## Homework \#5

## Problem 5:

We are told that the phase of the plane wave given by

$$
\begin{equation*}
\mathbf{k . r}-\omega t, \tag{1}
\end{equation*}
$$

is a scalar in Minkowski space and we know that

$$
\begin{equation*}
x^{\mu}=\left(c t, x^{1}, x^{2}, x^{3}\right) \tag{2}
\end{equation*}
$$

is a prototype 4 -vector. We need to show that

$$
\begin{equation*}
k^{\mu}=\left(\frac{\omega}{c}, k^{1}, k^{2}, k^{3}\right) \tag{3}
\end{equation*}
$$

is a 4 -vector. Let us try to contract (2) with (3). Using that

$$
\begin{equation*}
g^{\mu \nu}=(1,-1,-1,-1) \tag{4}
\end{equation*}
$$

we obtain

$$
\begin{equation*}
k_{\mu}=g^{\mu \nu} k_{\nu}=\left(\frac{\omega}{c},-k_{1},-k_{2},-k_{3}\right) \tag{5}
\end{equation*}
$$

Then

$$
\begin{equation*}
k_{\mu} x^{\mu}=\omega t .-\mathbf{k . r} . \tag{6}
\end{equation*}
$$

Since the result is a known scalar and we know that $x^{\mu}$ is a 4 -vector, the quotient rule indicates that $k^{\mu}$ is also a 4-vector in Minkowski space.

