Not in book (counting of states):

$$\uparrow\uparrow \qquad \uparrow\uparrow$$

$$\uparrow\downarrow \qquad \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$$

$$\downarrow\uparrow \qquad \downarrow\downarrow$$

$$\downarrow\downarrow \qquad \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

$2^2=4$ states sort of random

4 states grouped as S=1 (3) and S=0 (1)

Note: after finding S=1, there was only 1 state left, thus **had to be singlet** and **had to be orthogonal**, thus fixing the "-"

$$\uparrow\uparrow\uparrow$$

$$\uparrow\uparrow\downarrow,\ \uparrow\downarrow\uparrow,\ \downarrow\uparrow\uparrow$$

$$\uparrow\downarrow\downarrow,\ \downarrow\uparrow\downarrow,\ \downarrow\downarrow\uparrow$$

$$\downarrow\downarrow\downarrow$$

$2^3 = 8$ states sort of random

$$\uparrow\uparrow\uparrow,\ S\_\uparrow\uparrow\uparrow,\ S^2\_\uparrow\uparrow\uparrow,\ S^3\_\uparrow\uparrow\uparrow$$

4 states form S total 3/2

The 4 states left form TWO S total $\frac{1}{2}$ states.

$3/2 \oplus 1/2 \oplus 1/2$

Not in book (and FYI only):

FYI: spins can interact among themselves, not only with magnetic fields.

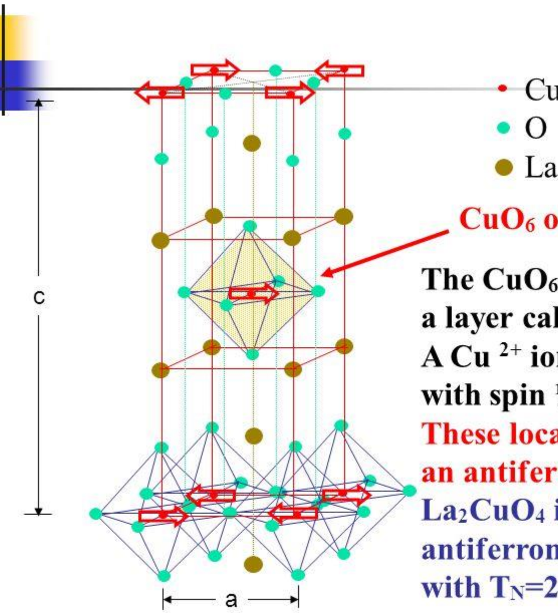
It is as if other spins "j" produce an effective magnetic field on the spin "i" you are looking at.

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Ground state? Number of states grows like 2^N (=2,4,8,16, …)

Record done exactly N~ 40.
2^{40} = 1,099,511,627,776 states

Crystal structure of undoped La_2CuO_4



The CuO_6 octahedrons form a layer called a CuO_2 layer. A Cu^{2+} ion has one d hole with spin ½.
These localized spins form an antiferromagnetic phase.
La_2CuO_4 is the antiferromagnetic insulator with T_N=240K.

WITHOUT PROOF, this is what happens when you combine a spin s_1 and a spin s_2 (each individually 0,1/2,1,3/2, ...).

The total spin s of the combination can be:

$$s = (s_1 + s_2),\ (s_1 + s_2 - 1),\ (s_1 + s_2 - 2),\ \ldots,\ |s_1 - s_2|$$

Example 1: for $s_1=1/2$ and $s_2=1/2$, then s runs from $s_1+s_2 = 1$ to $|s_1-s_2| = 0$, with nothing in between.

Example 2: for $s_1=3/2$ and $s_2=2$, then s runs from $s_1+s_2 = 7/2$ to $|s_1-s_2| = 1/2$, with 5/2 and 3/2 in between.

Example 3: this unproven theorem holds also for the addition of orbital angular momentum l and spin s. For $l=2$ and $s=1/2$, then total j runs from for $l+s = 5/2$ to $|s_1-s_2|= 3/2$, with nothing in between.

Example 4: if you have three particles with $s_1=1/2$, $s_2=1/2$, and $s_3=1/2$, then first you add two, such as s_1 and s_2, finding $s_{partial} = 1,0$ and then add $s_{partial}$ with s_3 finding 3/2,1/2 (for $s_{partial}=1$) and another 1/2 (for $s_{partial}=0$). So there are two independent combinations with total spin $\frac{1}{2}$.

