PROBLEM 1: Consider an electron in the Hydrogen atom potential, located in the state with quantum numbers n=2, l=1, m=-1. Using the formula sheet tables, construct the explicit wave function $\Psi_{2,1,-1}(r,\theta,\phi)$.

$$R_{21} = \frac{1}{\sqrt{24}}\, a^{-3/2} \left(\frac{r}{a}\right) e^{-r/2a}$$

$$Y_1^{-1} = \sqrt{\frac{3}{8\pi}}\, \sin\theta\, e^{-i\phi}$$

$$\boxed{\psi_{21-1}(r,\theta,\phi) = \frac{1}{\sqrt{24}}\sqrt{\frac{3}{8\pi}}\, a^{-5/2}\, r\, e^{-r/2a} \sin\theta\, e^{-i\phi}}$$

PROBLEM 2: Consider an electron in the Hydrogen atom potential, located in the state with quantum numbers n=3, l=0, m=0.

(a) Using the formula sheet tables, construct the explicit wave function $\Psi_{3,0,0}(r,\theta,\phi)$.

(b) Make a crude sketch by hand, as in the book and as in one of the lectures, of the probability $|\Psi_{3,0,0}(r,\theta,\phi)|^2$, describing the nodes and the angular dependence, if any, in the plane (x,z).

(a) $\psi_{300} = R_{30}(r)\, Y_0^0(\theta,\phi)$

From table in formula sheet.

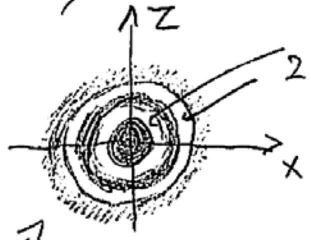
$$R_{30} = \frac{2}{\sqrt{27}}\, a^{-3/2}\left(1 - \frac{2}{3}\frac{r}{a} + \frac{2}{27}\left(\frac{r}{a}\right)^2\right) e^{-r/3a}$$

where a = Bohr radius ~ 0.5 Å

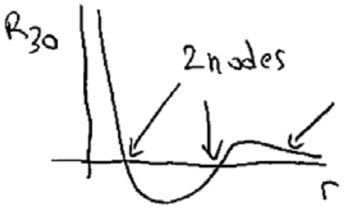
$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

Then: $$\boxed{\psi_{300}(r,\theta,\phi) = \frac{2}{\sqrt{27}}\,\frac{a^{-3/2}}{\sqrt{4\pi}}\left(1 - \frac{2}{3}\frac{r}{a} + \frac{2}{27}\left(\frac{r}{a}\right)^2\right) e^{-r/3a}}$$

(b) The polynomial is order 2, thus 2 roots. Because $l=0$, there is a strong spot at $r=0$. The shape is:



2 regions of low intensity where the nodes are.



exponential decay $e^{-r/3a}$

Sketched is $|\psi_{300}|^2$.

No angular dependence because $Y_0^0 = \frac{1}{\sqrt{4\pi}}$

positive,

close to 0.

PROBLEM 3: Consider the follow
linear combination of eigenstate
of the hydrogen atom:

Ψ=A [2 $\Psi_{4,0,0}(r,\theta,\phi)$ + i $\Psi_{2,1,1}(r,\theta,\phi$

(a) Find A so that Ψ is normalize
to 1. *Only* use the orthonormality
property of the eigenstates
$\Psi_{n,l,m}(r,\theta,\phi)$, **not** the explicit state
from formula sheet.

(b) If the Ψ given above is the sta
at time t=0, write the state at an
arbitrary time t. For the energies
simple use the notation E_n, **not** t
explicit values.

This entire problem should be do
in just a few lines. Do **not** explicit
write $\Psi_{4,0,0}(r,\theta,\phi)$ and $\Psi_{2,1,1}(r,\theta,\phi$
just keep the notation as it is.

