

Homework #12

Problem 6 - 20.3.6:

We need to solve the equation

$$-D \frac{d^2 \phi(x)}{dx^2} + K^2 D \phi(x) = Q \delta(x). \quad (1)$$

FT into k-space we obtain:

$$\frac{-D}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{d^2}{dx^2} (\phi_k e^{-ikx}) dk + \frac{K^2 D}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi_k e^{-ikx} dk = \frac{Q}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} dk. \quad (2)$$

For each k we have the equation:

$$-Dk^2 \phi_k + K^2 D \phi_k = \frac{Q}{\sqrt{2\pi}}. \quad (3)$$

Then

$$\phi_k = \frac{Q}{\sqrt{2\pi} D (K^2 + k^2)}. \quad (4)$$

Antitransforming (4) we obtain:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi_k e^{-ikx} dk = \frac{Q e^{-|Kx|}}{2DK}. \quad (5)$$