DETOUR after grading HW1:

If 50/60 or higher, do not worry.
Just check in detail the answers provided.

"See Prof's solution" means it is difficult to find a small error in a long algebra (too many HWs to grade). Better if student does it.

"Provide more detail next time" means your answer was so terse that leads me to believe that (perhaps) you copied it from somewhere.

"Be more clear next time" likely means your handwriting was difficult to understand, your presentation was messy, … Prof problem? Maybe, but likely employers will think similarly.

"Problem 1.17 missing". Most students noticed this extra problem was given when I moved the deadline to later, but a few perhaps forgot.

"Frame answers". Makes grader happier to see a clearly presented exam or HW. Do it for you!

Two main physics issues in HW1:

If you are asked about a probability, it requires an integral. The prob density $|\psi(x)|^2$, is not the answer. You cannot read a probability from $\psi(x)$, you must integrate in an interval.

THE MOST IMPORTANT: think if your answer makes sense. For instance, if $\langle x \rangle$ is say a/b, then the dimensions are wrong. If you are asked for a probability and your answer is say hbar, then the dimensions are wrong. If the standard deviation square is negative, then it is wrong, etc, etc, etc, etc, etc, etc.

Return to main lecture ...

There is more we can learn from this example.

For instance, $\Sigma_n |c_n|^2 = 1$. You can verify that adding, say, the first 10 terms. Reason?

$$1 = \int |\Psi(x,0)|^2 dx = \int \left(\sum_{m=1}^{\infty} c_m \psi_m(x) \right)^* \left(\sum_{n=1}^{\infty} c_n \psi_n(x) \right) dx$$

$$= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_m^* c_n \int \psi_m(x)^* \psi_n(x) \, dx$$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_m^* c_n \delta_{mn} = \sum_{n=1}^{\infty} |c_n|^2.$$

This happens only if the given $\Psi(x,0)$ is already normalized to 1.

Thus, $\sum_{n=1}^{\infty} |c_n|^2 = 1$ is equivalent to normalization 1, which is probability of finding the particle somewhere in the well 100%.

Moreover, $|c_1|^2 = 0.99855...$ implying other coefficients are very small. Why? Because $\Psi(x,0)$ resembles the ground state!

If you measure the energy, it will be shown that $|c_n|^2$ is the probability that you will find E_n as result. Here, the chance of measuring E_1 is high.

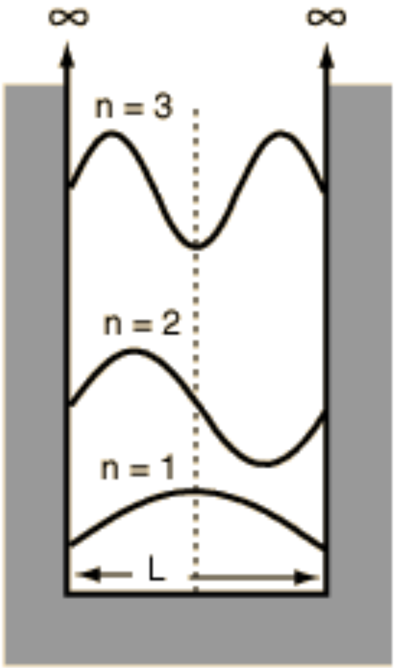
It can also be shown (book p37) that:

$$\langle H \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n$$

In general for <O>, with O any operator, this is not true.

Moreover, here <H> is only slightly above E_1 compatible with $|c_1|^2 \sim 1$

Summary



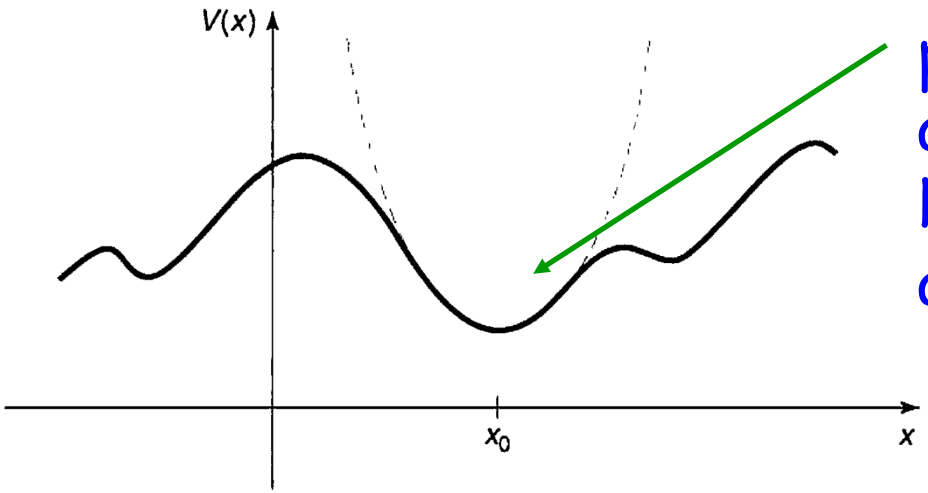
x = 0 at left wall of box.

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) ,$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \overbrace{e^{-i(n^2\pi^2\hbar/2ma^2)t}}^{e^{-iE_n t/\hbar}}$$

The harmonic oscillator



Any minimum of a potential can be approximated by a harmonic oscillator as long as oscillations are small.

We wish to study the harmonic oscillator from the QM perspective:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\underbrace{\frac{1}{2}m\omega^2x^2}_{V(x)=\frac{1}{2}m\omega^2x^2}\psi=E\psi$$

Two methods will be used to arrive to the same answer:

Algebraic method (uses a+ and a- operators

Analytic method (uses polynomials)

Preliminary exercise: commutation relations

We define the commutator between operators *A* and B as:

$$[A, B] \equiv AB - BA$$

Suppose *A*=*x* and B=*p*. Thus, we want $[x, p] = (xp - px)$.
This is NOT zero because *p* is the derivative operator.

To know its value you need a general "test function" *f(x)*.

$$[x, p]f(x) = \left[x\frac{\hbar}{i}\frac{d}{dx}(f) - \frac{\hbar}{i}\frac{d}{dx}(xf)\right] = \frac{\hbar}{i}\left(x\frac{df}{dx} - x\frac{df}{dx} - f\right) = i\hbar f(x)$$

$$\boxed{[x, p] = i\hbar}$$

In general, operators do not commute.

Algebraic method

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi \longrightarrow \frac{1}{2m}\underbrace{[p^2 + (m\omega x)^2]}_{\text{Sum of squares}}\psi = E\psi$$

$p \equiv (\hbar/i)d/dx$

For numbers $u^2 + v^2 = (iu + v)(-iu + v)$ but not for operators.

This factor is for future convenience. Not obvious

By analogy try $\boxed{a_{\pm} \equiv \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)}$ and cross fingers.

$$a_- a_+ = \frac{1}{2\hbar m\omega}(ip + m\omega x)(-ip + m\omega x) = \frac{1}{2\hbar m\omega}[p^2 + (m\omega x)^2 - im\omega(xp - px)]$$

$$a_- a_+ = \frac{1}{2\hbar m\omega}[p^2 + (m\omega x)^2] - \frac{i}{2\hbar}[x, p] = \frac{1}{\hbar\omega}H + \frac{1}{2}$$

$[x, p] = i\hbar$

$$H = \hbar\omega\left(a_{-}a_{+} - \frac{1}{2}\right)$$