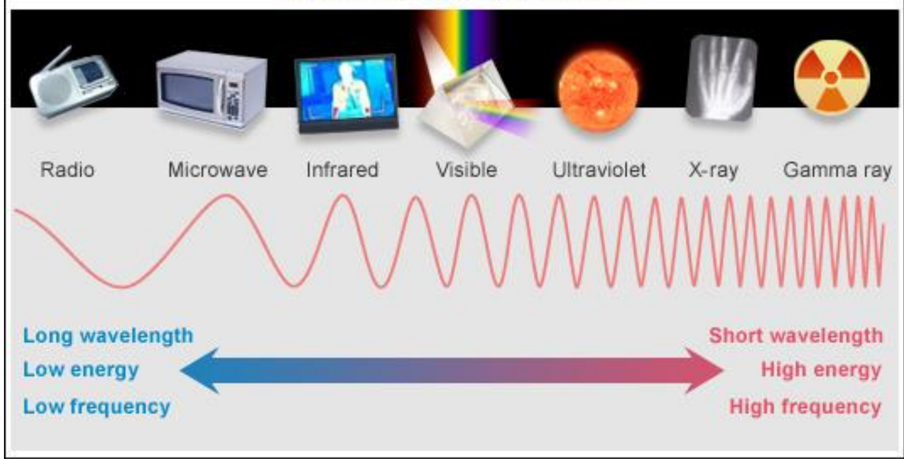
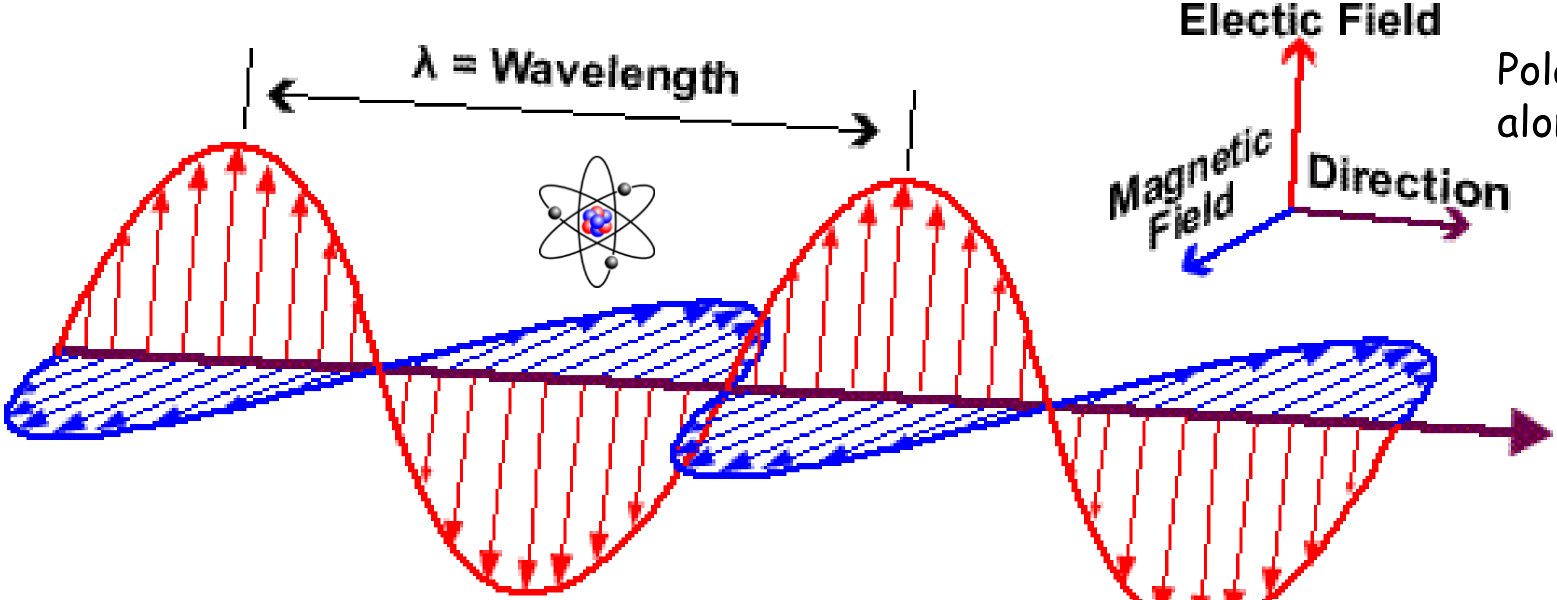
9.2: Emission and absorption of radiation
9.2.1: Electromagnetic waves



