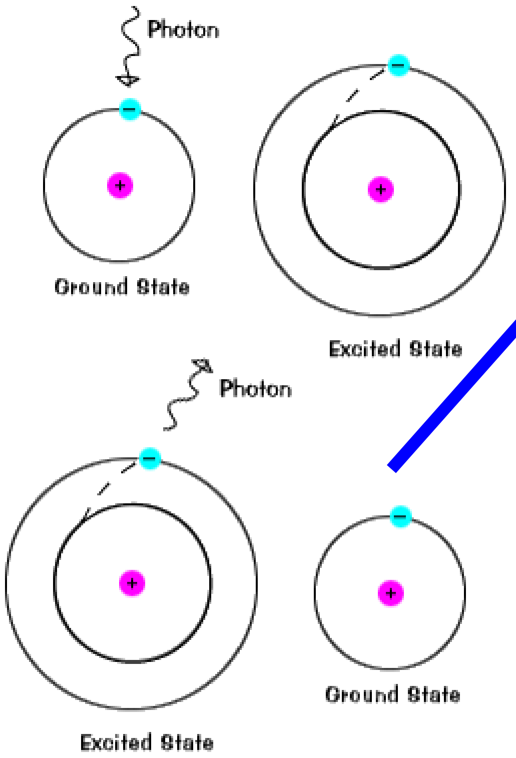
# 4.2.2: The Spectrum of Hydrogen

$n_i$=1, $n_f$=2, absorption




$n_i$=2, $n_f$=1
emission

Example, emission of photons:

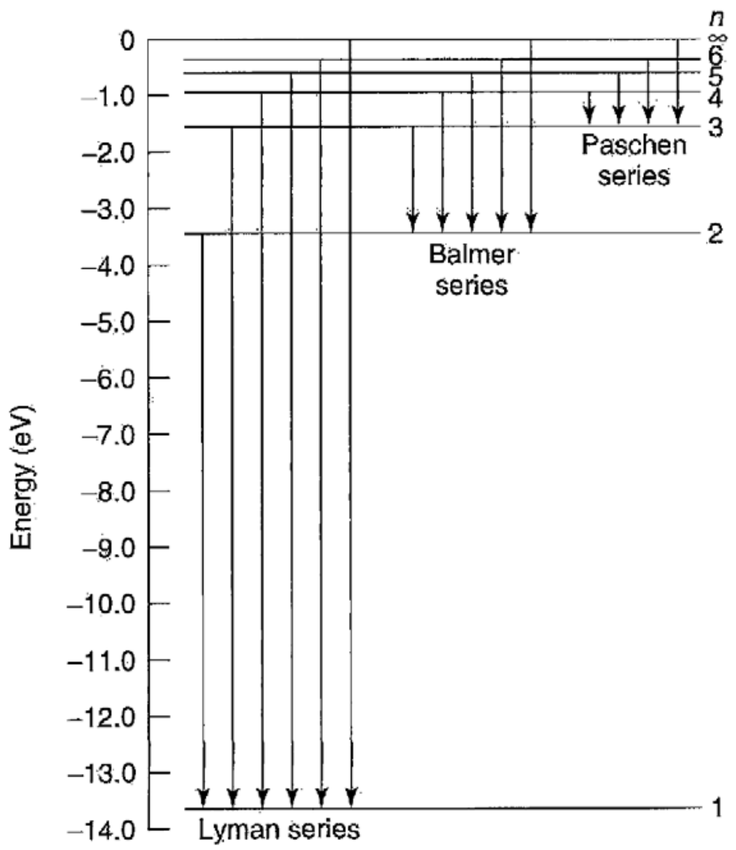
$$E_\gamma = E_i - E_f = -13.6 \text{ eV} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

