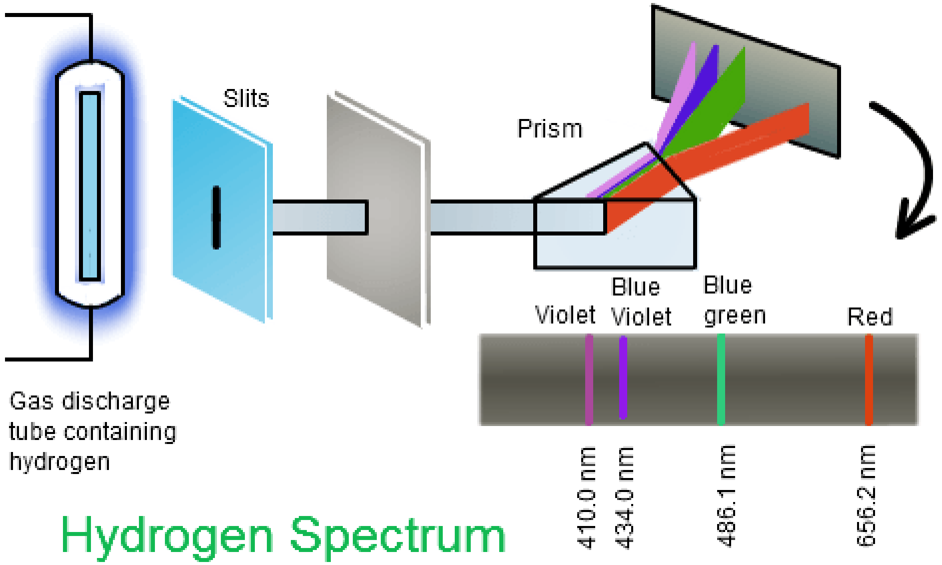
In addition, when hydrogen atoms inside a tube absorb energy, and then return the energy as light, the spectrum is found to be discrete, with just a few lines (Modern Physics class).



Classical physics has no explanation for this result at all.

We need a new physics **drastically** different from classical …

Chapter 1

Classical Mechanics must be replaced by Quantum Mechanics at short distances.

Instead of Newton's equation we will have the Schrödinger equation (Sch. Eq.)

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

V is V(x,t) in general

New fundamental constant of Nature is introduced. The Planck's constant:

$$\hbar = \frac{h}{2\pi} = 1.054572 \times 10^{-34} \text{ J s}$$

Energy x time

The "wave function" $\Psi(x,t)$.

This will be the most important quantity to calculate the whole semester.

Imaginary unit: we will deal with complex numbers.

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

(plus boundary conditions and $\Psi(x,t=0)$)

Planck's constant.

Mass of particle, typically electron.

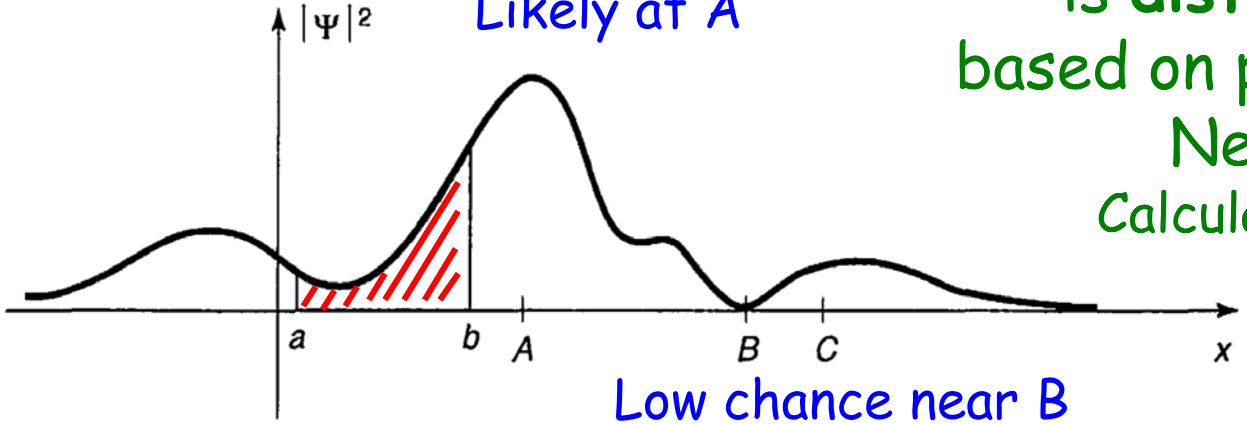
Potential $V(x,t)$ given to you (e.g. harmonic oscillator)

