Homework \#8

Problem 2-7.2.2:

We need to find $f(s)$ that solves the differential equation

$$
\begin{equation*}
\left(s^{2}+1\right) f^{\prime}(s)+s f(s)=0 \tag{1}
\end{equation*}
$$

From (1) we obtain:

$$
\begin{equation*}
f^{\prime}(s)=\frac{d f}{d s}=-\frac{s f(s)}{s^{2}+1} \tag{2}
\end{equation*}
$$

We can rearrange factors and integrate both sides of the equation

$$
\begin{equation*}
\int \frac{d f}{f(s)}=-\int \frac{s}{s^{2}+1} d s \tag{3}
\end{equation*}
$$

then,

$$
\begin{equation*}
\ln f=-\frac{1}{2} \ln \left(s^{2}+1\right)+\ln C \tag{4}
\end{equation*}
$$

where $\ln C$ is an integration constant. Exponentiating both sides we obtain:

$$
\begin{equation*}
f(s)=\frac{C}{\sqrt{s^{2}+1}} \tag{5}
\end{equation*}
$$

