$|s\,m\rangle$

Then, we have confirmed that indeed we form a **triplet** and a **singlet**, out of two spins $\frac{1}{2}$.

$$\left\{\begin{array}{ll} |1\,1\rangle & = \uparrow\uparrow \\ |1\,0\rangle & = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ |1\,-1\rangle & = \downarrow\downarrow \end{array}\right\} \quad s = 1$$

triplet

$$\left\{|0\,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\right\} \quad s = 0$$

singlet

Not in book (counting of states for 2 and 3 spins):

$\uparrow\uparrow$ $\qquad$ $\uparrow\uparrow$

$\uparrow\downarrow$ $\qquad$ $\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)$

$\downarrow\uparrow$ $\qquad$ $\downarrow\downarrow$

$\downarrow\downarrow$ $\qquad$ $\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)$

2^2=4 states sort of random

4 states grouped as S=1 (3) and S=0 (1)

Note: after finding S=1, there was only 1 state left, thus **had to be singlet** and **had to be orthogonal**, thus fixing the "-"

$\uparrow\uparrow\uparrow$

$\uparrow\uparrow\downarrow$, $\uparrow\downarrow\uparrow$, $\downarrow\uparrow\uparrow$

$\uparrow\downarrow\downarrow$, $\downarrow\uparrow\downarrow$, $\downarrow\downarrow\uparrow$

$\downarrow\downarrow\downarrow$

2^3 =8 states sort of random

$\uparrow\uparrow\uparrow$, $S_-\uparrow\uparrow\uparrow$, $S^2_-\uparrow\uparrow\uparrow$, $S^3_-\uparrow\uparrow\uparrow$

4 states form S total 3/2

The 4 states left form TWO S total $\frac{1}{2}$ states.

$3/2 \oplus 1/2 \oplus \frac{1}{2}$ (more about this in a few pages)

Not in book (and FYI only):

FYI: spins can interact among themselves, not only with external magnetic fields.

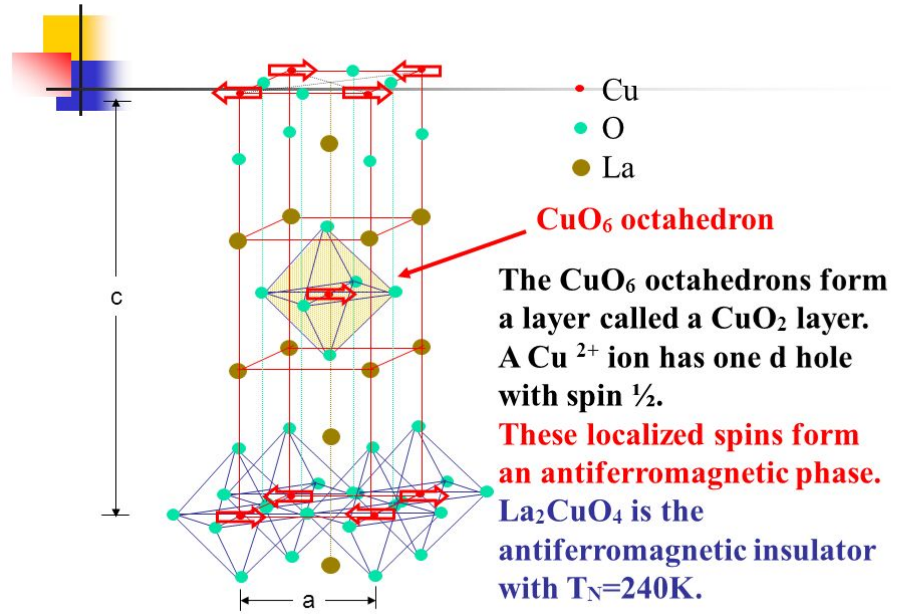
It is as if other spins "*j*" produce an ***effective*** magnetic field on the spin "*i*" you are looking at. This is the famous Heisenberg model:

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Ground state? Number of states grows like 2^N (=2,4,8,16, …)

Record done exactly N~ 40.
2^{40} = 1,099,511,627,776 states

Crystal structure of undoped La_2CuO_4



WITHOUT PROOF, this is what happens when you combine a spin s_1 and a spin s_2 (each individually 0, 1/2, 1, 3/2, …).

The total spin s of the combination can be Eq.(4.182):

$$s = (s_1 + s_2),\ (s_1 + s_2 - 1),\ (s_1 + s_2 - 2),\ \ldots,\ |s_1 - s_2|$$

Example 1: for $s_1 = 1/2$ and $s_2 = 1/2$, then s runs from $s_1 + s_2 = 1$ to $|s_1 - s_2| = 0$, with nothing in between. We already confirmed this is true.

