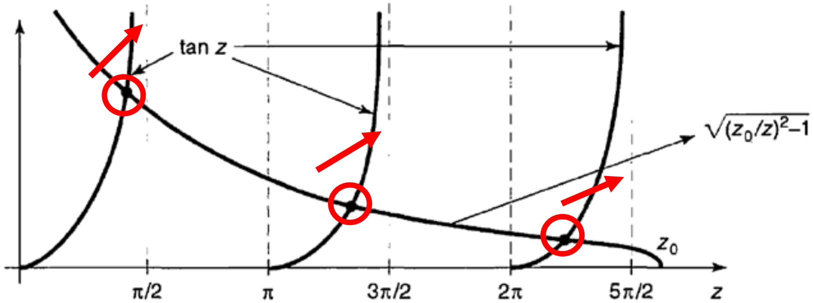
For each value of $z_0 \equiv \frac{a}{\hbar}\sqrt{2mV_0}$ there will be a different finite number of solutions. Consider two special limits:

(1) V_0 large i.e. a **very deep well**. This means z_0 large.

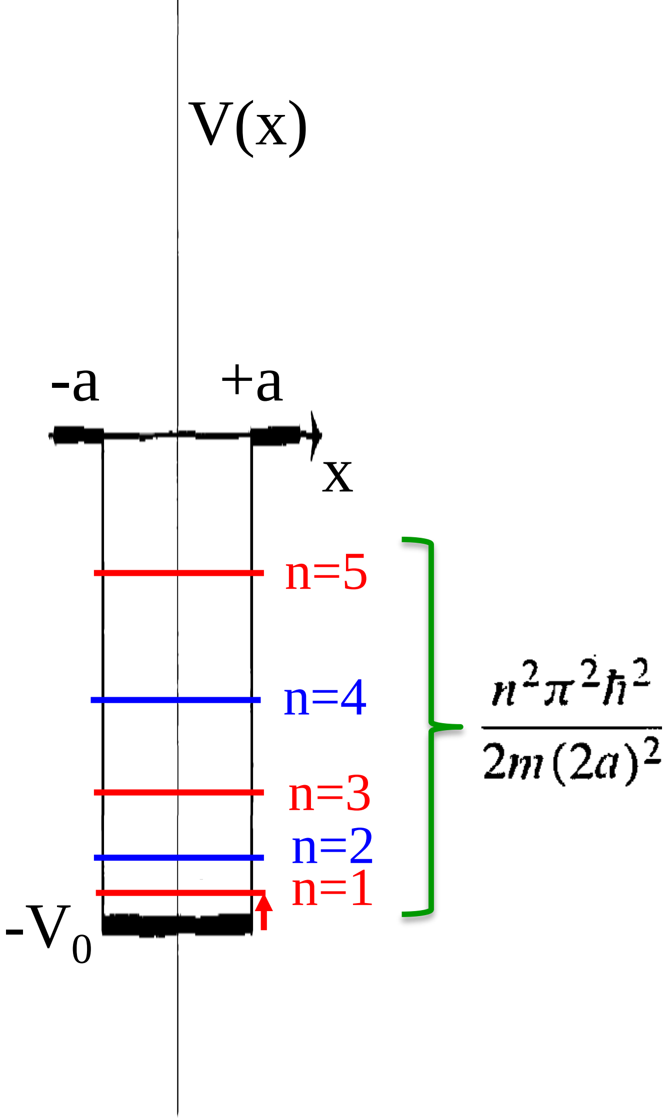


→ solutions are (n=1,3,5,... odd)

$$z_n = n\pi/2$$

Because $z \equiv la$, then $z_n = n\pi/2$ means $l_n = n\pi/2a$, and

$$l_n \equiv \frac{\sqrt{2m(E_n + V_0)}}{\hbar}$$ thus $$E_n + V_0 \cong \frac{n^2\pi^2\hbar^2}{2m(2a)^2}$$

