin{cases} \frac{\pi}{2} & \text{if } 1 > |x|, \\ 0 & \text{if } 1 < |x|, \\ \frac{\pi}{4} & \text{if } |x| = 1. \end{cases} \tag{5}$$