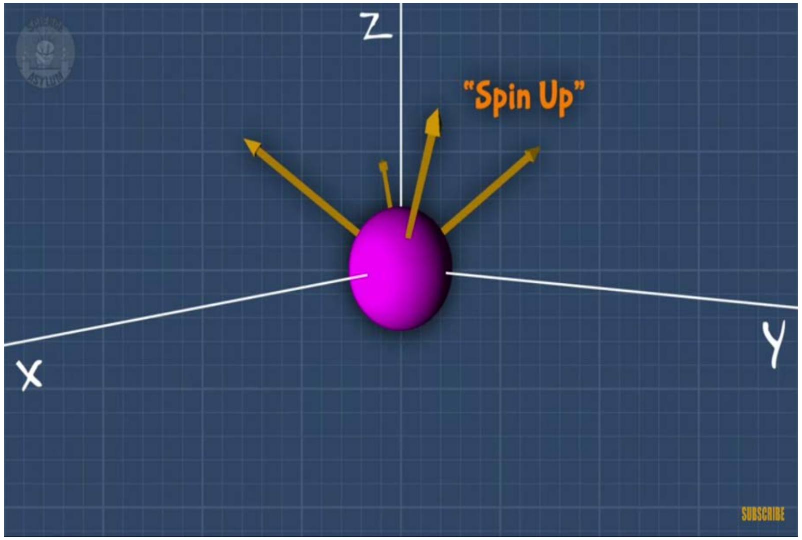
Example 2: for $s_1 = 3/2$ and $s_2 = 2$, then s runs from $s_1 + s_2 = 7/2$ to $|s_1 - s_2| = 1/2$, with 5/2 and 3/2 in between.

Example 3: this unproven theorem holds also for the addition of orbital angular momentum l and spin s. For l=2 and s=1/2, then total j runs from for $l+s$ = 5/2 to $|l-s|$= 3/2, with nothing in between.

Example 4: if you have three particles with s_1=1/2, s_2=1/2, and s_3=1/2, then first you add two, such as s_1 and s_2, finding $s_{partial}$ = 1,0 and then add $s_{partial}$ with s_3 finding 3/2,1/2 (for $s_{partial}$ =1) and another 1/2 (for $s_{partial}$=0). So there are two independent combinations with total spin $\frac{1}{2}$. We already saw that in previous pages: $3/2 \oplus 1/2 \oplus 1/2$

We will NOT deal with the **Clebsch-Gordan** coefficients, but with the foundation given already, it should be easy for you to learn from the book.

Detour: Explanation of video about "spin"



$$\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\cdot\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}}_{+\hbar/2 \text{ for } \sigma_x} + \frac{1}{\sqrt{2}}\cdot\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}}_{-\hbar/2 \text{ for } \sigma_x}$$

$\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} = 50\%$ chance of measuring "up" when measuring S_x.

From HW 4.32

$$= \frac{1}{\sqrt{2}}\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}}_{+\hbar/2 \text{ for } \sigma_y} + \frac{1}{\sqrt{2}}\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}}_{-\hbar/2 \text{ for } \sigma_y}$$

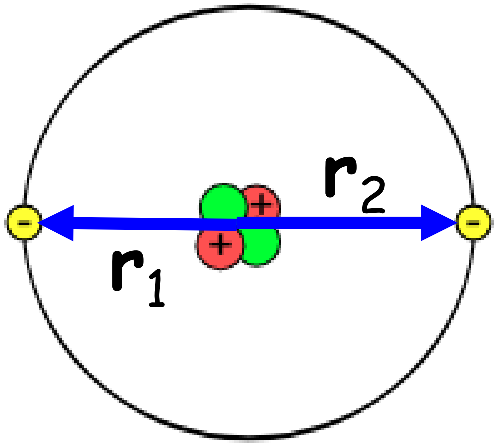
again 50% of measuring "up" when measuring S_y.

Chapter 5: Identical Particles

For one particle, like one electron in the H atom, we simply need the wave function $\Psi(\mathbf{r}_1,t)$ where $\mathbf{r}_1$ is the coordinate of electron "1".

Consider now two particles as warm up.

For two particles, e.g. two electrons in the He atom, in QM we need the wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2,t)$ where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of the two electrons.

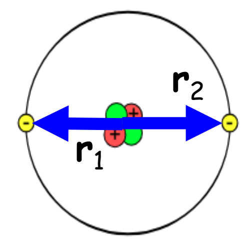


Mathematically, the Sch. Eq. $i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$ has a more complicated Hamiltonian.

$$\boxed{H = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\mathbf{r}_1, \mathbf{r}_2, t)}$$

The potential V typically has terms like e-p attraction, but also e-e repulsion.

Example, for the He atom:



$$H = \left\{-\frac{\hbar^2}{2m}\nabla_1^2 \underbrace{- \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_1}}_{\substack{\text{p-e}_1\text{ attraction} \\ \text{is } V(\mathbf{r}_1)}}\right\} + \left\{-\frac{\hbar^2}{2m}\nabla_2^2 \underbrace{- \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_2}}_{\substack{\text{p-e}_2\text{ attraction} \\ \text{is } V(\mathbf{r}_2)}}\right\} + \underbrace{\frac{1}{4\pi\epsilon_0}\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}}_{\substack{\text{e}_1\text{-e}_2\text{ repulsion} \\ \text{is } V(\mathbf{r}_1,\mathbf{r}_2)}}$$