Homework #5

Problem 5:

We are told that the phase of the plane wave given by

$$\mathbf{k}.\mathbf{r} - \omega t, \tag{1}$$

is a scalar in Minkowski space and we know that

$$x^\mu = (ct, x^1, x^2, x^3), \tag{2}$$

is a prototype 4-vector. We need to show that

$$k^\mu = (\frac{\omega}{c}, k^1, k^2, k^3), \tag{3}$$

is a 4-vector. Let us try to contract (2) with (3). Using that

$$g^{\mu\nu} = (1, -1, -1, -1) \tag{4}$$

we obtain

$$k_\mu = g^{\mu\nu} k_\nu = (\frac{\omega}{c}, -k_1, -k_2, -k_3). \tag{5}$$

Then

$$k_\mu x^\mu = \omega t. - \mathbf{k}.\mathbf{r}. \tag{6}$$

Since the result is a known scalar and we know that x^μ is a 4-vector, the quotient rule indicates that k^μ is also a 4-vector in Minkowski space.