Homework #2

Problem 4 - 3.5.2:

$$\phi = x^2 + y^2 + z^2 = r^2 = 3. \tag{1}$$

a) Since $\nabla\phi \perp S$ where S is the surface defined by (1) then

$$\hat{\mathbf{n}} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{r}. \tag{2}$$

If $(x, y, z) = (1, 1, 1)$ then

$$\hat{\mathbf{n}} = \frac{\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}}{\sqrt{3}}. \tag{3}$$

b) Any point (x, y, z) on the tangent plane through (x_0, y_0, z_0) satisfies that

$$(x - x_0, y - y_0, z - z_0).\hat{\mathbf{n}} = 0. \tag{4}$$

Using for the normal the value found in (3) and knowing that $(x_0, y_0, z_0) = (1, 1, 1)$ (4) becomes

$$x + y + z - 3 = 0, \tag{5}$$

then, the equation that desfines the tangent plane is $x + y + z = 3$.