7.4 Zeeman Effect:

Electron in an atom in the presence of a magnetic field. Before, only "spin" **S** in a magnetic field was considered. Now we add the orbital angular momentum **L** because electron is orbiting the proton.

$$H'_Z = -(\boldsymbol{\mu}_l + \boldsymbol{\mu}_s) \cdot \mathbf{B}_{ext}$$

Missing 2 in spin is due to relativity.

$$\boldsymbol{\mu}_s = -\frac{e}{m}\mathbf{S} \qquad \boldsymbol{\mu}_l = -\frac{e}{2m}\mathbf{L}$$

$$H'_Z = \frac{e}{2m}(\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}_{ext}$$

But the electron is immersed in two magnetic fields. The external one that can be turn on and off, and the internal one, always there.

$\mathbf{B}_{ext}$ vs $\mathbf{B}_{in} = \frac{1}{4\pi\epsilon_0}\frac{e}{mc^2r^3}\mathbf{L}$

$$B_{ext} \ll B_{int}$$

If external field is very small, then the fine structure constant must be considered as part of the H_0 before adding magnetic field.

$$B_{ext} \gg B_{int}$$

If external field not too small, then the fine structure constant is neglected in H_0 before adding magnetic field.

(1) $B_{ext} \ll B_{int}$

Fine structure perturbation is important.
Good quantum numbers are (n, l, s, j, m_j)

First order in perturbation theory

$$E_Z^1 = \langle nljm_j | H_Z' | nljm_j \rangle = \frac{e}{2m} \mathbf{B}_{ext} \cdot \langle \mathbf{L} + 2\mathbf{S} \rangle$$

It can be shown that:

$$\langle \mathbf{L} + 2\mathbf{S} \rangle = \left[1 + \frac{j(j+1) - l(l+1) + 3/4}{2j(j+1)} \right] \langle \mathbf{J} \rangle$$

Called "Lande' g_J factor"

Consider external magnetic field along z-axis:

Then:

$$E_Z^1 = \mu_B g_J B_{ext} m_j$$

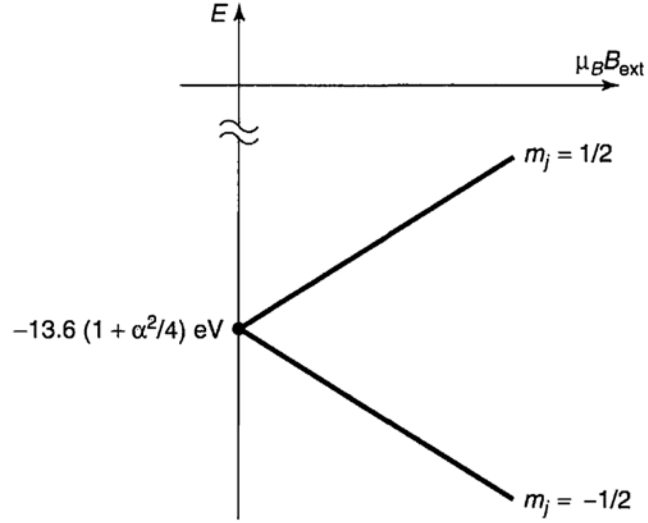
In a few words, a simple split linear with B_{ext}: some levels up, others down.

where the Bohr magneton is defined as (using $\langle J_z \rangle$= hbar m_j):

$$\mu_B \equiv \frac{e\hbar}{2m} = 5.788 \times 10^{-5} \text{ eV/Tesla}$$

Example: ground state has n=1, l=0, s=1/2, j=1/2, m_j = ±1/2 (g_J=2; check!)

$$E_{\text{ground state}} = -13.6\ \text{eV}(1+\alpha^2/4) \pm \mu_B B_{\text{ext}}$$



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(2) $B_{\text{ext}} \gg B_{\text{int}}$ External magnetic field dominates. Good quantum numbers are (n, l, s, m_l, m_s) because external magnetic field is larger than fine structure correction.

$$E_{nm_lm_s} = -\frac{13.6\ \text{eV}}{n^2} + \mu_B B_{\text{ext}}(m_l + 2m_s)$$

$$E^1_{\text{fs}} = \langle n l\, m_l\, m_s | (H'_r + H'_{\text{so}}) | n l\, m_l\, m_s \rangle$$