Check that all factors multiplying Ψ have the same units

What is the wave function? In classical mechanics we need x(t), but $\Psi(x,t)$ is a function of x and t. It is **spread**. So it cannot be the position of the electron …

1.2 Born's statistical interpretation

$$\int_a^b |\Psi(x,t)|^2\,dx = \left\{ \begin{array}{l} \text{probability of finding the particle} \\ \text{between } a \text{ and } b, \text{ at time } t. \end{array} \right\}$$



Likely at A

Low chance near B
Unlikely at C

The statistical interpretation is **disturbing**, because it's based on probabilities, unlike Newtonian mechanics.
Calculating $\Psi(x,t)$ is the best you can do ☹.

TOO early to start philosophical discussions, but following the book we will address: if I measure the position of a particle and it is at x=c, where was an instant **before**?

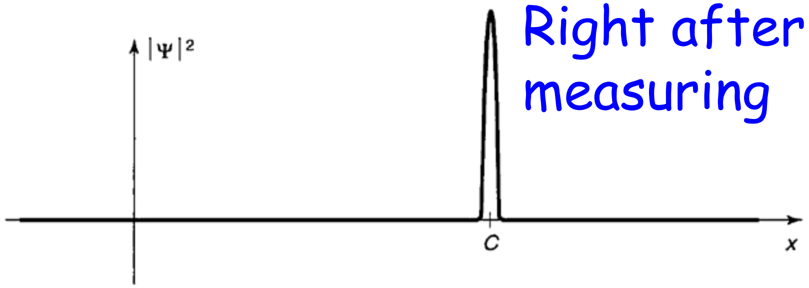
Realistic view: the particle was at x=c or very close. Thus, QM is incomplete since it did not know it. There must be a more fundamental theory (Einstein's view: QM is incomplete).

Orthodox view: the wave function is the particle. "Measuring" is something peculiar … always done with a macroscopic object (virtually everybody accepts this view, plus confirmed by Bell's argument Ch. 12).

Agnostic view: the question cannot be verified experimentally, thus it is methaphysics.

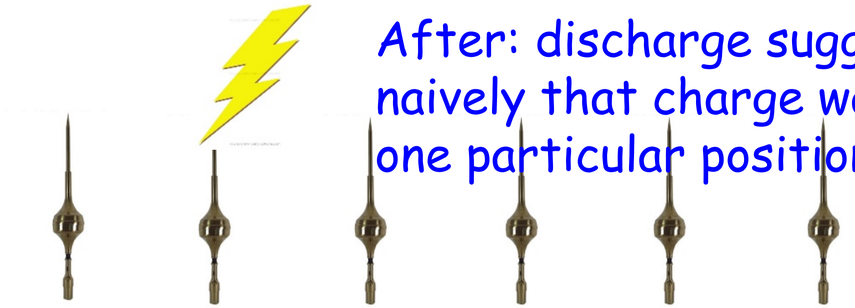
… read about collapse of the wave function … if you are brave … page 6 book. We will return to this later.

It addresses the interaction of a quantum object, the electron, with a classical and large object, the measuring device. VERY difficult. Measuring is not trivial!



Right after measuring

Before: Cloud with charge uniformly distributed



After: discharge suggests naively that charge was at one particular position

Lightning rods

Definitely QM is against "common sense".
At short distances, weird things happens!

Any one who is not shocked by quantum mechanics has not fully understood it. Niels Bohr

If you think you understand quantum mechanics, then you're not trying hard enough. Richard Feynman

Similar anti-common-sense behavior near large masses (general relativity), at huge distances (accelerating expanding universe), or at huge velocities (c is max).

The best approach is to become familiar with the formalism, understand the concepts and how to calculate, and … slowly … you get used to quantum mechanics.

