Introduction

In non-relativistic classical mechanics, Newton's law says

$$ma = F$$

As a second order differential equation and assuming the force is conservative (arises from a potential energy function $V(x)$, unlike friction), then (in 1D for simplicity):

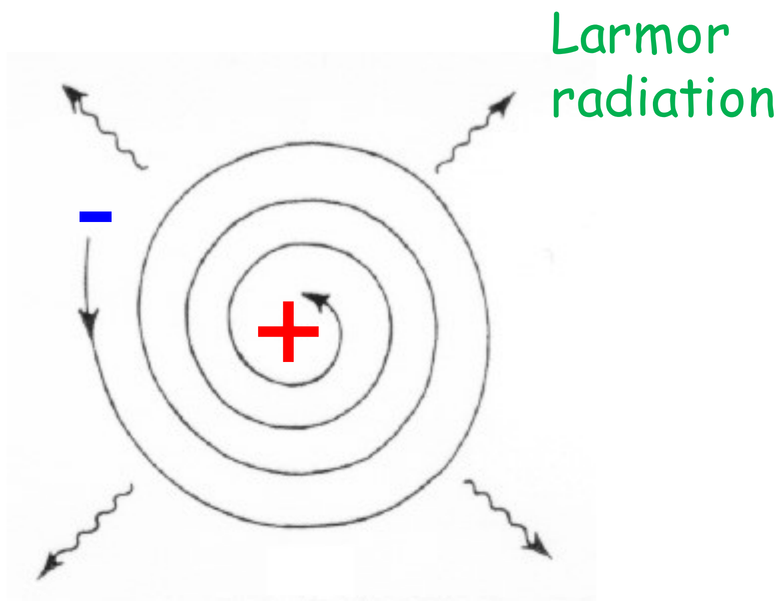
$$\boxed{m\, d^2x/dt^2 = -\partial V/\partial x}$$

Then, typically we solve this 2nd order differential eq. with some initial conditions at $t=0$, such as $x(0)$ and $dx/dt\ (t=0)$, and find $x(t)$.

From $x(t)$ we get position, velocity, acceleration, kinetic energy, etc.

In addition, we have Maxwell's equations for electrodynamics.
All seems very nice and clear, right?

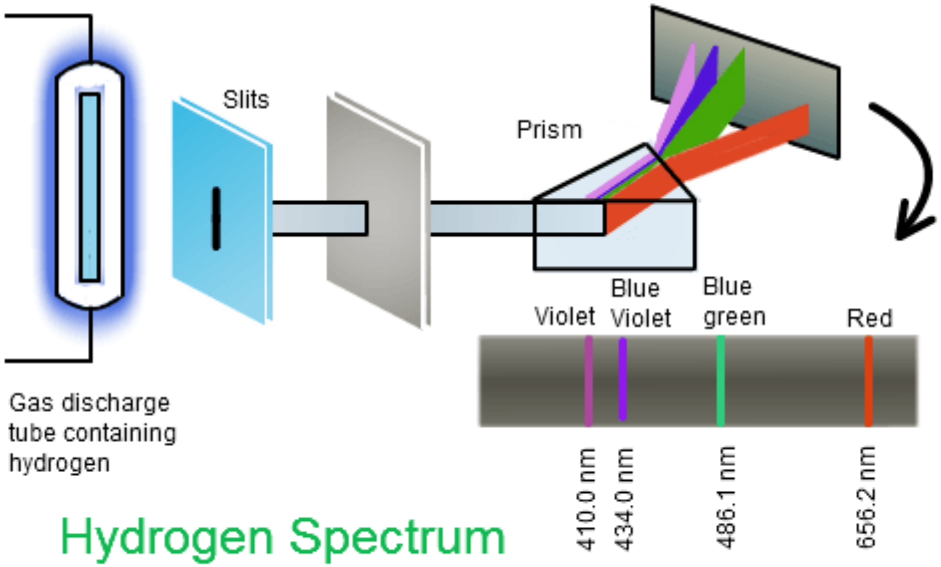
<u>However, at the atomic level classical physics does not work</u>



The classical view of an atom as a miniature "solar system" does not work because electron and proton are charged.

Within classical electromagnetism (Maxwell equations) charged particles in a circular orbit loose energy because they emit Larmor radiation. Lifetime estimated to be 10^{-10} seconds.

In addition, when hydrogen atoms inside a tube absorb energy, and then returns the energy as light, the spectrum is found to be discrete, with just a few lines.



Classical physics has no explanation for this result at all.

We need something drastically different ...

Chapter 1

Classical Mechanics must be replaced by Quantum Mechanics at short distances.

Instead of Newton's equation we will have the Schrödinger equation (Sch. Eq.)

