## Problem 4:

We want to write the continuity equation

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\nabla . \mathbf{J}=0 \tag{1}
\end{equation*}
$$

in covariant form knowing that $J^{\mu}=(c \rho, \mathbf{J})$ is a 4-vector.
We know that the covariant derivative is given by

$$
\begin{equation*}
\partial_{\alpha}=\left(\frac{\partial}{c \partial t}, \nabla\right) . \tag{2}
\end{equation*}
$$

We see that

$$
\begin{equation*}
\partial_{\alpha} J^{\alpha}=\frac{\partial \rho}{\partial t}+\nabla . \mathbf{J} . \tag{3}
\end{equation*}
$$

Then, the covariant form of Eq.(1) is

$$
\begin{equation*}
\partial_{\alpha} J^{\alpha}=0 \tag{4}
\end{equation*}
$$

