Homework #8

Problem 3 - 7.2.12:

The differential equation that describes the falling body is

$$m\frac{dv}{dt} = mg - bv, \tag{1}$$

we have to find $v(t)$ knowing that $v(0) = 0$. Let work with (1):

$$\frac{dv}{dt} = g - \frac{bv}{m}. \tag{2}$$

Now put all the terms that depend on v on one side and the ones that depend on t on the other and let integrate:

$$\int_{v(0)}^{v} \frac{dv}{g - \frac{bv}{m}} = \int_0^t dt. \tag{3}$$

Integrating we obtain:

$$-\frac{m}{b} ln(g - \frac{bv}{m})|_{v(0)=0}^{v} = t \tag{4}$$

Then

$$-\frac{m}{b} ln(g - \frac{bv}{m}) + \frac{m}{b} ln(g) = t. \tag{5}$$

Rearranging terms:

$$ln(g - \frac{bv}{m}) = -\frac{b}{m}t - ln(g). \tag{6}$$

Exponentiating both terms we obtain

$$g - \frac{bv}{m} = ge^{-\frac{b}{m}t}. \tag{7}$$

Rearranging terms:

$$v = \frac{mg}{b}(1 - e^{-\frac{b}{m}t}). \tag{8}$$