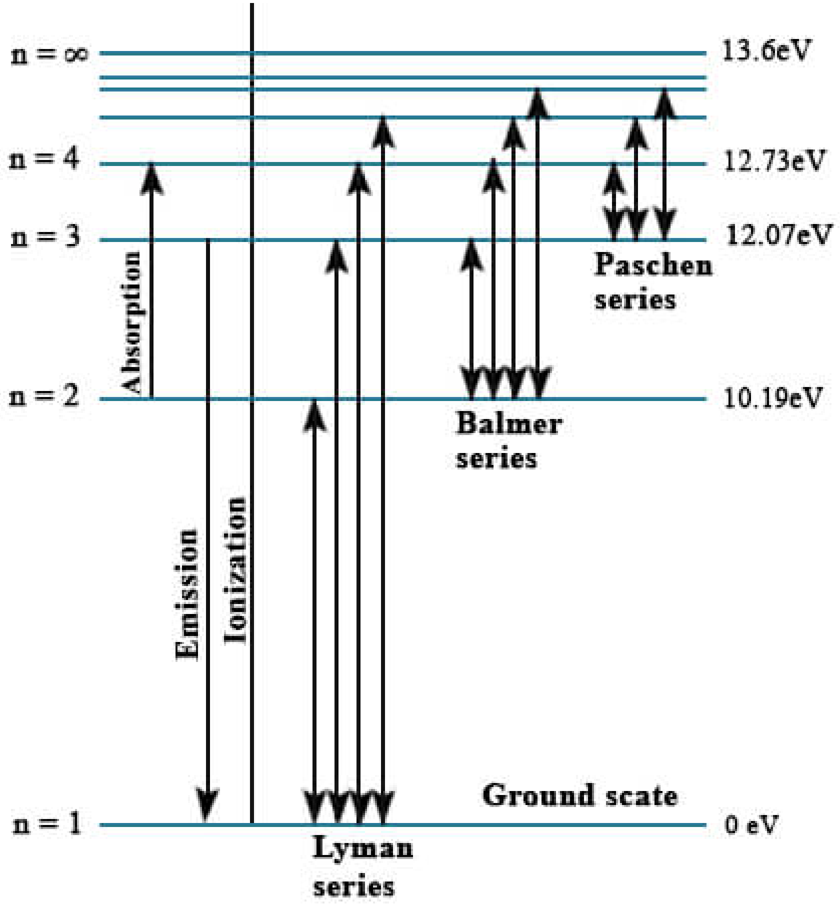
$$E_\gamma = h\nu \qquad \lambda = c/\nu$$

$$\boxed{\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)}$$

$$R \equiv \frac{m}{4\pi c \hbar^3} \left( \frac{e^2}{4\pi \epsilon_0} \right)^2 = 1.097 \times 10^7 \text{ m}^{-1}$$

R= Rydberg constant; this formula was found before Sch. Eq., just empirically.