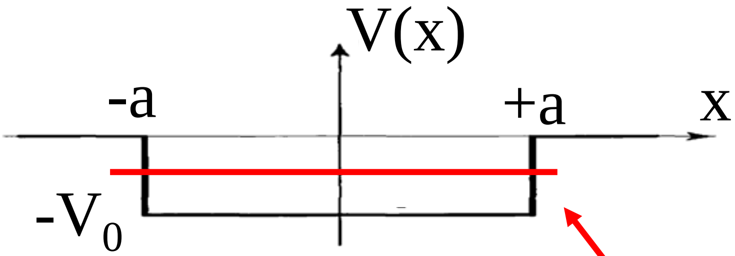


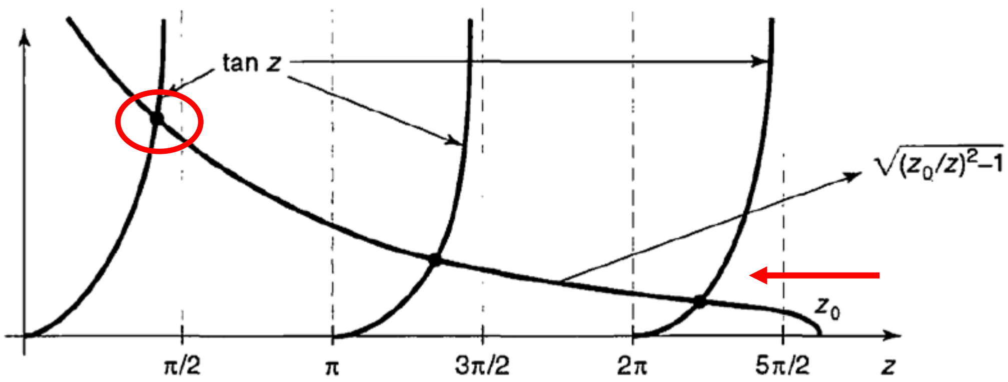
In red are "half" the solutions (those "even" under x→-x, i.e. n odd) that we found before for the infinite square well of width "2a".

In blue are the "other half" that you will do in HW6.

For any large but finite V_0 the number of solutions will be large but finite as well.

(2) V_0 small i.e. a **very shallow well**. This means z_0 small.





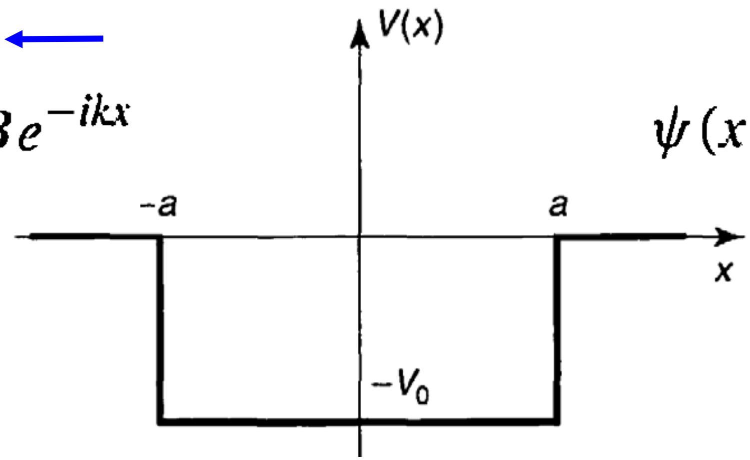
As z_0 is reduced, the number of solutions decreases but always one survives no matter how weak the potential is!

Consider now the scattering states E>0 (unrestricted).

