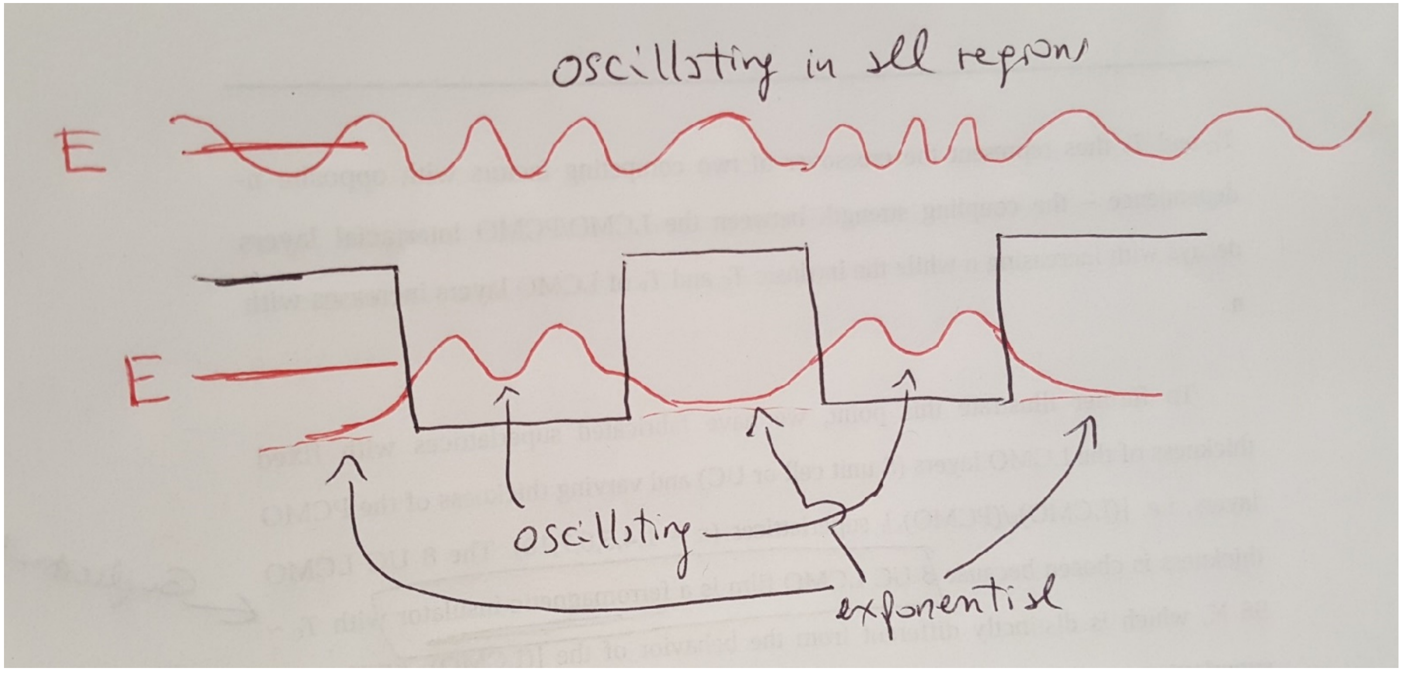
About Test2:



Problem 1 was in practice. I just flipped the V(x).
Problem 2 was in HW6, just changing V0 to 2V0
Problem 3 was very similar to practice.
Problem 4 was very similar to practice.
Problem 5 was identical to practice (changed a to b)

How to find your grade?

Assume in HW1,2,3,4,5,6,7 your numbers are 40+38+25+16+21+18+20= 178. The max number of points was 60+40+30+20+30+30+30=240. Then your HW contribution is (178/240)x40=29.67~30.

Suppose your test scores were 35+33=68. Max number of points 50+50=100. Then your Test contribution to the grade is (68/100)x60=40.8~41.

Then your % number of points is 30+41=71 from a max of 100.

Go to the syllabus in the web page. 71 corresponds to B-.

Last day to drop with a W : November 10.

Should you take P412 in Spring? If you wish to consult with me, go ahead. I recommend grade B or higher.

In a compact form:

$$Y_l^m(\theta,\phi) = \epsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}\, e^{im\phi}\, P_l^m(\cos\theta)$$

with $\epsilon = (-1)^m$ for $m \geq 0$ and $\epsilon = 1$ for $m \leq 0$

and the orthonormality condition

$$\int_0^{2\pi}\int_0^{\pi} [Y_l^m(\theta,\phi)]^*[Y_{l'}^{m'}(\theta,\phi)]\sin\theta\, d\theta\, d\phi = \delta_{ll'}\delta_{mm'}$$

"*l*" is the azimuthal quantum number (or angular momentum)
"m" is the magnetic quantum number (or z-axis projection of the angular momentum)

4.1.3: The Radial Equation

For the angular component we are DONE. But the radial portion depends on V(r), changing from problem to problem.

