Homework #8

Problem 1:

a) We know that $\epsilon_{\alpha\beta\gamma\delta}$ is a pseudotensor antisymmetric upon exchange of any pair of indices and that $\epsilon_{0123} = 1$. In order to obtain the contravariant pseudotensor we need to use the metric tensor $g^{\mu\nu}$ to raise each of the four indices:

$$\epsilon^{\rho\sigma\tau\phi} = g^{\rho\alpha} g^{\sigma\beta} g^{\tau\gamma} g^{\phi\delta} \epsilon_{\alpha\beta\gamma\delta} = det[g^{\mu\nu}] \delta_{\rho}{}^{\alpha} \delta_{\sigma}{}^{\beta} \delta_{\tau}{}^{\gamma} \delta_{\phi}{}^{\delta} \epsilon_{\alpha\beta\gamma\delta} = -\epsilon_{\rho\sigma\tau\phi}.$$

Where we have used that $g^{\mu\nu}$ is a diagonal matrix whose diagonal elements are $(1, -1, -1, -1)$ and all of them must appear in the non-vanishing terms of the transformation indicated above.

b) From part a) we see that $\epsilon^{0123} = -1$. $\epsilon_{3012} = -1$ because we need to exchange indices 3 times to go from 0123 to 3012 and $\epsilon^{3012} = 1$ because of the result found in (a).