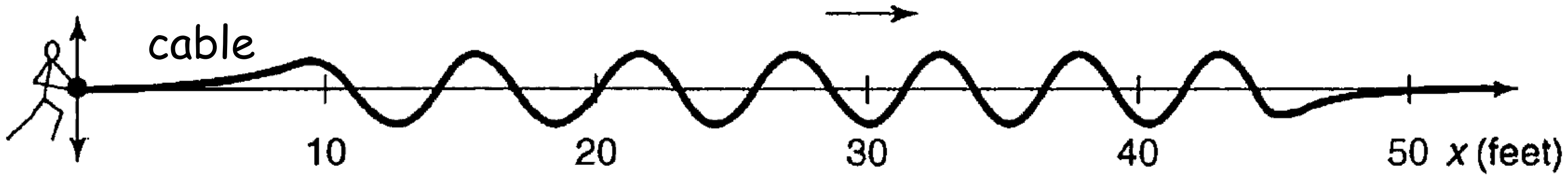
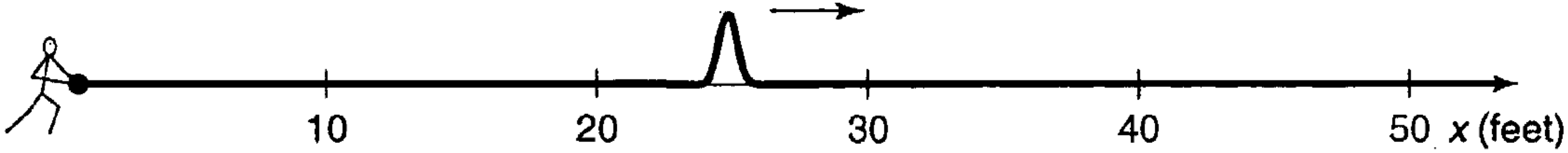
Preliminaries to the uncertainty principle

Caution: this is not an independent principle but it arises entirely from Sch. Eq. (see Ch. 3). Thus, if you do the calculations right, it is always satisfied. But intuitively it is interesting to discuss it.



Wavelength? Clear; Position? Unclear



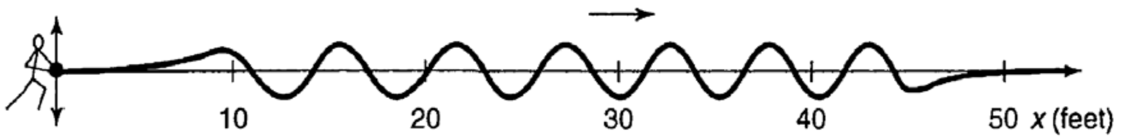
Wavelength? Unclear; Position? Clear

This is true for any wave-like phenomenon, thus it has to apply to the Sch. Eq. somehow as well.

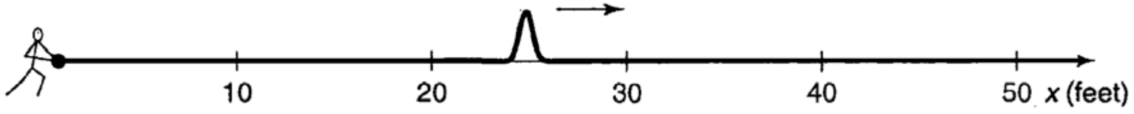
De Broglie formula (2 years before Sch. Eq.) said that electrons have wave-like features, like photons do:

$$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda}$$

Thus, if wavelength is known accurately, p is known accurately. If wavelength is unknown, p is unknown.



Momentum? Sharp
Position? Not sharp



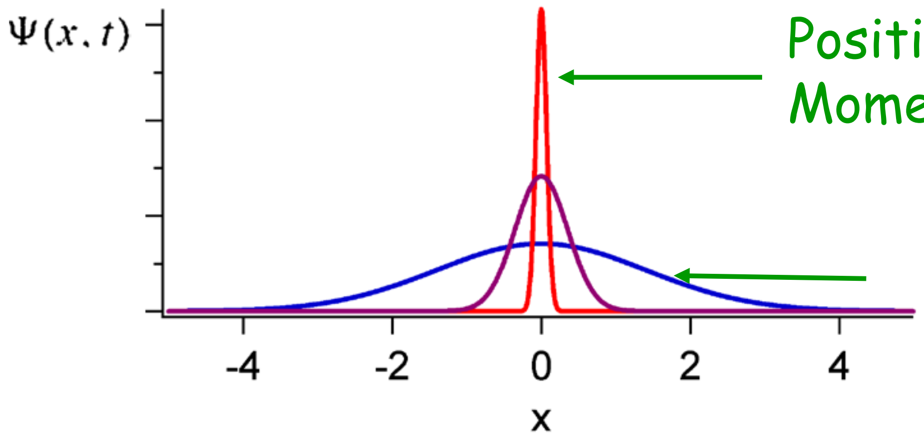
Momentum? Not sharp
(if you Fourier decompose a spike, it has all k values!)
Position? Sharp