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

left

$$k \equiv \frac{\sqrt{2mE}}{\hbar}$$



$$\psi(x) = Fe^{ikx}$$

right

$$\psi(x) = C\sin(lx) + D\cos(lx) \qquad l \equiv \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

inside

Continuity of ψ and $d\psi/dx$ at x=-a:

$$Ae^{-ika} + Be^{ika} = -C\sin(la) + D\cos(la)$$

$$ik[Ae^{-ika} - Be^{ika}] = l[C\cos(la) + D\sin(la)]$$

Continuity of ψ and $d\psi/dx$ at x=+a:

$$C\sin(la) + D\cos(la) = Fe^{ika}$$

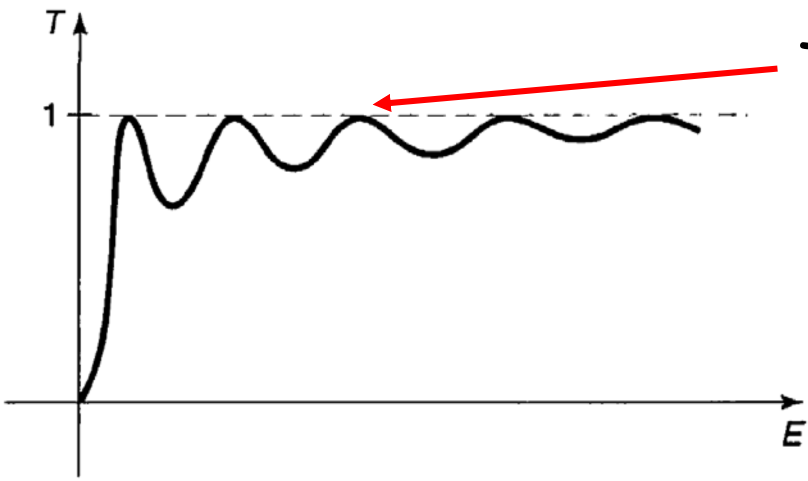
$$l[C\cos(la) - D\sin(la)] = ikFe^{ika}$$

This system of 4 equations and 5 unknowns can be solved as for the delta potential via ratios.

Recall $T = |F|^2/|A|^2$ is the transmission coefficient

The result is

$$T^{-1} = 1 + \frac{V_0^2}{4E(E+V_0)} \sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right)$$



T=1 if $\frac{2a}{\hbar}\sqrt{2m(E_n+V_0)} = n\pi$

Thus, for some particular energies there is perfect transmission !! And they happen to be the solutions of the infinite square well. Weird QM!

$$E_n + V_0 = \frac{n^2\pi^2\hbar^2}{2m(2a)^2}$$

There are "remnants" of the infinite well ...

SOME MOVIES SHOWN TO STUDENTS …

Movie 1: probability shown, intermediate weird state with oscillations, plotting prob and potential together is confusing, not same units.

https://www.youtube.com/watch?v=4-PO-RHQsFA

From web site: "*FDTD simulation of a Gaussian wave packet with kinetic energy of 500 eV. The potential barrier has a height of 600 eV, and a thickness of 25 pm. The wave packet partially tunnels through the barrier, giving a total probability of about 17% of finding the particle on the other side.*"

Movie 2: Re and Im out of phase causing smooth probability, until collision occurs. Note vgroup and vphase are different.

https://www.youtube.com/watch?v=_3wFXHwRP4s

From web site: *"FDTD simulation of a Gaussian wave packet with kinetic energy of 500 eV. The potential barrier has a height of 600 eV, and a thickness of 25 pm. The black line represents the real part of the wave function, while the red line represents the imaginary part. The actual probability amplitude is found by taking the magnitude-square of the total wave function. The wave packet partially tunnels through the barrier, giving a total probability of about 17% of finding the particle on the other side. This simulation was written and rendered in Matlab".*

Movie 3: well and barrier of same depth/height behave similar for scattering state (not for bound states of course). https://www.youtube.com/watch?v=cV2fkDscwvY

Movie 4: Besides middle obstacle, walls far right and left added creating a huge box that can reflect. Wave packet widths grows, eventually particle spreads uniformly. https://www.youtube.com/watch?v=YaEvanuJcz4

Movie 5: Case where width is smaller than obstacle, then wave packet can collide twice within the same obstacle! And actually many times. https://www.youtube.com/watch?v=GjCUlcOyhLc

Movie 6: 2D collision. Now some aspects seem like tunneling and other aspects seem like classical scattering at an angle. https://www.youtube.com/watch?v=8qOXof9_34M