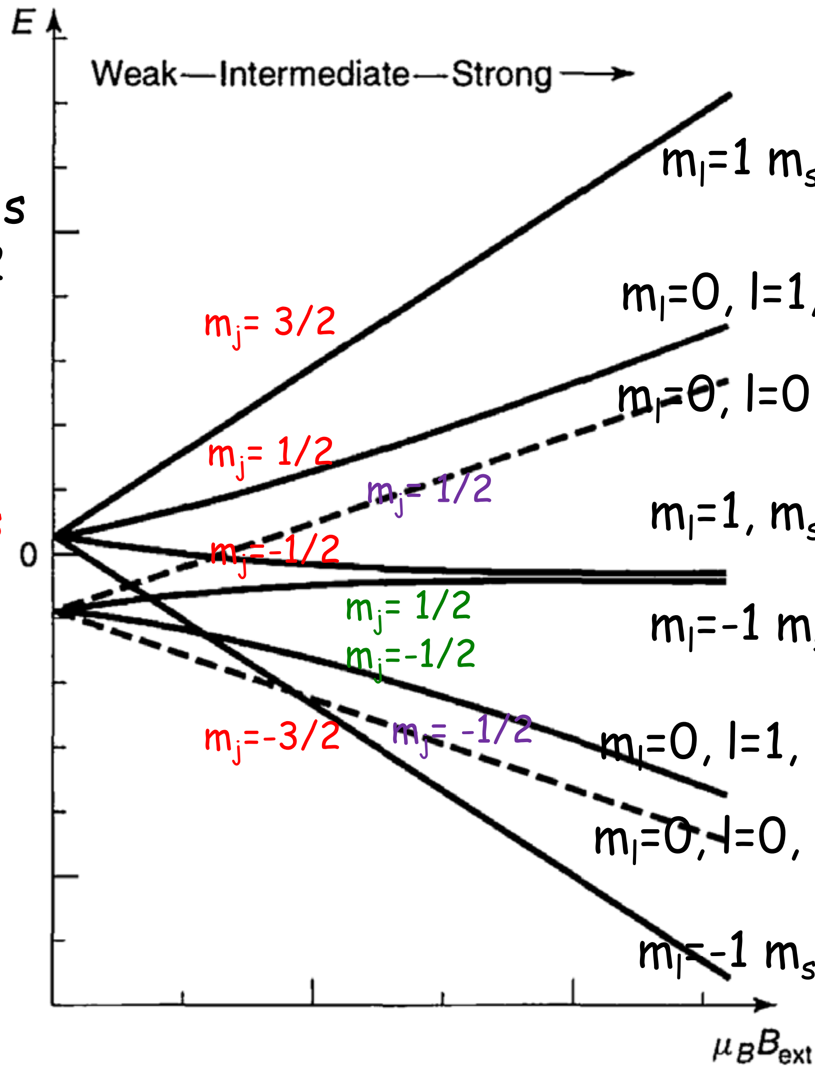
$$E^1_{\text{fs}} = \frac{13.6\ \text{eV}}{n^3}\alpha^2 \left\{ \frac{3}{4n} - \left[\frac{l(l+1) - m_l m_s}{l(l+1/2)(l+1)} \right] \right\}$$

EXAMPLE:

n=2 states of H atom are 8 total because 4 orbitals (s,px,py,pz) and 2 spins

j=3/2, deg=4, l=1, s=1/2, j=l+s

j=1/2, deg=2, l=1, s=1/2, j=l-s or l=0 s=1/2



	m_l+2m_s
m_l=1 m_s=1/2	+2
m_l=0, l=1, m_s=+1/2	1
m_l=0, l=0, m_s=+1/2	1
m_l=1, m_s=-1/2	0
m_l=-1 m_s=+1/2	0
m_l=0, l=1, m_s=-1/2	-1
m_l=0, l=0, m_s=-1/2	-1
m_l=-1 m_s=-1/2	-2

n=2 without fine structure correction has:

3 l=1 px,py,pz - states where spin can go up or down

and 1 l=0 s-state where spin can go up or down

What separates weak from strong magnetic fields? See portion of the book where the "intermediate field" is described. They define two quantities called γ and β.

$$\gamma = (\alpha/8)^2\ (13.6\ \text{eV}) \qquad \beta = \mu_B\ B_{ext}$$

$$\gamma = (\alpha/8)^2\ (13.6\ \text{eV}) << \beta = \mu_B\ B_{ext}$$

$$\gamma = (\alpha/8)^2\ (13.6\ \text{eV}) >> \beta = \mu_B\ B_{ext}$$

$$\alpha = 1/137,\ \mu_B \sim 6\text{x}10^{-5}\ \text{eV/Tesla}$$

$B_{ext} << 0.2$ Teslas is weak field

$B_{ext} >> 0.2$ Teslas is strong field

7.5 Hyperfine Splitting:

These are corrections to the H atom that are even much smaller than the fine structure corrections, yet they are very important.

$$\mu_e = -\frac{e}{m_e}\mathbf{S}_e \qquad \mu_p = \frac{g_p e}{2m_p}\mathbf{S}_p$$

The proton also has a spin, like the electron. The magnitude of the dipole moment is much smaller because of m_p in the denominator. Still, μ_p is nonzero. g_p is ~5 because the proton is composed of 3 quarks.