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi}$$

New fundamental constant of Nature is introduced. The Planck's constant:

$$\hbar = \frac{h}{2\pi} = 1.054572 \times 10^{-34}\,\text{J s}$$

Energy x time

Imaginary unit: we will deal with complex numbers.

The "wave function" $\Psi(x,t)$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

(plus boundary conditions and $\Psi(x,t=0)$)

Planck's constant.

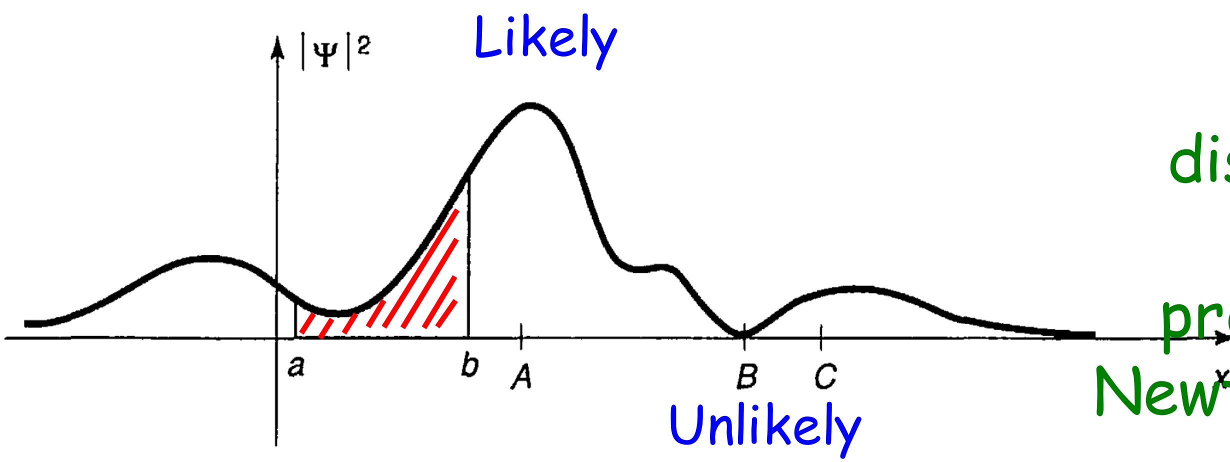
Mass of particle, typically electron.

Potential $V(x,t)$ given to you (e.g. harmonic oscillator)

What is the wave function? In classical mechanics we need x(t), but $\Psi(x,t)$ is a function of x and t.
So it cannot be the position of the electron …

Born's statistical interpretation

$$\boxed{\int_a^b |\Psi(x,t)|^2\,dx = \left\{ \begin{array}{l} \text{probability of finding the particle} \\ \text{between } a \text{ and } b\text{, at time } t. \end{array} \right\}}$$



The statistical interpretation is disturbing, because it's based on probabilities, unlike Newtonian mechanics.

TOO early to start philosophical discussions, but following the book we will address: if I measure the position of a particle and it is at x=c, where was an instant before?

Realistic view: the particle was at x=c or very close. Thus, QM is incomplete since it did not know it. There must be a more fundamental theory (Einstein's view).

Orthodox view: the wave function is the particle. Measuring is something peculiar … (virtually everybody accepts this view).

Agnostic view: the question cannot be verified experimentally, thus it is methaphysics.

Definitely QM is against "common sense".
At short distances, weird things happens!

Any one who is not shocked by quantum mechanics has not fully understood it. Niels Bohr

If you think you understand quantum mechanics, then you're not trying hard enough. Richard Feynman

Similar anti-common-sense behavior near large masses (general relativity), at huge distances (accelerating expanding universe), or at huge velocities (c is max).

The best approach is to become familiar with the formalism, understand the concepts and how to calculate, and … slowly … you get used to quantum mechanics.