1.3 Probability

$$N = \sum_{j=0}^{\infty} N(j) = 14$$

Number of people of age j

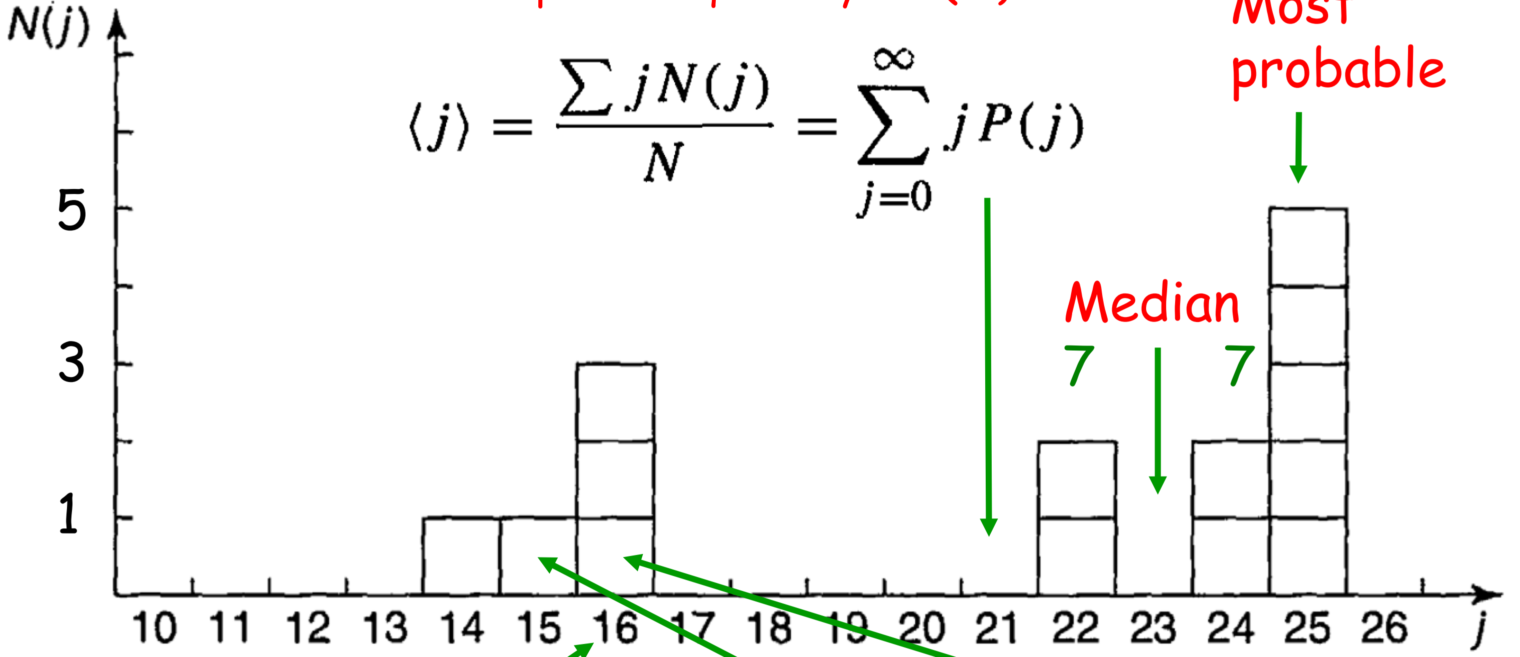
Average or mean or "expectation value" (most important quantity in QM)