We will prove later (not a new law, but it is consequence of Sch. Eq.) that the standard deviations satisfy:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$



Position? Sharp i.e. σ_x=0 if δ function.
Momentum? Totally unknown $\sigma_p=\infty$

Position? Less sharp $\sigma_x>0$
Momentum? Less unknown $\sigma_p<\infty$

END CHAPTER 1

1. Students that are “isolated”, i.e. they do not know other students in the class, send me email and I will place all together for QM interactions.

2. I sent you a video about even-odd functions and how it can simplify integration.

3. Test 1 will be 9/27, not 9/29.

4. There is a good chance that class Oct.4 will be by Zoom due to obligations on my side, plus it is Fall break week.

Was the Zoom connection good? Only 31 of 36+ students connected.

5. How to return HW without violating privacy rules?

START CHAPTER 2

Chapter 2: time-indep Sch. Eq.

Now the real work starts ☺: how do we solve the Sch. Eq.?

In general, the Sch. Eq. has a *time-dependent* potential V(x,t) in

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V\Psi$$

For example, the V(x,t) of an oscillating electric field **E**(x,t).

We start with something *simpler*: a *time independent* potential V=V(x). Then use "separation of variables". Assume a product solution:

$$\Psi(x,t)=\psi(x)\,\varphi(t)$$

$$\frac{\partial\Psi}{\partial t}=\psi\frac{d\varphi}{dt},\qquad \frac{\partial^2\Psi}{\partial x^2}=\frac{d^2\psi}{dx^2}\varphi$$

$$i\hbar\psi\frac{d\varphi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}\varphi + V\psi\varphi.$$

Divide by $\psi\varphi$ to get:

$$i\hbar\frac{1}{\varphi}\frac{d\varphi}{dt} = -\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V(x) \quad = \; E$$

Function of t only

Function of x only

(must be constant, with important physical meaning to be discussed)

Time dependence is easy here!

$$i\hbar \frac{1}{\varphi}\frac{d\varphi}{dt} = E \quad \Rightarrow \quad \frac{d\varphi}{dt} = -\frac{iE}{\hbar}\varphi$$

$$\varphi(t) = e^{-iEt/\hbar}$$

Coordinate dependence is not easy! ☹

$$-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V(x) = E$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\,\psi(x)$$

Constant to be found

The solutions of this time-indep. Sch. Eq. are important for three reasons [(1,2) given here, (3) later in this presentation]:

(1) They are "stationary" states: $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$

$$|\Psi(x,t)|^2 = \Psi^*\Psi = \psi^* e^{+iEt/\hbar}\psi e^{-iEt/\hbar} = |\psi(x)|^2$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} x|\Psi(x,t)|^2\, dx$$

Constant in time

$$\langle p \rangle = m\frac{d\langle x \rangle}{dt} = 0$$

All expectation values of generic operators O(x,p) are constant in time.

(2) They have "sharp" energies.

operators are denoted by ^

$$H(x,p) = \frac{p^2}{2m} + V(x) \xrightarrow[\text{rules}]{\text{QM}} \hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

Classical "Hamiltonian"

$\hat{p} \rightarrow (\hbar/i)(\partial/\partial x)$

$\hat{x} \rightarrow x$

Quantum "Hamiltonian"

Now the time-indep. Sch. Eq. looks "simple":

$$\boxed{\hat{H}\psi = E\psi}$$

Then, the constant E is the energy! Proof:

$$\langle \hat{H} \rangle = \int \psi^* \underbrace{\hat{H}\psi}_{E\psi} \, dx = E \int |\psi|^2 \, dx = E \underbrace{\int |\Psi|^2 \, dx}_{=1} = E$$

Why *E* is "sharp"?

$$\hat{H}^2\psi = \hat{H}(\hat{H}\psi) = \hat{H}(E\psi) = E(\hat{H}\psi) = E^2\psi$$

$$\langle \hat{H}^2 \rangle = \int \psi^* \hat{H}^2 \psi \, dx = E^2 \underbrace{\int |\psi|^2 \, dx}_{=1} = E^2$$

Thus: $\sigma_H^2 = \langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2 = E^2 - E^2 = 0$

sharp!