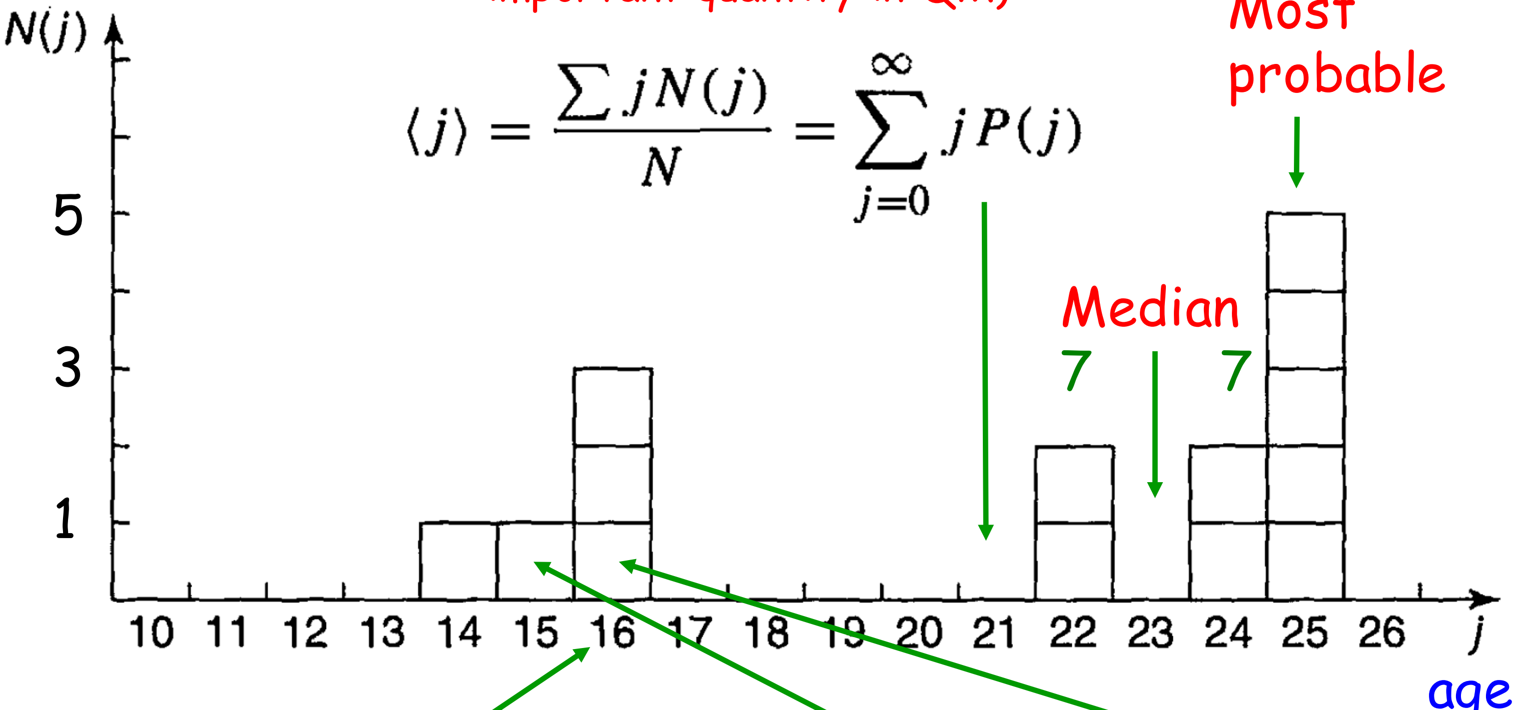
$$N = \sum_{j=0}^{\infty} N(j) = 14$$

Number of people of age j

Average or mean or expectations value (most important quantity in QM)

$$\langle j \rangle = \frac{\sum j N(j)}{N} = \sum_{j=0}^{\infty} j P(j)$$

Most probable



example

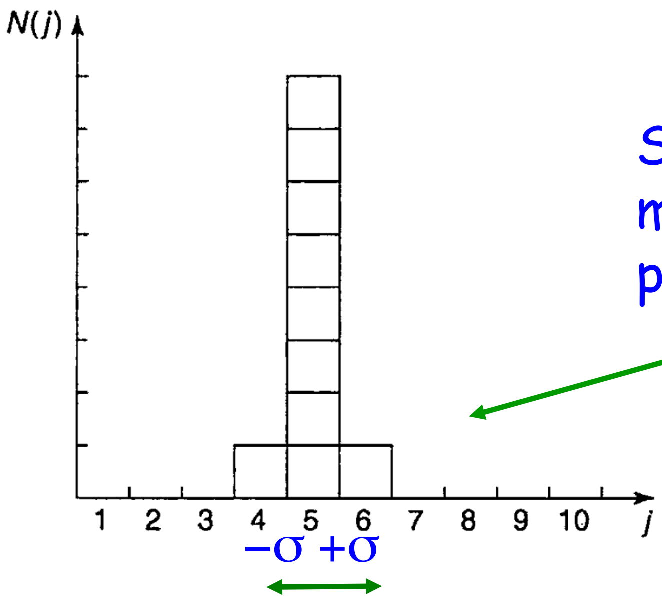
$$P(j) = \frac{N(j)}{N} \quad P(16) = 3/14$$