Reminder:

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{2mr^2}{\hbar^2}[V(r) - E]R = l(l+1)R$$

Before proceeding, another redefinition!

$u(r) = r\,R(r)$

$dR/dr = d[u(r)/r]/dr = (1/r)du/dr - u/r^2$

$d/dr[\, r^2 dR/dr] = r\, d^2u(r)/dr^2$

It is indeed a simplification! (check formula)

The new radial equation becomes … (make sure you **do the math** to prove that this is correct; you have to multiply all by $-\hbar^2/2mr$)

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u = Eu$$

Mathematically identical to a 1D problem (if r->x, u->ψ) with an effective potential that includes a centrifugal term:

$$V_{\text{eff}} = V + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}$$

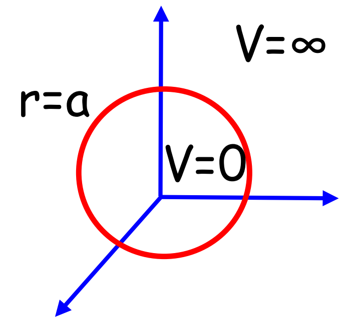
Normalization becomes

$$\int_0^\infty |u|^2\, dr = 1$$

[because $u(r) = r\,R(r)$]

Example 4.1: infinite spherical well

$$V(r) = \begin{cases} 0, & \text{if } r \le a, \\ \infty, & \text{if } r > a \end{cases}$$



(in a HW problem you will solve the infinite cubic well)

Steps very similar to 1D. Same "*k*" etc., but with a centrifugal component

$$\frac{d^2u}{dr^2} = \left[\frac{l(l+1)}{r^2} - k^2\right]u \qquad k \equiv \frac{\sqrt{2mE}}{\hbar}$$

Difference between 1D and 3D equations

If l=0, then it is the exact **same** Sch. Eq. of the 1D infinite square well! We know the general solution:

$$\frac{d^2u}{dr^2} = -k^2 u \Rightarrow u(r) = A\sin(kr) + B\cos(kr)$$

But the boundary conditions are different. The condition u(r=a)=0 is as before. But u(r=0) is a bit different.

The true function we need is R(r) = u(r)/r. Thus, we must choose *B*=0 to avoid a divergence at r=0
A nonzero "*A*" is ok because limit r→ 0, sin(kr)/kr = 1.

Then, at "the end of the day" it is all the same as in 1D:
sin(ka)=0 → *ka*=*N*π, with
N=1,2,3, … →

$$E_{N0} = \frac{N^2\pi^2\hbar^2}{2ma^2}$$

Now we have to place all together!

In general: $\psi_{Nlm}(r,\theta,\phi) = R_{Nl}(r)Y_l^m(\theta,\phi)$

$R_{Nl}(r) = u_{Nl}(r)/r$

For l=0: $Y_0^0(\theta,\phi) = 1/\sqrt{4\pi}$

for l=0, $u_{Nl=0}(r) = A\sin(kr)$ with normalization $A = \sqrt{2/a}$

Then, the final answer for arbitrary N, l=0, m=0 is:

