Returning to the general combination:

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_+ + b\chi_-$$

What is the probability that the spin points say along the *positive x axis*?

To answer this question, first you have to diagonalize the 2x2 Pauli matrix "x". Turns out the "eigenspinors" are:

$$\chi_+^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \left(\text{eigenvalue} + \frac{\hbar}{2}\right); \quad \chi_-^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}, \left(\text{eigenvalue} - \frac{\hbar}{2}\right)$$

The general combination …

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_+ + b\chi_-$$

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

… can now be written (check!) as:

$$\chi = \left(\frac{a+b}{\sqrt{2}}\right)\chi_+^{(x)} + \left(\frac{a-b}{\sqrt{2}}\right)\chi_-^{(x)}$$

$$(1/2)|a+b|^2$$

is the probability of measuring spin up along x.

$$(1/2)|a-b|^2$$

is the probability of measuring spin down along x.

"Spin" practice problem for Test3. Start with:

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_+ + b\chi_-$$

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Consider example 4.2 book, page 169. An arbitrary normalized (check!) spin state is given (in Test3 may not be normalized)

$$\chi = \frac{1}{\sqrt{6}} \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$$

Then, $a = (1+i)/\sqrt{6}$ and $b = 2/\sqrt{6}$

prob. of $+\hbar/2$ if S_z measured $= |(1+i)/\sqrt{6}|^2 = 1/3$

prob. of $-\hbar/2$ if S_z measured $= |2/\sqrt{6}|^2 = 2/3$

Use: (1-i)(1+i)=1+(-i)(i)=1+1=2

In the previous page we used the eigenspinors of the *z*-axis Pauli matrix: $\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix}$

Before, we only mentioned the eigenspinors of the *x* Pauli matrix: $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

To confirm this result you diagonalize a 2x2 matrix:

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \longrightarrow \begin{vmatrix}-\lambda & \hbar/2\\ \hbar/2 & -\lambda\end{vmatrix} = 0 \longrightarrow \lambda^2 = \left(\frac{\hbar}{2}\right)^2 \Rightarrow \lambda = \pm\frac{\hbar}{2}$$

x Pauli matrix

construct determinant for Sx

solve determinant (check!); find eigenvalues

Finally find eigenvectors (check!):

$$\frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \pm\frac{\hbar}{2}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \longrightarrow \begin{pmatrix}\beta\\ \alpha\end{pmatrix} = \pm\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$\beta = \pm\alpha$$

... and finally normalize to 1 (check!).

Consider now the SAME given spinor but from the perspective of the eigenspinors of the *x* Pauli matrix. You can use any basis after all.

$$\chi = \left(\frac{a+b}{\sqrt{2}}\right)\chi_+^{(x)} + \left(\frac{a-b}{\sqrt{2}}\right)\chi_-^{(x)} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$+\hbar/2$ $\quad$ $-\hbar/2$

$$a = (1+i)/\sqrt{6}$$
$$b = 2/\sqrt{6}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\left|\frac{a+b}{\sqrt{2}}\right|^2$ = prob. of getting $+\hbar/2$ if S_x is measured $= (1/2)|(3+i)/\sqrt{6}|^2 = 5/6$

Check!

$$\left|\frac{a-b}{\sqrt{2}}\right|^2$$ = prob. of getting $-\hbar/2$ if S_x is measured $= (1/2)|(-1+i)/\sqrt{6}|^2 = 1/6$

prob. of getting $+\hbar/2$ or $-\hbar/2$ if S_x is measured has to be 1.

Indeed 5/6 + 1/6 =1.

What is the "expectation value" of S_x ?

$$\frac{5}{6}\left(+\frac{\hbar}{2}\right)+\frac{1}{6}\left(-\frac{\hbar}{2}\right)=\frac{\hbar}{3}$$

Alternatively, you get the same ħ/3 as follows:

$$\mathbf{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

σ_x Pauli matrix

Check last step! $(e,f)\begin{pmatrix} c \\ d \end{pmatrix} = e.c+f.d$

$$\frac{\hbar}{2}\begin{pmatrix} 2/\sqrt{6} \\ (1+i)/\sqrt{6} \end{pmatrix}$$

$$\langle S_x \rangle = \chi^\dagger \mathbf{S}_x \chi = \left(\frac{(1-i)}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)\begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix}\begin{pmatrix} (1+i)/\sqrt{6} \\ 2/\sqrt{6} \end{pmatrix} = \frac{\hbar}{3}$$

horizontal spinor with each component conjugated

vertical spinor as given; 2x2 acts over spinor (check!)