## Homework \#8

Problem 3-7.2.12:

The differential equation that describes the falling body is

$$
\begin{equation*}
m \frac{d v}{d t}=m g-b v \tag{1}
\end{equation*}
$$

we have to find $v(t)$ knowing that $v(0)=0$. Let work with (1):

$$
\begin{equation*}
\frac{d v}{d t}=g-\frac{b v}{m} \tag{2}
\end{equation*}
$$

Now put all the terms that depend on $v$ on one side and the ones that depend on $t$ on the other and let integrate:

$$
\begin{equation*}
\int_{v(0)}^{v} \frac{d v}{g-\frac{b v}{m}}=\int_{0}^{t} d t \tag{3}
\end{equation*}
$$

Integrating we obtain:

$$
\begin{equation*}
-\left.\frac{m}{b} \ln \left(g-\frac{b v}{m}\right)\right|_{v(0)=0} ^{v}=t \tag{4}
\end{equation*}
$$

Then

$$
\begin{equation*}
-\frac{m}{b} \ln \left(g-\frac{b v}{m}\right)+\frac{m}{b} \ln (g)=t . \tag{5}
\end{equation*}
$$

Rearranging terms:

$$
\begin{equation*}
\ln \left(g-\frac{b v}{m}\right)=-\frac{b}{m} t-\ln (g) \tag{6}
\end{equation*}
$$

Exponentiating both terms we obtain

$$
\begin{equation*}
g-\frac{b v}{m}=g e^{-\frac{b}{m} t} \tag{7}
\end{equation*}
$$

Rearranging terms:

$$
\begin{equation*}
v=\frac{m g}{b}\left(1-e^{-\frac{b}{m} t}\right) . \tag{8}
\end{equation*}
$$