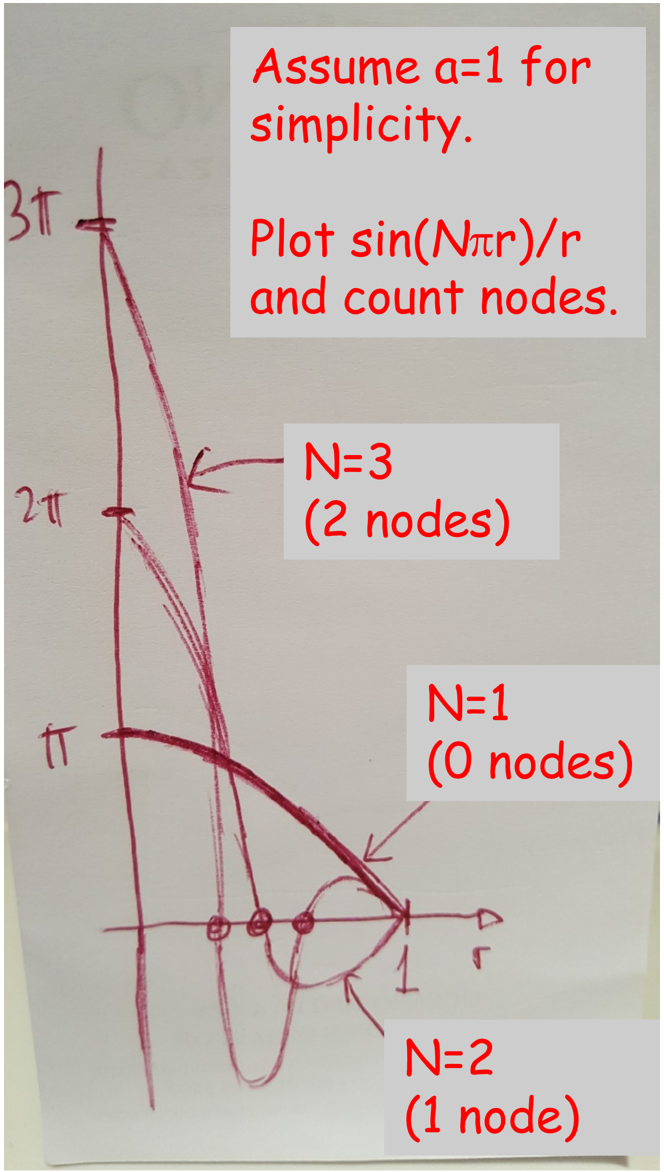
$$\psi_{N00} = \frac{1}{\sqrt{2\pi a}}\frac{\sin(N\pi r/a)}{r}$$

Common error: forgetting this r.

$$\psi_{N00} = \frac{1}{\sqrt{2\pi a}} \frac{\sin(N\pi r/a)}{r}$$

These l=0 wave functions have no angular dependence (spherical harmonic is a constant).
Only r dependence for l=0.

But they have nodes in the r axis [for index N, there are N-1 nodes (i.e. spherical surfaces where the wave function vanishes)].



THIS PORTION IS FYI ONLY. For arbitrary "*l*" the solution is complicated because of the centrifugal term.

$$R_{Nl}(r) = A j_l(kr)$$

Spherical Bessel function of order "*l*" denoted as $j_l(x)$

Energies?

$$k \equiv \frac{\sqrt{2mE}}{\hbar}$$

Boundary condition equivalent to sin(ka)=0 is now:

$$j_l(ka) = 0$$

k's that cancel j_l no longer "easy" (Eq.4.49). And they depend on l.

$$j_0 = \frac{\sin x}{x}$$

$$j_1 = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$

$$j_2 = \left(\frac{3}{x^3} - \frac{1}{x}\right)\sin x - \frac{3}{x^2}\cos x$$

Do NOT worry about the **Spherical Neumann function** of order "*l*" denoted as $n_l(x)$. They diverge at r=0.

As x->0, *l*=0 converges to 1.
This "s" wave is nonzero at r=0

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$$j_1 = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$

