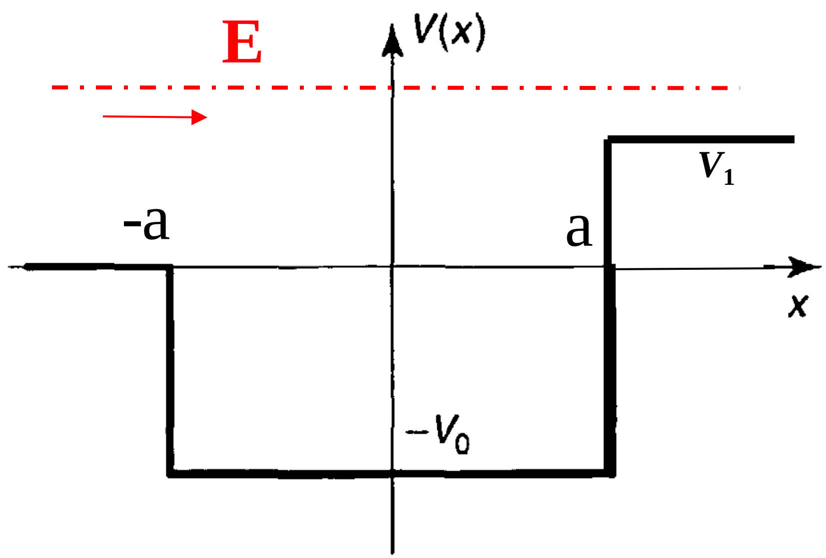
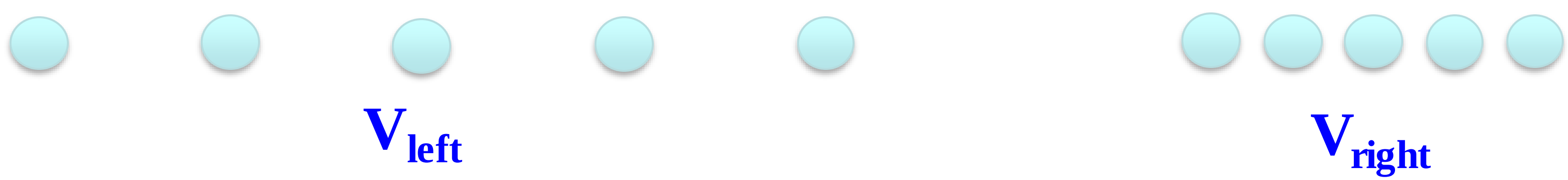
Note about problem 2.34, HW6: R+T=1 will work in the asymmetric case -- where the potential far left and far right are different -- only if velocities are incorporated.



It is like counting cars in a highway before and at a construction zone: if you use the same "dt" you may think you have less cars in a traffic jam.



SOME MOVIES …

Movie 1: Probability shown, intermediate weird state with oscillations (plotting prob and potential together is confusing, not same units).

Movie 2: Re and Im out of phase causing smooth probability, until collision occurs. Note v_{group} and v_{phase} are different.

Movie 3: Well and barrier of same depth or height behave similarly for scattering state (not for bound states of course).

Movie 4: Walls far right and left added, widths grows, eventually particle spreads uniformly.

Movie 5: Case where width is smaller than obstacle, then wave packet can collide twice within the same obstacle! And actually many times.

Movie 6: 2D collision. Now some aspects seem like tunneling and other aspects seem like classical scattering at an angle.

Chapter 3

Consider a 3D vector:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\mathbf{a} = \sum_{n=1}^{3} c_n \, \boldsymbol{e}_n$$

A "dot product" can be defined as:

$$\mathbf{a.b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

It is not difficult to imagine a generalization to N dimensions and to complex numbers:

$$|\alpha\rangle \rightarrow \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$$

$$\mathbf{a} = \sum_{n=1}^{N} c_n \, \boldsymbol{e}_n$$

$$\langle\alpha|\beta\rangle = a_1^* b_1 + a_2^* b_2 + \cdots + a_N^* b_N$$

Linear transformations, represented by matrices, act on vectors. For instance, a rotation.

$$|\beta\rangle = T|\alpha\rangle \rightarrow \mathbf{b} = \mathbf{Ta} = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1N} \\ t_{21} & t_{22} & \cdots & t_{2N} \\ \vdots & \vdots & & \vdots \\ t_{N1} & t_{N2} & \cdots & t_{NN} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$$

In Quantum Mechanics, wave functions expressed as a sum over stationary states, is like an N → infinite dimensional vector

$$\Psi(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

Like full vectors **a**

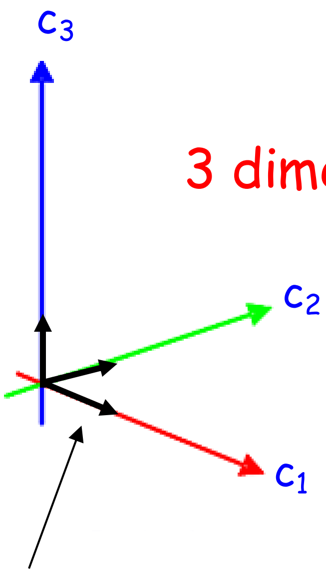
Like unit vectors $\mathbf{e}_n$

Of the set of possible functions, the subset that can be normalized ("square-integrable functions") is called a Hilbert space.

From many lectures back …

Cartesian axes

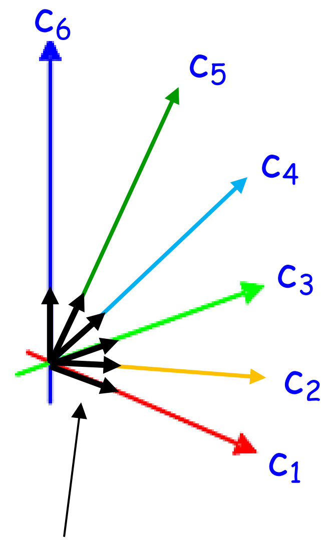
$$r = \sum_{n=1}^{3} c_n \, e_n$$



3 dimensions

Unit vectors e_1, e_2, e_3.

Any vector can be expanded in the orthonormal basis e_1, e_2, e_3.



$$f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

Square well solutions

∞ dimensions

"Unit vectors" are $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \ldots$

Any wave function can be expanded in the orthonormal basis ψ_n

All these properties are not pathological of the square well or harmonic oscillator but generic.