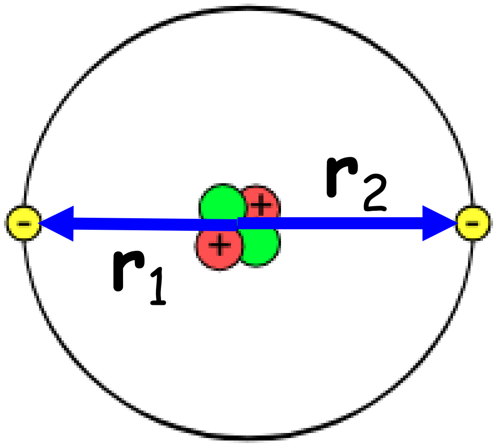
We will NOT deal with the **Clebsch-Gordan** coefficients, but with the foundation given already, it should be easy for you to learn from the book.

Chapter 5: Identical Particles

For one particle, like one electron in the H atom, in QM we simply need the wave function $\Psi(\mathbf{r}_1,t)$ where $\mathbf{r}_1$ is the coordinate of electron "1".

Consider now two particles as warm up.

For two particles, e.g. two electrons in the He atom, in QM we need the wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2,t)$ where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the two coordinates.

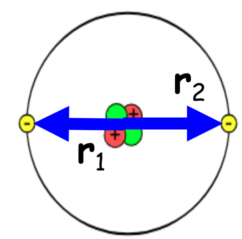


Mathematically, the Sch. Eq. $i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$ has a more complicated Hamiltonian.

$$\boxed{H = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\mathbf{r}_1, \mathbf{r}_2, t)}$$

The potential *V* typically has terms like e-p attraction, but also e-e repulsion.

Example, for the He atom:



$$H = \left\{ -\frac{\hbar^2}{2m}\nabla_1^2 \underbrace{- \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_1}}_{\text{p-e1 attraction}} \right\} + \left\{ -\frac{\hbar^2}{2m}\nabla_2^2 \underbrace{- \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_2}}_{\text{p-e2 attraction}} \right\} + \underbrace{\frac{1}{4\pi\epsilon_0}\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}}_{\text{e1-e2 repulsion}}$$

The last term, the e-e repulsion, makes everything "complicated" because it "correlates" the electrons:

When one e goes one way, the other e tries to avoid it, because they repel. Ignoring this e-e repulsion is an approx. that "sometimes is good, sometimes not".

Finally, as usual, we must normalize to 1 because of the probabilistic interpretation:

$$\boxed{\int |\Psi(\mathbf{r}_1, \mathbf{r}_2, t)|^2\, d^3\mathbf{r}_1\, d^3\mathbf{r}_2 = 1}$$

5.1.1 Bosons and Fermions

First, for simplicity, let us neglect the e-e repulsion. The energy levels are the **same** as in the H atom.

Assume one particle is in state "a" (e.g. 1s, spin up) and the other particle is in state "b" (e.g. 2s, spin down).

Then, ONLY in this particular case when e-e is neglected, the wave function is the product:

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$$

To prove this, use as potential V simply the sum of two H-atom terms, one per particle i.e. the p-e1 and p-e2 attractions.

However, the key new concept is that if electrons are identical, then we cannot say "electron 1 is in (1s,↑)". We can only say "**an electron** is in (1s,↑) ".

In classical physics we can always "follow" particles and tell them apart, even if identical. READ discussion in book pages 203-204.

In quantum physics we **cannot** follow particles. We only know probabilities.

Thus, if a particle is in state "a" and a particle in state "b", we need to symmetrize the wave function to account for particles being identical.

One way to symmetrize is to add the two cases:

$$\boxed{\psi_+(\mathbf{r}_1, \mathbf{r}_2) = A[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) + \psi_b(\mathbf{r}_1)\psi_a(\mathbf{r}_2)]}$$

In what sense this has been "symmetrized"?
When $\mathbf{r}_1$ and $\mathbf{r}_2$ are exchanged, the first term becomes the second, and the second the first.

Then:

$$\psi_+(\mathbf{r}_1,\mathbf{r}_2) = \psi_+(\mathbf{r}_2,\mathbf{r}_1)$$

Elementary particles where the "+" applies are called **bosons**.

Because in QM we only care about the wave function in absolute value, there is another possible combination:

$$\boxed{\psi_{-}(\mathbf{r}_1, \mathbf{r}_2) = A[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) - \psi_b(\mathbf{r}_1)\psi_a(\mathbf{r}_2)]}$$

In this case, when $\mathbf{r}_1$ and $\mathbf{r}_2$ are exchanged, we collect a minus sign in front:

$$\psi_{-}(\mathbf{r}_1,\mathbf{r}_2) = -\psi_{-}(\mathbf{r}_2,\mathbf{r}_1)$$

Elementary particles where the "-" applies are called **fermions**.

In summary, we have to accept as another law of Nature that elementary particles are either **bosons**, for the + case, or **fermions**, for the – case.

Moreover, there is a link between the sign +- in the combination and the value of the spin:

bosons <-> integer spin (example: photon)

fermions <-> half-integer spin (example: electron).