The radiation behind radios, visible light and nuclear blasts is all exactly the same thing, just with different amounts of energy.



Polarized light along z axis

If atom is much smaller than wavelength, then the electric field is ~ uniform inside atom

$$\mathbf{E} = E_0 \cos(\omega t)\,\hat{k}$$

Visible light is ~5000 Å while an atom is ~ 1 Å

$$H' = -qE_0 z\cos(\omega t)$$

$$\mathbf{E} = -\nabla H'(\mathbf{r})/q$$

$$\mathbf{E} = E_0\cos(\omega t)\,\hat{k}$$

We will see that diagonal matrix elements of *H′* vanish by symmetry for atoms such as hydrogen. Thus the matrix elements that matter are (a≠b):

$$H'_{ba} = -\wp E_0\cos(\omega t)$$

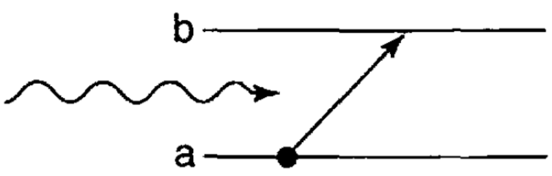
where the electric dipole moment →

$$\wp \equiv q\langle\psi_b|z|\psi_a\rangle$$

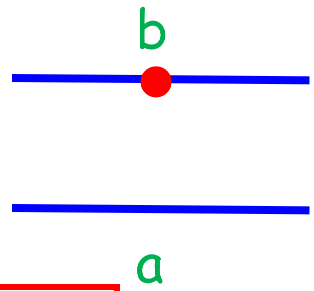
Relation with previous generic formulas require replacement →

$$V_{ba} = -\wp E_0$$

$$P_{a\to b}(t) = \left(\frac{|\wp|E_0}{\hbar}\right)^2 \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}$$



$$c_a(0) = 0, \; c_b(0) = 1$$

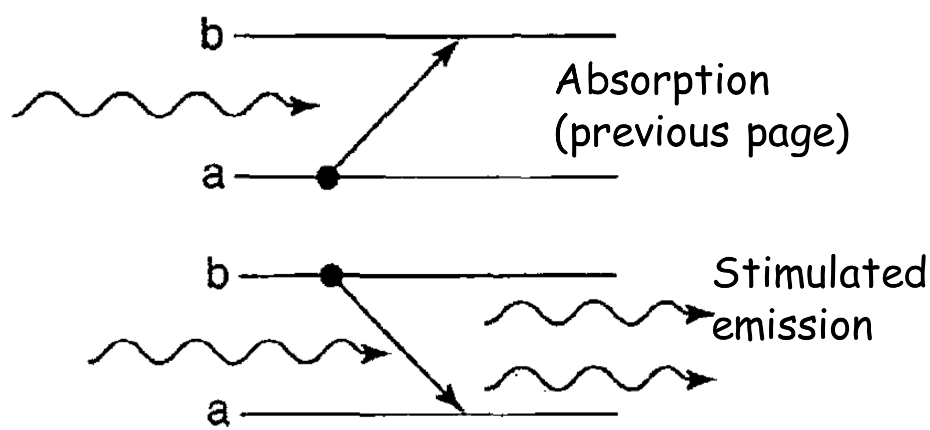


Suppose we repeat the same calculation as in the previous lecture but with the electron first at b. Result is the same just switching a < -- > b.

