## Problem 4-3.5.2:

$$
\begin{equation*}
\phi=x^{2}+y^{2}+z^{2}=r^{2}=3 \tag{1}
\end{equation*}
$$

a) Since $\nabla \phi \perp S$ where $S$ is the surface defined by (1) then

$$
\begin{equation*}
\hat{\mathbf{n}}=\frac{\nabla \phi}{|\nabla \phi|}=\frac{x \hat{\mathbf{x}}+y \hat{\mathbf{y}}+z \hat{\mathbf{z}}}{r} \tag{2}
\end{equation*}
$$

If $(x, y, z)=(1,1,1)$ then

$$
\begin{equation*}
\hat{\mathbf{n}}=\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}}{\sqrt{3}} \tag{3}
\end{equation*}
$$

b) Any point $(x, y, z)$ on the tangent plane through $\left(x_{0}, y_{0}, z_{0}\right)$ satisfies that

$$
\begin{equation*}
\left(x-x_{0}, y-y_{0}, z-z_{0}\right) \cdot \hat{\mathbf{n}}=0 \tag{4}
\end{equation*}
$$

Using for the normal the value found in (3) and knowing that $\left(x_{0}, y_{0}, z_{0}\right)=(1,1,1)(4)$ becomes

$$
\begin{equation*}
x+y+z-3=0 \tag{5}
\end{equation*}
$$

then, the equation that desfines the tangent plane is $x+y+z=3$.

