Announcements for Test 1 and general:

We will have **Zoom sessions** on Sunday afternoon and Monday noonish. I will announce time by email, with zoom links. Both sessions are expected to be almost identical: the goal of having two is in case you cannot make it to one, then you go to the other. Each Zoom will have time at the end for free formatted questions of any kind.

I will send **by email** a couple of practice problems maybe Friday or Saturday morning, with the solutions but play the game of trying to solve before checking solutions.

About grades and HW, if you feel that the grader was not fair, wait until I circulate my HW solution and then compare yours vs mine and judge on fair-unfair afterwards. Complain if you conclude too many points were removed, after that comparison. No problem. Same with the grades of the test BTW.

For the exam, the solutions of HW1,2,3 (my solutions) + my lectures (focus on examples) are enough for a good preparation for test.

(a)You are given a QM problem with potential $V(x)$ unknown. But you are told that the first and second solution of the time-independent Sch. Eq. are $\psi_1(x)$ and $\psi_2(x)$ (both are normalized to 1 already). Then, you are told you have an electron in the initial wave function $\psi(x,t=0)= A\,[\psi_1(x)+ 2\,i\,\psi_2(x)]$. Find A such that $\psi(x,t=0)$ is normalized to 1.

$$\psi^* = A^* (\psi_1^* - 2i\psi_2^*)$$

$$\psi^*\psi = |A|^2(\psi_1^* - 2i\psi_2^*)(\psi_1 + 2i\psi_2) = |A|^2\left(|\psi_1|^2 - 2i\psi_2^*\psi_1 + 2i\psi_1^*\psi_2 + 4|\psi_2|^2\right)$$

Using orthonormality

$$\int|\psi|^2dx = 1 = |A|^2\Big[\underbrace{\int|\psi_1|^2dx}_{=1} - 2i\underbrace{\int\psi_2^*\psi_1\,dx}_{=0} + 2i\underbrace{\int\psi_1^*\psi_2\,dx}_{=0} + 4\underbrace{\int|\psi_2|^2dx}_{=1}\Big]$$

$$\int|\psi|^2dx = 1 = |A|^2(1+0+0+4) \rightarrow \boxed{A = \frac{1}{\sqrt{5}}}$$

(b) Time dependence of ψ if energies are E_1 and E_2?

$$\psi(x,t) = \frac{1}{\sqrt{5}}\left[\psi_1(x)e^{-\frac{iE_1t}{\hbar}} + 2i\,\psi_2(x)e^{-\frac{iE_2t}{\hbar}}\right]$$

$$\hat{a}_+ = \frac{1}{\sqrt{2\hbar m\omega}}\left(-i\hat{p} + m\omega\hat{x}\right)$$

Replace $\hat{p}$ by $-i\hbar\frac{d}{dx}$ and $\hat{x}$ by x and write $\hat{a}_+$ in a less mysterious manner. Write $(\hat{a}_+)^2$ also in a simple manner.

Answer:

$$\hat{a}_+ = \frac{1}{\sqrt{2\hbar m\omega}}\Big(\underbrace{-i\left(-i\hbar\frac{d}{dx}\right)}_{+i^2\hbar\frac{d}{dx} = -\hbar\frac{d}{dx}} + m\omega x\Big)$$

$$= \boxed{\frac{1}{\sqrt{2\hbar m\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)}$$

$$\boxed{(\hat{a}_+)^2 = \frac{1}{2\hbar m\omega}\left(-\hbar\frac{d}{dx} + m\omega x\right)\left(-\hbar\frac{d}{dx} + m\omega x\right)}$$

Note that d/dx of the first (…) acts on anything to the right, including the “mωx” term of the second parenthesis as well as on any wave function (not written) that this full operator may act on.

The product $\left(e^{\frac{i\pi}{4}}\right)^{*} e^{\frac{i\pi}{4}}$ is equal to (a) 1 or equal to (b) $e^{i\pi/2}$? Answer: 1

Consider wave function $\psi(x) = -A x e^{-2x^2}$. Find $\frac{d\psi(x)}{dx}$.

How many nodes $d\psi/dx$ has? Sketch $d\psi/dx$.

$$\frac{d\psi}{dx} = -A\frac{d}{dx}\left(xe^{-2x^2}\right) = -A\left[\frac{d(x)}{dx}\cdot e^{-2x^2} + x\frac{d}{dx}\left(e^{-2x^2}\right)\right] = -A\left[1\cdot e^{-2x^2} + x(-4x)e^{-2x^2}\right]$$

$$= -A\left[e^{-2x^2} - 4x^2 e^{-2x^2}\right] = \boxed{Ae^{-2x^2}(-1+4x^2)}$$

Nodes? $-1+4x_{node}^2 = 0 \rightarrow x_{node}^2 = \frac{1}{4}$, $\boxed{x_{node} = \pm\frac{1}{2}}$.

even