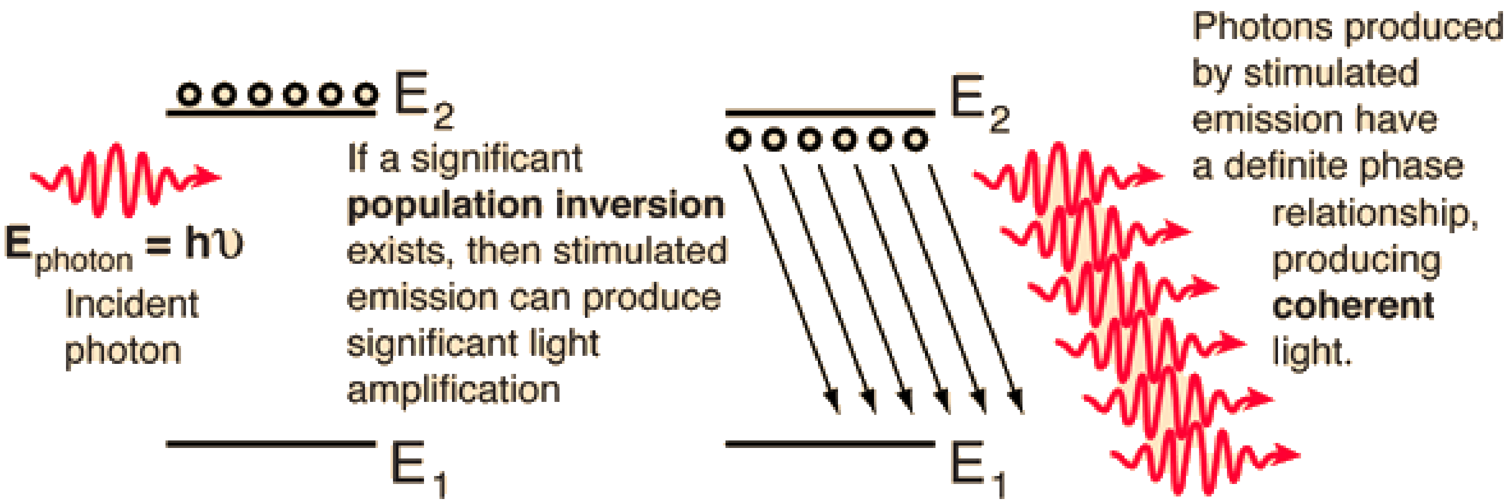
$$P_{b \to a}(t) = \left(\frac{|\wp| E_0}{\hbar} \right)^2 \frac{\sin^2[(\omega_0 - \omega)t/2]}{(\omega_0 - \omega)^2}$$

As expressed before, even though you are originally at "b", by merely being immersed in a radiation field (i.e. a lot of photons) the electron can decay to a lower energy.

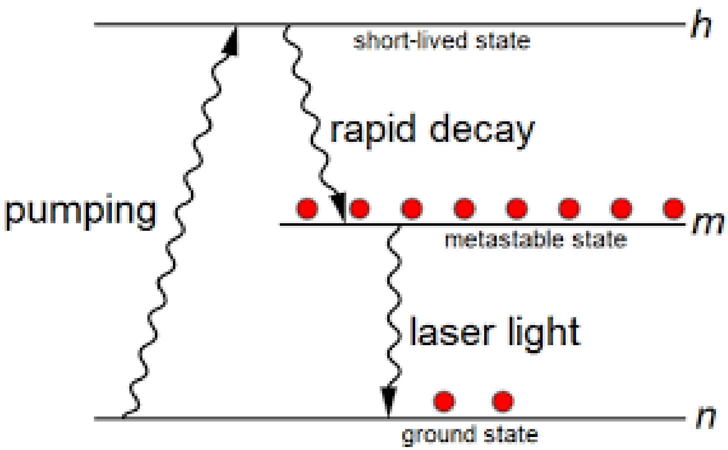
This is called **stimulated emissio** The electron in "b" is "unstable", and any perturbation (like a shower of photons) may drop it to "a". Actually the probability a->b and b->a is the same.



