Homework #5

Problem 3 - 4.1.10:

In S

$$K_{ij}A^iB^j = V. \tag{1}$$

In S'

$$K'_{ij}A'^iB'^j = V, \tag{2}$$

where we have used that $V' = V$ since V is a scalar.

Let's see that K_{ij} transforms like a covariant rank 2 tensor:

$$V = K_{ij}A^iB^j = K_{ij}\frac{\partial x^i}{\partial x'^r}A'^r\frac{\partial x^j}{\partial x'^s}B'^s = K'_{rs}A'^rB'^s. \tag{3}$$

From (3) we obtain:

$$(K_{ij}\frac{\partial x^i}{\partial x'^r}\frac{\partial x^j}{\partial x'^s} - K'_{rs})A'^rB'^s = 0 \tag{4}$$

Eq.(4) is satisfied if the expression in parenthesis vanishes which means that

$$K'_{rs} = \frac{\partial x^i}{\partial x'^r}\frac{\partial x^j}{\partial x'^s}K_{ij}, \tag{5}$$

which means that K_{ij} transforms like a covariant rank 2 tensor.