Any dipole moment is felt like a magnetic field by the other spin, thus an interaction arises which for the l=0 ground state is:

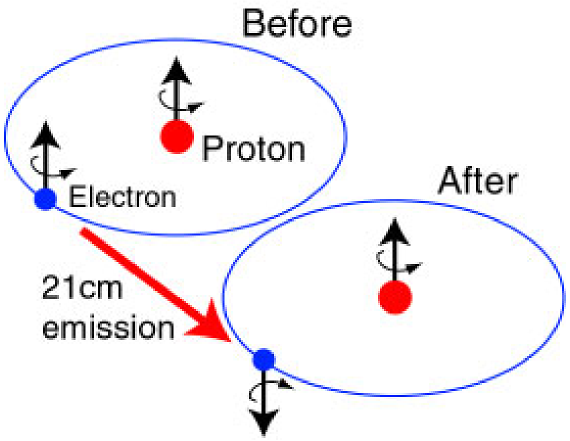
$$E^1_{\text{hf}} = \frac{\mu_0 g_p e^2}{3\pi m_p m_e a^3}\langle \mathbf{S}_p \cdot \mathbf{S}_e \rangle$$

Similar to: $\mathbf{S}\cdot\mathbf{L}$

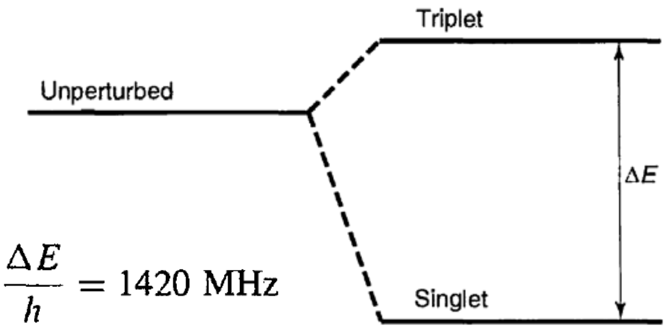
$$E^1_{\text{hf}} = \frac{4 g_p \hbar^4}{3 m_p m_e^2 c^2 a^4}\begin{cases} +1/4, & \text{(triplet);} \\ -3/4, & \text{(singlet).} \end{cases}$$

$$\Delta E = \frac{4 g_p \hbar^4}{3 m_p m_e^2 c^2 a^4} = 5.88 \times 10^{-6}\ \text{eV}$$

NOTE: There even more petite corrections! The correction coming from the finite size of the nucleus is even smaller.



Spin flip transition

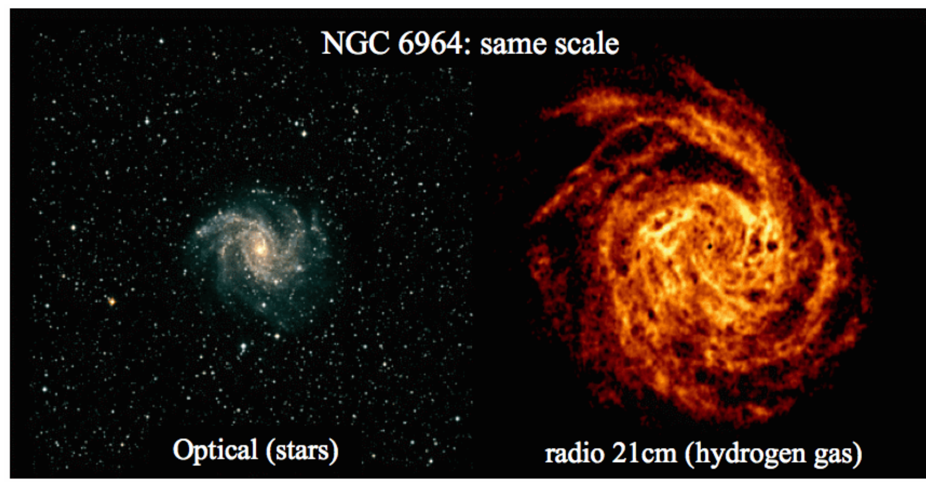


$$\nu = \frac{\Delta E}{h} = 1420 \text{ MHz}$$

$$c/\nu = 21 \text{ cm}$$

Lifetime 10M years.

The 21-cm line is a "fingerprint" of hydrogen gas, which is 90% of the gas between stars. The emitted radiation penetrates "dust" in space, that blocks visible light. The spiral arms of galaxies, made mostly of H, can be mapped studying this frequency.



Galaxy about 1/3 size Milky Way.