SOME MOVIES SHOWN TO STUDENTS …

Movie 1: probability shown, intermediate weird state with oscillations, plotting prob and potential together is a bit confusing, not same units.
http://sces.phys.utk.edu/~dagotto/QuantumMechanics/Animation/movie_1.mp4
From original web site: "*FDTD simulation of a Gaussian wave packet with kinetic energy of 500 eV. The potential barrier has a height of 600 eV, and a thickness of 25 pm. The wave packet partially tunnels through the barrier, giving a total probability of about 17% of finding the particle on the other side.*"

Movie 2: Re and Im out of phase causing smooth probability, until collision occurs. Note vgroup and vphase are different.
http://sces.phys.utk.edu/~dagotto/QuantumMechanics/Animation/movie_2.mp4
From original web site: *"FDTD simulation of a Gaussian wave packet with kinetic energy of 500 eV. The potential barrier has a height of 600 eV, and a thickness of 25 pm. The black line represents the real part of the wave function, while the red line represents the imaginary part. The actual probability amplitude is found by taking the magnitude-square of the total wave function. The wave packet partially tunnels through the barrier, giving a total probability of about 17% of finding the particle on the other side. This simulation was written and rendered in Matlab".*

Movie 3: well and barrier of same depth/height behave similar for scattering state (not for bound states of course).
http://sces.phys.utk.edu/~dagotto/QuantumMechanics/Animation/movie_3.mp4

Movie 4: Besides middle obstacle, walls far right and left added creating a huge box that can reflect. Wave packet widths grows, eventually particle spreads uniformly.
http://sces.phys.utk.edu/~dagotto/QuantumMechanics/Animation/movie_4.mp4

Movie 5: Case where wave packet width is smaller than obstacle, then wave packet can collide twice within the same obstacle! And actually many times.
http://sces.phys.utk.edu/~dagotto/QuantumMechanics/Animation/movie_5.mp4

Movie 6: 2D collision. Now some aspects seem like tunneling and other aspects seem like classical scattering at an angle.
http://sces.phys.utk.edu/~dagotto/QuantumMechanics/Animation/movie_6.mp4

Chapter 3: Formalism

Consider a 3D vector using real numbers:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\mathbf{a} = \sum_{n=1}^{3} a_n \mathbf{e}_n$$

The "dot product" is defined as:

$$\mathbf{a.b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

It is not difficult to imagine a generalization to N dimensions and to complex numbers:

$$|\alpha\rangle \rightarrow \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$$

$$\mathbf{a} = \sum_{n=1}^{N} c_n \mathbf{e}_n$$

$$\langle\alpha|\beta\rangle = a_1^* b_1 + a_2^* b_2 + \cdots + a_N^* b_N$$

Linear transformations, represented by matrices, act on vectors. For instance, a rotation.

$$|\beta\rangle = T|\alpha\rangle \rightarrow \mathbf{b} = \mathbf{Ta} = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1N} \\ t_{21} & t_{22} & \cdots & t_{2N} \\ \vdots & \vdots & & \vdots \\ t_{N1} & t_{N2} & \cdots & t_{NN} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$$

Like an "operator" acting on a vector **a**

In Quantum Mechanics, wave functions, expressed as a sum over stationary states, are like an $N \rightarrow \infty$ dimensional vector

$$\Psi(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

Infinite dimensional

Similar to a vector **a**

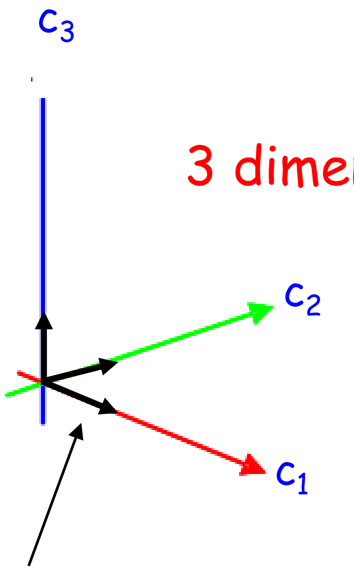
Similar to unit vectors $\mathbf{e}_n$

Of the set of possible functions, the subset that can be normalized ("square-integrable functions") is called a Hilbert space.

From many lectures back …

Cartesian axes

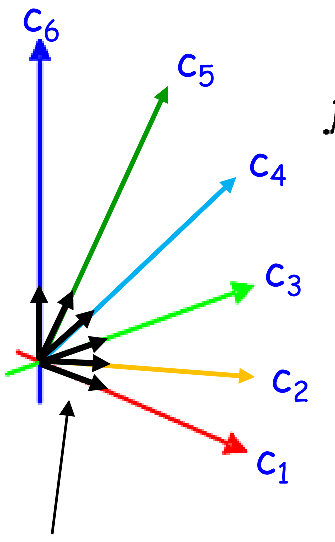
$$\boldsymbol{r} = \sum_{n=1}^{3} c_n \boldsymbol{e}_n$$



3 dimensions

Unit vectors e_1, e_2, e_3.

Any vector can be expanded in the orthonormal basis e_1, e_2, e_3.



$$f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

Square well solutions

∞ dimensions

"Unit vectors" are $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \ldots$

Any wave function can be expanded in the orthonormal basis ψ_n

All these properties are not pathological of the square well or the harmonic oscillator or the delta potential but generic: all have Hilbert Spaces.

For example, in the infinite square well of width "a" the set of functions that are normalizable i.e.

$$\int_0^a |f(x)|^2\, dx < \infty,$$

, and that are zero outside [0,a] because V(x)=∞,

define the "Hilbert space of the infinite square well".

In general, in the [a,b] interval (which could be [-∞,+∞]):

$$\int_a^b |f(x)|^2\, dx < \infty$$

If this condition is satisfied, we say the function is "square integrable"

$$\langle f|g\rangle \equiv \int_a^b f(x)^* g(x)\, dx$$

This is the "inner product" of *f*(x) with *g*(x) (analogous of the dot product for complex numbers in 3D)