Test 2 is Nov 1

Chapter 2: subjects to study start **after** Harmonic oscillator, i.e. covers free particle, bound and scattering states, delta function, finite square well.

Chapter 3: subjects include all that I will be teaching until lecture Oct 25 included.

HWs : 4,5,6,7,8 (4 is one problem, 7 is two problems, etc).

HW6: solution will circulate today, graded returned Oct. 25.

HW7: deadline Oct 25, solution will circulate Oct 25, returned Oct. 30.

HW8: deadline Oct.30, solution will circulate Oct. 30, returned Oct. 31 at noon, left in Physics Dept office.

For the definition of Hermitian operators most books require using two functions f(x) and g(x). Steps are the same as we did, no worries.

$$\langle f | \hat{Q} g \rangle = \langle \hat{Q} f | g \rangle$$

If $\hat{Q}$ is NOT Hermitian, like d/dx, then the **definition of Hermitian of an operator** (a.k.a. Hermitian conjugate, or adjoint) is the operator $\hat{Q}^{\dagger}$ that satisfies:

$$\langle f | \hat{Q} g \rangle = \langle \hat{Q}^{\dagger} f | g \rangle$$

Examples: $(d/dx)^{\dagger} = -(d/dx)$, $(i)^{\dagger} = -i$, ...

"Determinate" States

The "stationary states" of the $\hat{H}$ Hamiltonian, Ψ_n, had a sharp energy E_n . Can we do the same for other operators $\hat{Q}$?

Similarly as when we used to write $\hat{H}\Psi_n(x)=E_n\Psi_n(x)$, we want *eigenfunctions of* $\hat{Q}$, $f_q(x)$, such that

$$\hat{Q}\, f_q(x) = q\, f_q(x) \; [q=q_1,q_2,q_3, \ldots]$$

q is a number: the "eigenvalue" of the operator $\hat{Q}$. The reason for the language is the similarity to a matrix operation $\mathbf{Ta} = \lambda \mathbf{a}$. (see page 449). There are many $\lambda = \lambda_1, \lambda_2, \ldots$

Example: in Chapter 4 we will discuss eigenfunctions of the angular momentum operator $\hat{L}$, the "spin" $\hat{S}$, etc.

Example, how about the momentum operator $\hat{p}$.

First we need an eigenfunction $f_p(x)$ such that

$$\frac{\hbar}{i}\frac{d}{dx} f_p(x) = p\, f_p(x)$$

$\hat{p}$ operator → $\frac{\hbar}{i}\frac{d}{dx}$; eigenfunction → $f_p(x)$; eigenvalue → p

Solution is very easy (but normalization is complicated because they are not normalizable -> read book 103):

$$f_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$$

Then, given an arbitrary wave function $\Psi(x,t)$, we should calculate (see later)

$$c(p) = \langle f_p | \Psi \rangle = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar}\, \Psi(x,t)\, dx$$

$f_p^*(x)$

For continuous eigenvalues c(p) no longer "a number from 0 to 1".

There are many theorems for Hermitian operators, the operators that matter for observables, that I will NOT prove:

(1) The eigenvalues q are real, like the energies E_n were. If you measure $\hat{Q}$ in any $\Psi(x,t)$, you will get one of the q's.

(2) The eigenfunctions $f_q(x)$ for different q's are orthonormal, like $\Psi_n(x)$ for energies were.

(3) The eigenfunctions are complete, like $\Psi_n(x)$ for energies were.

Caveat: careful with degenerate states i.e. those with the same eigenvalue q.

Generalized statistical interpretation

Suppose the electron is in a state $\Psi(x,t)$. We know that the probability of measuring E_n is $|c_n|^2 = | < \Psi_n(x)| \Psi(x,t) > |^2$.

Suppose in the **same** state $\Psi(x,t)$ I measure say the momentum, or angular momentum, etc. What will I find?

If I measure the Hermitian (observable) operator $\hat{M}$, the probability of finding "m" is $|c_m|^2 = | < f_m(x) | \Psi(x,t) > |^2$ if the eigenvalues are discrete, like in angular momentum.

Like with the energy, the total probability of measuring "some" value for operator $\hat{M}$ must be 1.

$$\Sigma_m |c_m|^2 = 1$$

For eigenvalues that are continuous, the probability of measuring "p", like a linear momentum, requires a tiny width for its definition

$$|c_p|^2\, dp = |\langle f_p(x) \mid \Psi(x,t) \rangle|^2\, dp,$$

dimensionless

and then you must integrate in a finite range from say p_a to p_b.

For the operator $\hat{x}$, we recover the old result (see book page 108; warning a bit complicated):

$$\int_a^b |\Psi(x,t)|^2\, dx = \left\{ \begin{array}{l} \text{probability of finding the particle} \\ \text{between } a \text{ and } b \text{, at time } t. \end{array} \right\}$$