(a)

$$\psi = A\left[2\psi_{400} + i\psi_{211}\right]$$

$$1 = |A|^2 \int |\psi|^2 d^3r = |A|^2 \Big[4\int|\psi_{400}|^2 d^3r + \int|\psi_{211}|^2 d^3r +$$

$$\boxed{(2\psi_{400}^* - i\psi_{211}^*)(2\psi_{400} + i\psi_{211})}$$

$$+2i\int \psi_{400}^* \psi_{211}\, d^3r$$

$$-2i\int \psi_{211}^* \Psi_{400}\, d^3r \Big] =$$

orthonormality

$$= |A|^2\left[4 + 1 + 2i.0 - 2i.0\right] = 5|A|^2 \rightarrow \boxed{A = \frac{1}{\sqrt{5}}}$$

(b)

$$\boxed{\psi(r,\theta,\phi,t) = \frac{1}{\sqrt{5}}\left[2\,\psi_{400}(r,\theta,\phi)e^{-iE_4t/\hbar} + i\,\psi_{211}(r,\theta,\phi)e^{-iE_2t/\hbar}\right]}$$

PROBLEM 4: Calculate the commutator [Lz,y] by the procedure in one of the HW problems and in a lecture i.e. using simpler commutators such as $[y,p_y]$.

$$[y, L_z] = [y, xp_y - yp_x] = [y, xp_y] - [y, yp_x] =$$

$$= x\underbrace{[y, p_y]}_{i\hbar} - p_x\underbrace{[y, y]}_{=0} = i\hbar x. \text{ Thus, } [L_z, y] = \boxed{-i\hbar x}$$

PROBLEM 5: Choose the correct answer. No calculations allowed or needed, but you should provide a simple *brief* reason for your choice.

(i) If we only know that a wave function is *zero* at the origin of coordinates r=0, what can we conclude with certainty about the angular momentum l? Explain.

(a) l=0 or (b) $l \neq 0$ or (c) l=2?

(ii) Of the R_{nl} functions with quantum numbers given below, which one has two nodes located at different values of r? Use formula sheet.

(a) (n,l)=(3,2) or (b) (n,l)=(3,0) or
(c) (n,l)=(4,3)

(iii) If we only know that an eigenstate of the hydrogen Hamiltonian, as those we have studied in Ch. 4, is independent of the angle ϕ, what can we conclude?

(a) m=-1 or (b) l=0 or (c) m=0

(i) $l=0$ has a wave function not zero at r
Thus (a) is incorrect.
If $\psi(r=0)$ is zero we only know that l
$l=2$ is only a special case.
Answer: (b).

(ii) To have 2 nodes at different values of r
we need a polynomial of order 2.
$(n,l)=(3,2)$ is incorrect, same $(n,l)=(4,3)$.
The only solution of those proposed with
a polynomial of second order and two
different roots is (n,l)=(3,0).
Answer: (b)

(iii) From the $e^{\pm im\phi}$ portion of wave functi
we know that no dependence with ϕ means
$m=0$. Any value of l is allowed, so long
as $m=0$. Thus "$l=0$" cannot be inferred.

Answer: (c)

(iv) Consider a hydrogen atom. Make the list of all possible states with n=4, namely write all the possible quantum numbers (l,m) allowed if n=4.

(iv) For $n=4$, the allowed values of l are $0, 1, 2, 3$

For $l=3$, $m = -3, -2, -1, 0, 1, 2, 3$ 7

For $l=2$, $m = -2, -1, 0, 1, 2$ 5

For $l=1$, $m = -1, 0, 1$ 3

For $l=0$, $m = 0$ 1

$16 = 4^2 = n^2$

(v) What is the full degeneracy of the level with quantum number n=3?

(v) n=3 means l=2,1,0: $5+3+1 = \boxed{9}$

(vi) What are the possible values of the angular momentum square, in units of ħ^2, that you could find as an outcome of an experiment if the electron is in a Hydrogen atom orbital with n=3?

(vi) $\langle L^2 \rangle / \hbar^2 \begin{cases} 2(2+1) = 6 \\ 1(1+1) = 2 \\ 0(0+1) = 0 \end{cases}$