Problem 6:

Let's consider how V transforms from S to S'. First let's write an expression in terms of the vectors' components:

$$\mathbf{V}_n = [\mathbf{A} \times (\mathbf{B} \times \mathbf{C})]_n = \epsilon_{nip} A_i (\mathbf{B} \times \mathbf{C})_p = \epsilon_{nip} A_i [\epsilon_{pjk} B_j C_k] =$$

$$\epsilon_{nip}\epsilon_{pjk} A_i B_j C_k = \epsilon_{pni}\epsilon_{pjk} A_i B_j C_k = (\delta_{nj}\delta_{ik} - \delta_{nk}\delta_{ij}) A_i B_j C_k = B_n (A_i C_i) - C_n (A_i B_i) \quad (1)$$

Then

$$V_n = B_n (A^i C_i) - C_n (A^i B_i). \quad (2)$$

Now let's consider the 3 cases:

a)

$$V_n' = B_n' (A'^i C_i') - C_n' (A'^i B_i') = a_n^j B_j (a_k^i A^k a_i^l C_l) - a_n^j C_j (a_k^i A^k a_i^l B_l) = a_n^j a_k^i a_i^l (B_j A^k C_l - C_j A^k B_l) = V_n. \quad (3)$$

Since there are no $|a|$ we see that $\mathbf{V}$ transforms a a tensor.

b)

$$V_n' = B_n' (A'^i C_i') - C_n' (A'^i B_i') = |a| a_n^j B_j (|a| a_k^i A^k |a| a_i^l C_l) - |a| a_n^j C_j (|a| a_k^i A^k |a| a_i^l B_l) =$$

$$|a|^3 a_n^j a_k^i a_i^l (B_j A^k C_l - C_j A^k B_l) = |a| V_n. \quad (3)$$

Since there is a $|a|$ we see that $\mathbf{V}$ transforms as a pseudotensor.

a)

$$V_n' = B_n' (A'^i C_i') - C_n' (A'^i B_i') = |a| a_n^j B_j (a_k^i A^k |a| a_i^l C_l) - |a| a_n^j C_j (a_k^i A^k |a| a_i^l B_l) =$$

$$|a|^2 a_n^j a_k^i a_i^l (B_j A^k C_l - C_j A^k B_l) = V_n. \quad (3)$$

Since there are two factors of $|a|$ their product is always +1 and we see that $\mathbf{V}$ transforms as an tensor.