Homework #12

Problem 4 - 20.2.7:

We know that

$$\delta(t-x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega(t-x)}d\omega. \tag{1}$$

To represent the δ with a cosine expansion we need to assume that it is an even function, i.e., that the function that we are representing is actually

$$f(x)=\delta(t-x)+\delta(t+x). \tag{2}$$

From the representation for the δ given in (1) we know that

$$\delta(t+x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega(t+x)}d\omega. \tag{3}$$

Then plugging (1) and (3) into (2) we obtain:

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(e^{i\omega(t-x)}+e^{i\omega(t+x)})d\omega=$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}(cos\omega(t+x)+isin\omega(t+x)+cos\omega(t-x)+isin\omega(t-x))d\omega=$$

$$\frac{2}{2\pi}\int_{-\infty}^{\infty}(cos\omega tcos\omega x+isin\omega tcos\omega x)d\omega. \tag{4}$$

Now we notice that the first term in (4) is even while the second term is odd and thus its integral will vanish and we are left with:

$$\delta(t-x)=\frac{2}{\pi}\int_{0}^{\infty}cos\omega tcos\omega xd\omega. \tag{5}$$

For the sine expansion we need to assume that our function is odd so now we are going to expand:

$$f(x)=\delta(t-x)-\delta(t+x). \tag{6}$$

Then plugging (1) and (3) into (5) we obtain:

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(e^{i\omega(t-x)}-e^{i\omega(t+x)})d\omega=$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}(cos\omega(t+x)+isin\omega(t+x)-cos\omega(t-x)+isin\omega(t-x))d\omega=$$

$$\frac{2}{2\pi}\int_{-\infty}^{\infty}(sin\omega tsin\omega x+icos\omega tsin\omega x)d\omega. \tag{7}$$

Now we notice that the first term in (7) is even while the second term is odd and thus its integral will vanish and we are left with:

$$\delta(t-x)=\frac{2}{\pi}\int_{0}^{\infty}sin\omega tsin\omega xd\omega. \tag{8}$$