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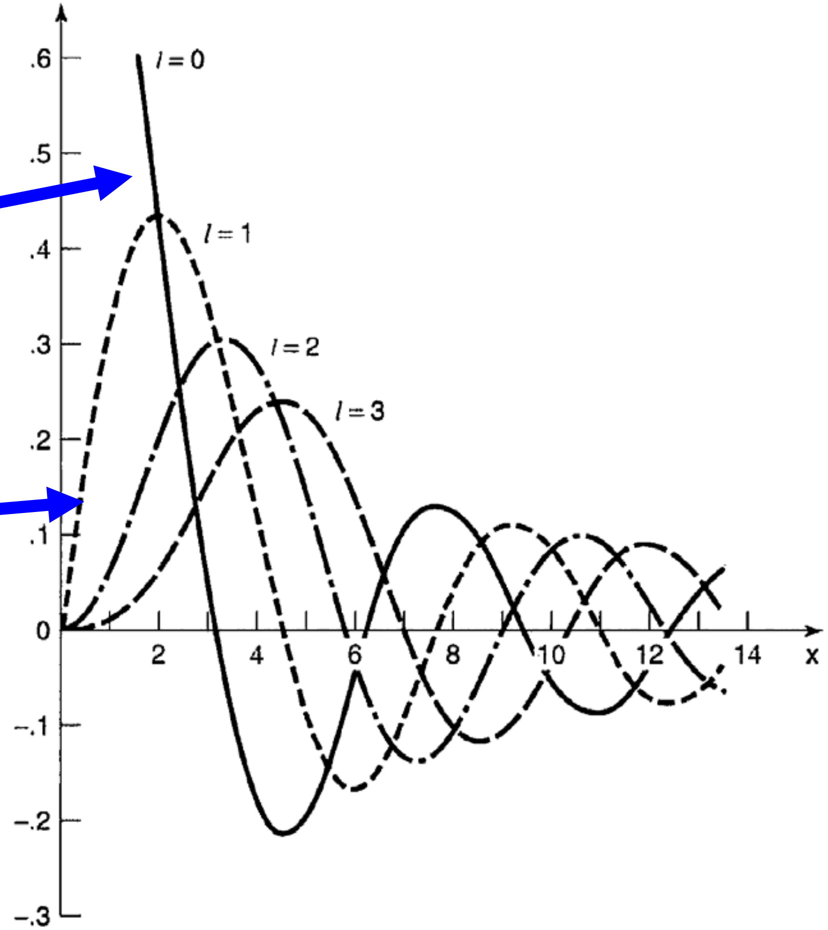
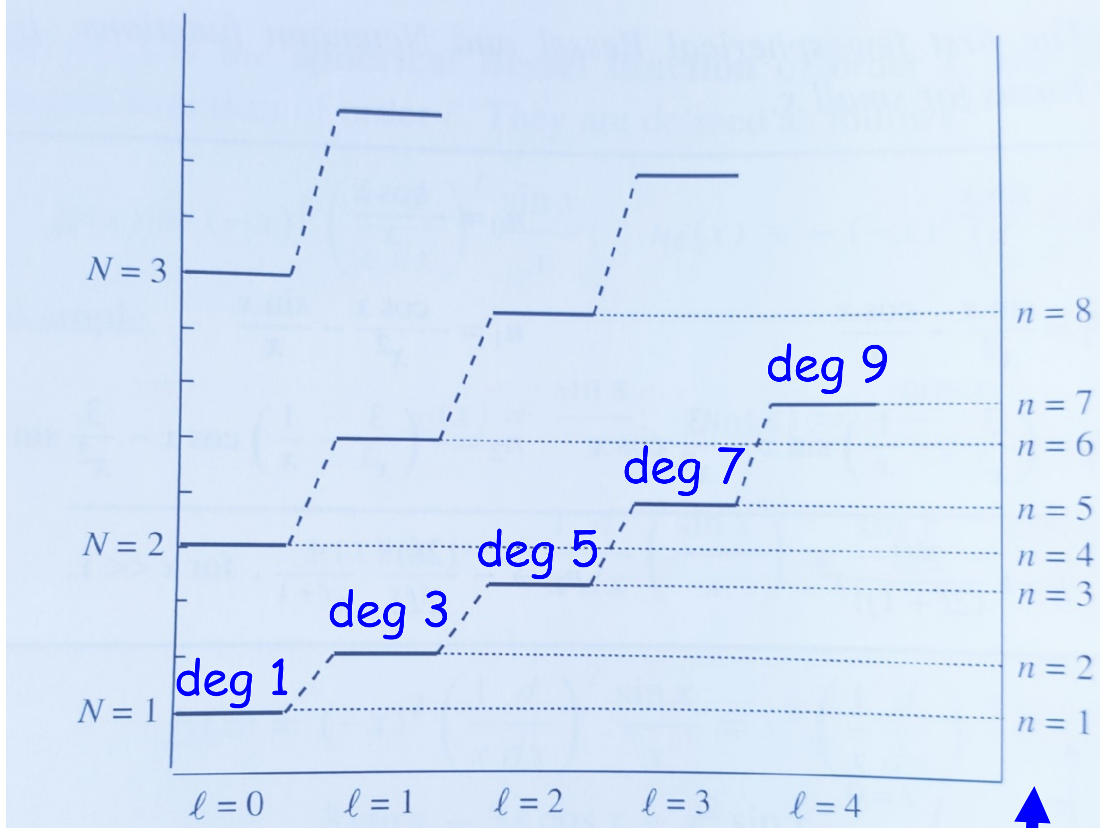


As x->0, *l*=1,2, … converge to 0.
"p,d,…" are zero at r=0.

We have to start thinking in terms of multiple "quantum numbers" and the labels become complicated.



For the "spherical well" we have to use the labels *N* and *l*. For each *l* (such as 0,1,2,...), then *N*=1,2,3, ... labels solutions from the bottom up

We can also count with a single index "n", but here is not illuminating.

Here each level has a degeneracy 2*l*+1 due to m