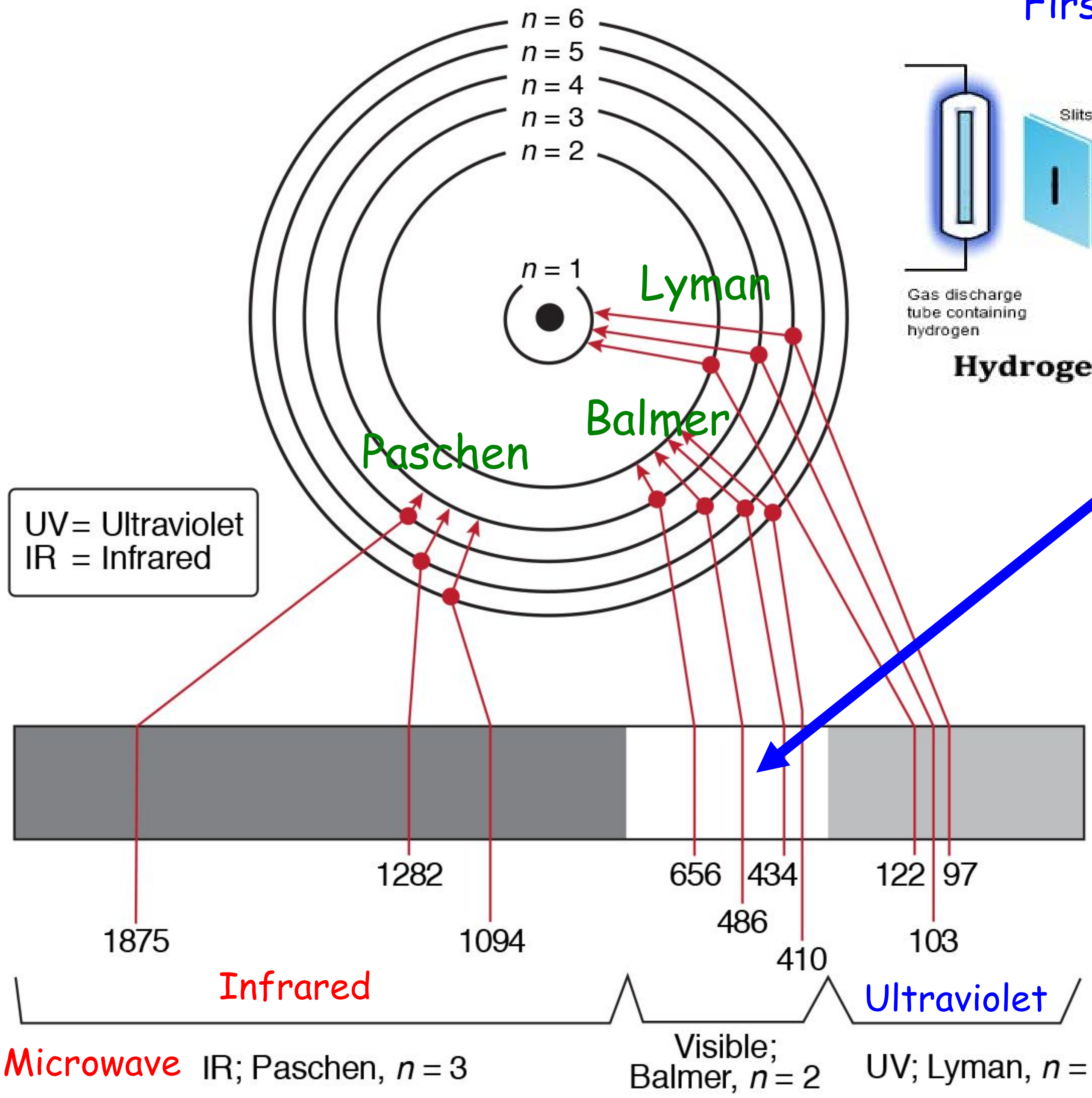

Emission of photons.

Emission and absorption of photons.

Ionization of H atom requires
13.6 eV or more.

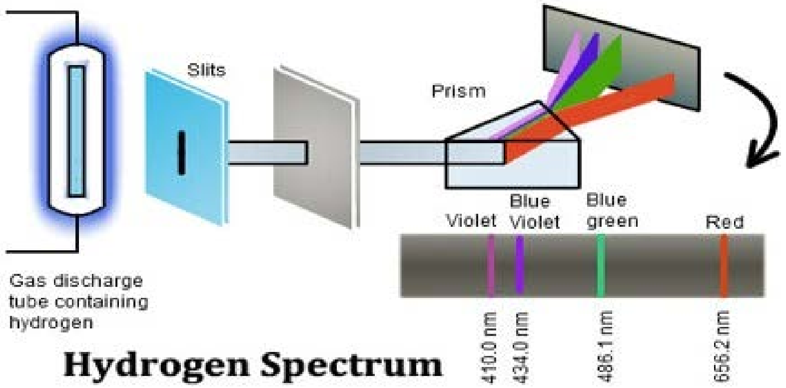
## Bohr Model for Hydrogen Atom

(measurement in nanometers)




<- Microwave

First lecture Aug 2022!