$$P(15) = 1/14, \; P(16) = 3/14$$

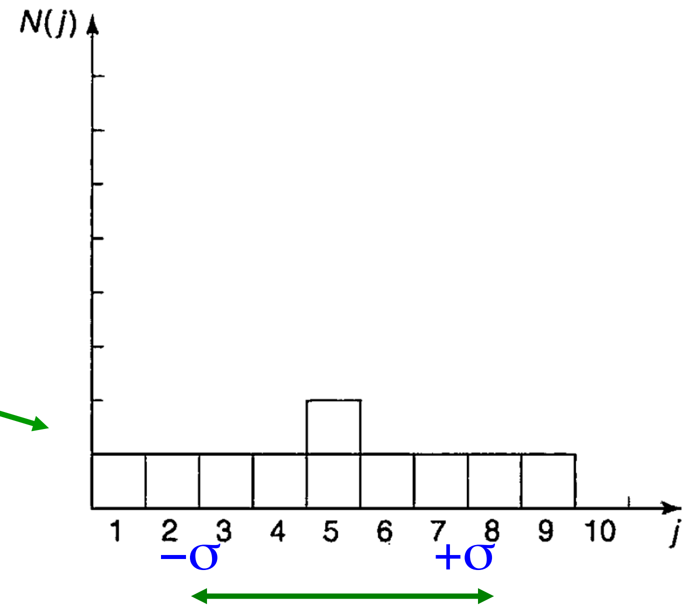
$$P(15)+P(16)=4/14$$

$$\sum_{j=0}^{\infty} P(j) = 1.$$

In addition to average, median, and most probable, there is another very important quantity to characterize a histogram: the standard deviation (or width).



Same average, median, most probable = 5



$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$$

Standard deviation

$$\langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j)$$

When we use continuous variables (say x instead of j) then we have to talk about a probability density.

$$\left\{ \begin{array}{l} \text{probability that an individual (chosen} \\ \text{at random) lies between } x \text{ and } (x + dx) \end{array} \right\} = \rho(x)\, dx$$

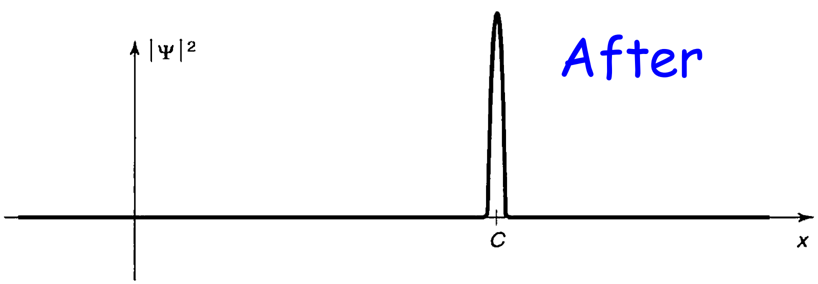
$$P_{ab} = \int_a^b \rho(x)\, dx \qquad 1 = \int_{-\infty}^{+\infty} \rho(x)\, dx$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} x\rho(x)\, dx \qquad \sigma^2 \equiv \langle (\Delta x)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

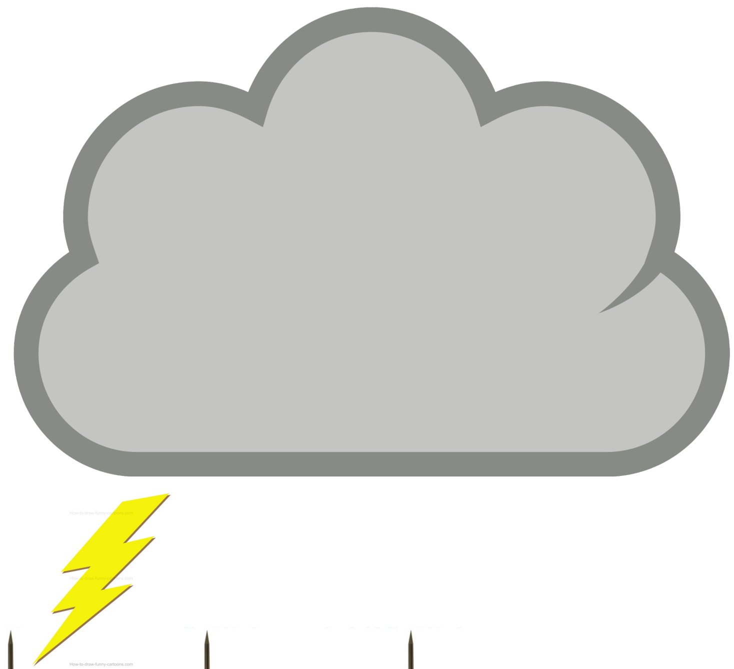
So $|\Psi(x,t)|^2$ is a probability density.

… read about collapse of the wave function … if you are brave … in the book. We will return to this later.

It addresses the interaction of a quantum object, the electron, with a classical and large object, the measuring device. VERY difficult. Measuring is not trivial!



After



Lightning rods