Homework #12

Problem 5 - 20.2.8:

We know that

$$A(t) = A_0 e^{-\frac{\omega_0 t}{2Q}} e^{-i\omega_0 t}, \tag{1}$$

for $t > 0$ and $A(t) = 0$ for $t < 0$. We need to find

$$|a(\omega)|^2 = a^*(\omega)a(\omega). \tag{2}$$

Then we are going to evaluate the FT of A:

$$a(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(t)e^{i\omega t} dt =$$

$$\frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} A_0 e^{-\frac{\omega_0 t}{2Q}} e^{i(\omega-\omega_0)} t dt =$$

$$\frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} A_0 e^{-\frac{\omega_0 t}{2Q}} (cos(\omega-\omega_0)t + isin(\omega-\omega_0)t) dt =$$

$$\frac{A_0\omega_0}{\sqrt{2\pi}2Q} \frac{1}{\frac{\omega_0^2}{4Q^2} + (\omega-\omega_0)^2} + i\frac{A_0}{\sqrt{2\pi}} \frac{(\omega-\omega_0)}{\frac{\omega_0^2}{4Q^2} + (\omega-\omega_0)^2} \tag{3}$$

Plugging (3) in (2) we obtain:

$$|a(\omega)|^2 = \frac{A_0^2}{2\pi[\frac{\omega_0}{2Q}^2 + (\omega-\omega_0)^2]}. \tag{4}$$