$$\langle j \rangle = \frac{\sum jN(j)}{N} = \sum_{j=0}^{\infty} jP(j)$$

Most probable

Median

7 7



age

$$P(j) = \frac{N(j)}{N}$$

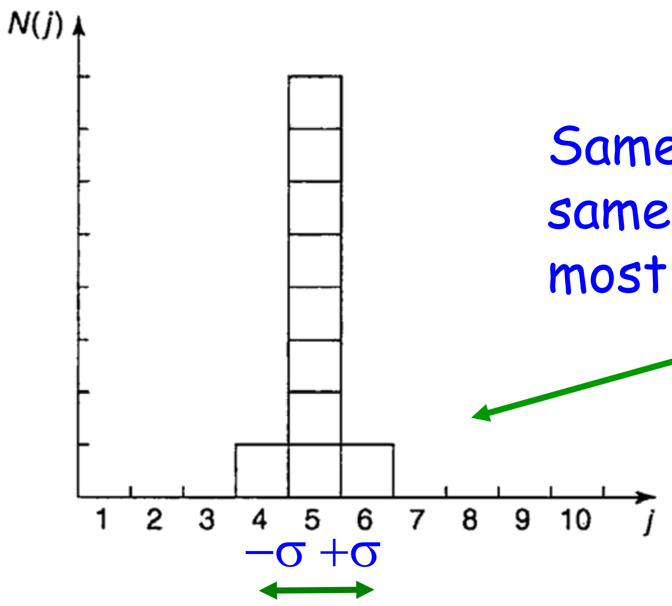
example $P(16) = 3/14$

$P(15) = 1/14$, $P(16) = 3/14$

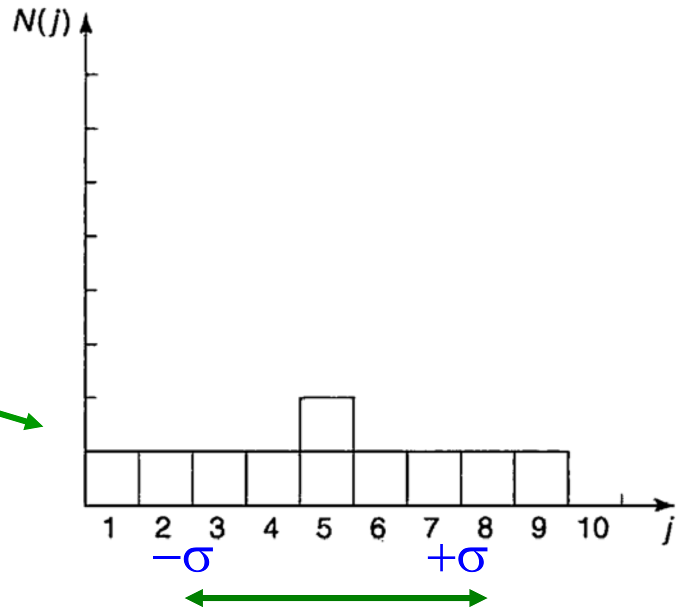
$P(15)+P(16)=4/14$

$$\sum_{j=0}^{\infty} P(j) = 1.$$

In addition to average, median, and most probable, there is another very important quantity to characterize a histogram: the standard deviation (or width).



Same average, same median, same most probable = 5



$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$$

$$\langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j)$$

Standard deviation (it will be like an "error bar" in an experiment)

When we use continuous variables (say x instead of j) then we have to talk about a probability density.

$$\left\{ \begin{array}{l} \text{probability that an individual (chosen} \\ \text{at random) lies between } x \text{ and } (x + dx) \end{array} \right\} = \rho(x)\,dx$$

$$P_{ab} = \int_a^b \rho(x)\,dx \qquad 1 = \int_{-\infty}^{+\infty} \rho(x)\,dx$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} x\rho(x)\,dx \qquad \sigma^2 \equiv \langle (\Delta x)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

So $|\Psi(x,t)|^2$ is a probability density.
Do Example 1.2, page 12 book.

1.4 Normalization

Based on the statistical interpretation of $|\psi(x,t)|^2$, its integral has to be 1 because the particle must be somewhere.

$$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2\, dx = 1$$

Thus, normalizing to 1 is just common sense.

If we are given a not normalized wave function f(x,t), we simply choose a multiplicative constant *A* such that

$$|A|^2 \int_{-\infty}^{+\infty} |f(x,t)|^2\, dx = 1$$

The normalization is up to a constant phase factor that, usually, has no physical importance.

Notes: If $\psi=0$, then the integral can never be 1.
If the integral $\int_{-\infty}^{+\infty} |\Psi(x,t)|^2\, dx$ diverges it cannot be normalized.

We will, mainly, deal with **square integrable** wave functions.