Problem 4:

We want to write the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \mathbf{J} = 0, \tag{1}$$

in covariant form knowing that $J^{\mu} = (c\rho, \mathbf{J})$ is a 4-vector. We know that the covariant derivative is given by

$$\partial_{\alpha} = \left(\frac{\partial}{c\partial t}, \nabla\right). \tag{2}$$

We see that

$$\partial_{\alpha}J^{\alpha} = \frac{\partial\rho}{\partial t} + \nabla \mathbf{J}.$$
(3)

Then, the covariant form of Eq.(1) is

$$\partial_{\alpha}J^{\alpha} = 0. \tag{4}$$