When an electron is at rest inside a uniform magnetic field $\mathbf{B}$, or for example in a *1s* level of H, the spin Hamiltonian is

$$H = -\boldsymbol{\mu} \cdot \mathbf{B} = -\gamma \mathbf{B} \cdot \mathbf{S}$$

In lecture and book $\mathbf{B} = (0,0,1) B_0$. Then,

$$\mathbf{H} = -\gamma B_0 \mathbf{S}_z = -\frac{\gamma B_0 \hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Eigenfunctions, or eigenspinors, were in that case:

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Consider the same Hamiltonian but now with the magnetic field $\mathbf{B} = (2,\text{-}1,0)B_0$. Note sign change γ->-γ for electrons.

$$H = -\boldsymbol{\mu} \cdot \mathbf{B} = +\gamma \mathbf{B} \cdot \mathbf{S}$$

(a) Write the 2x2 matrix you need to diagonalize. Leave γ, ħ/2, and B_0 as overall constants in front.
(b) Find the eigenvalues.
(c) Find the eigenvectors normalized to 1.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Answers:

$$\mathbf{H} = \gamma(\hbar/2) B_0 (2\sigma_x - \sigma_y) = \gamma(\hbar/2) B_0 \begin{pmatrix} 0 & 2+i \\ 2-i & 0 \end{pmatrix}$$

$$\lambda = \pm\, \gamma(\hbar/2) B_0 \sqrt{5}$$

For $+\sqrt{5}$: $$\chi = (1/\sqrt{2}) \begin{pmatrix} 1 \\ (2-i)/\sqrt{5} \end{pmatrix}$$

For $-\sqrt{5}$: $$\chi = (1/\sqrt{2}) \begin{pmatrix} 1 \\ -(2-i)/\sqrt{5} \end{pmatrix}$$