Example:

$(1/\lambda)=R(1/1^2 - 1/2^2)$

$\sim 1/(122\text{ nm})$

X rays $\gamma$ rays …

# 4.3: Angular Momentum

We wish to find out what is the meaning of "*l*" and "*m*" in the quantum numbers *(n,l,m)*.

Let us start with the classical formula for angular momentum: $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

Component by component in Cartesian coordinates this is:

$$L_x = y p_z - z p_y, \quad L_y = z p_x - x p_z, \quad L_z = x p_y - y p_x.$$

To move into quantum mechanics we follow the usual recipe:

$$p_x \rightarrow -i\hbar \partial/\partial x, \; p_y \rightarrow -i\hbar \partial/\partial y, \; p_z \rightarrow -i\hbar \partial/\partial z$$

# Do these operators commute? (HW)

$$[L_x, L_y] = [yp_z - zp_y, zp_x - xp_z]$$

$$= \underbrace{[yp_z, zp_x]}_{yp_x[p_z,z]} - \underbrace{[yp_z, xp_z]}_{yx\underbrace{[p_z,p_z]}_{0}} - \underbrace{[zp_y, zp_x]}_{p_yp_x\underbrace{[z,z]}_{0}} + \underbrace{[zp_y, xp_z]}_{xp_y[z,p_z]}$$

$$= yp_x\underbrace{[p_z, z]}_{-i\hbar} + xp_y\underbrace{[z, p_z]}_{+i\hbar} = i\hbar\underbrace{(xp_y - yp_x)}_{L_z} = i\hbar L_z$$

$$\boxed{[L_x, L_y] = i\hbar L_z; \quad [L_y, L_z] = i\hbar L_x; \quad [L_z, L_x] = i\hbar L_y}$$

If operators do not commute, then we cannot know them simultaneously, as shown in the general theorem of Ch. 3. For example:

$$\sigma_{L_x}^2 \sigma_{L_y}^2 \geq \left(\frac{1}{2i}\langle i\hbar L_z\rangle\right)^2 = \frac{\hbar^2}{4}\langle L_z\rangle^2$$

However, something special happens with the **square** of the angular momentum:

$$L^2 \equiv L_x^2 + L_y^2 + L_z^2$$

It commutes with $L_x$ (and with $L_y$ and with $L_z$):

$$[L^2, L_x] = [L_x^2, L_x] + [L_y^2, L_x] + [L_z^2, L_x]$$

... we need a "mini theorem" now ...

$[L_y^2, L_x] = L_y^2 L_x - L_x L_y^2$

$L_y [L_y, L_x] = L_y (L_y L_x - L_x L_y)$

$[L_y, L_x] L_y = (L_y L_x - L_x L_y) L_y$

Add these two and you get $[L_y^2, L_x]$

Then, $[L_y^2, L_x] = L_y [L_y, L_x] + [L_y, L_x] L_y$ to be used in *HW*

In general $[A^2, B] = A [A,B] + [A,B] A$. You will use this theorem in *HW*. Applying this theorem multiple times:

$$[L^2, L_x] = [L_x^2, L_x] + \underbrace{[L_y^2, L_x]} + \underbrace{[L_z^2, L_x]}$$

$$= L_y[L_y, L_x] + [L_y, L_x]L_y + L_z[L_z, L_x] + [L_z, L_x]L_z$$

$$= L_y(-i\hbar L_z) + (-i\hbar L_z)L_y + L_z(i\hbar L_y) + (i\hbar L_y)L_z$$

$$= 0.$$

The same holds for all components:

$$[L^2, L_x] = 0, \quad [L^2, L_y] = 0, \quad [L^2, L_z] = 0$$

Because $L^2$ commutes with at least one component (usually chosen to be $L_z$) then we should find **eigenstates of both operators simultaneously**.

$$L^2 f = \lambda f \qquad\qquad L_z f = \mu f$$

Then our mission is to find $\lambda$ and $\mu$ and $f$.

To solve this problem we will use a procedure very similar to that of the Harmonic Oscillator with the *lowering and raising operators.*