Amplification can occur. Almost instantly a huge number of photons in phase can be produced:



You need three states in practice:



This is the basis of Light Amplification by Stimulated Emission of Radiation (LASER).

9.2.1: Incoherent Perturbations

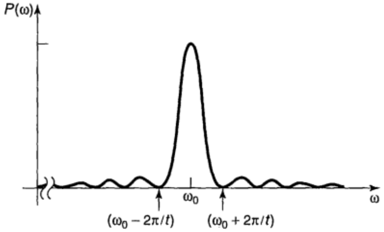
So far we have assumed the "external" field has only one frequency.

However, radiation is never perfectly monochromatic. There are always many frequencies contributing (non-monochromatic).

It can be shown that in the presence of many frequencies ω with a particular energy density ρ(ω) the probability for the transition becomes:

$$P_{b\to a}(t) = \frac{2}{\epsilon_0 \hbar^2}|\wp|^2 \int_0^\infty \rho(\omega)\left\{\frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}\right\} d\omega$$

Because {…} is sharply peaked, then:



$$P_{b\to a}(t) \cong \frac{2|\wp|^2}{\epsilon_0 \hbar^2}\rho(\omega_0)\int_0^\infty \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2} d\omega$$

Using the mathematical identity $$\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2}\, dx = \pi$$

we arrive to $$P_{b\to a}(t) \cong \frac{\pi |\wp|^2}{\epsilon_0 \hbar^2} \rho(\omega_0) t$$

and taking time derivative to obtain a probability rate $R \equiv dP/dt$ $$R_{b\to a} = \frac{\pi}{\epsilon_0 \hbar^2} |\wp|^2 \rho(\omega_0)$$

This is a constant (no longer an oscillatory function).

So far we also assumed the radiation was polarized with electric field along z. If we average in all directions (unpolarized):

$$R_{b\to a} = \frac{\pi}{3\epsilon_0 \hbar^2} |\boldsymbol{\wp}|^2 \rho(\omega_0)$$

$$\wp \equiv q \langle \psi_b | z | \psi_a \rangle \Rightarrow \boldsymbol{\wp} \equiv q \langle \psi_b | \mathbf{r} | \psi_a \rangle$$

9.3.3 Selection rules

Very often the matrix elements $\langle \psi_b | \mathbf{r} | \psi_a \rangle$ that appear in the rate are zero by symmetry.

Consider the hydrogen atom. In this case: $\langle n'l'm' | \mathbf{r} | nlm \rangle$

Commutators discussed time ago (Ch. 4)

$$[L_z, x] = i\hbar y, \quad [L_z, y] = -i\hbar x, \quad [L_z, z] = 0$$

allow us to arrive (page 360) to the first rule:

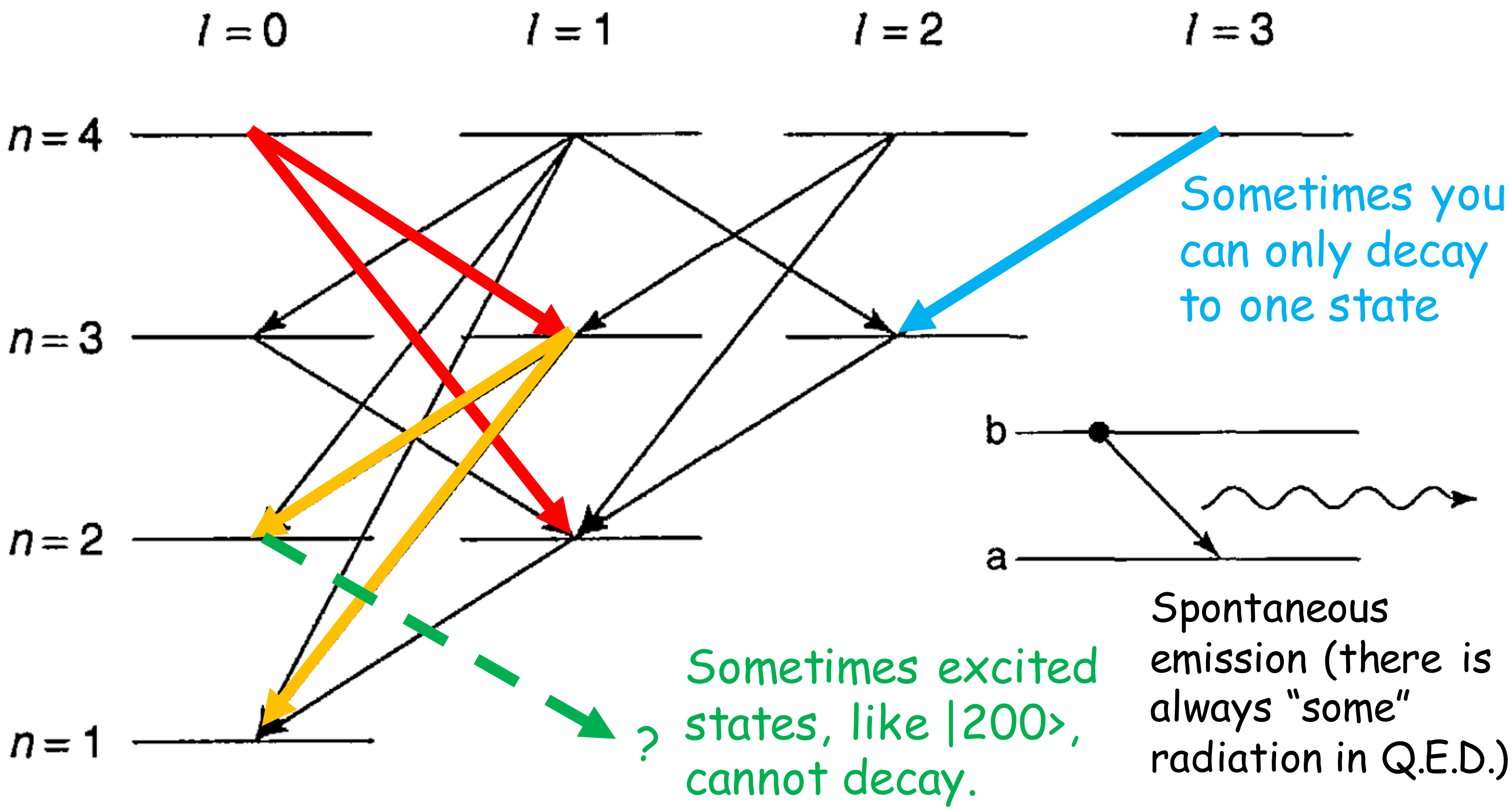
No transitions occur unless $\Delta m = \pm 1$ or 0

From many other commutators (see book pages 361-362), we can derive the second rule:

No transitions occur unless $\Delta l = \pm 1$

Although for us the external field is not quantum, intuitively the results are in agreement with "photons" emitted or absorbed, because photons have spin s=1 (bosons) and projections $m_s = -1, 0, 1$.

Conservation of angular momentum leads to these selections rules: whatever happens to the electron in the atom, must be compensated by the photon.



The states that cannot decay are "metastable" with long lifetimes. They eventually decay from atomic collisions or emitting two photons (much lower probability).