6.2 Degenerate Perturbation Theory:

What happens if the state you are correcting by perturbation theory is degenerate?

Consider first degeneracy two. We will drop the index "n" because degenerate states are not generic but special cases. This means:

$$H^0\psi_a^0 = E^0\psi_a^0 \qquad H^0\psi_b^0 = E^0\psi_b^0 \qquad \langle\psi_a^0|\psi_b^0\rangle = 0$$

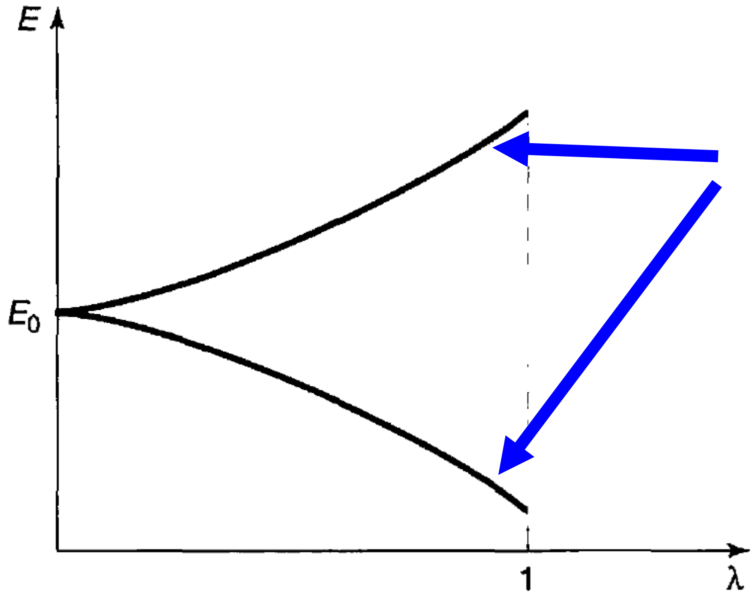
Same energy, like 2s and 2p levels in H atom.

Any linear combination has the same energy.

$$\psi^0 = \alpha\psi_a^0 + \beta\psi_b^0$$

$$H^0\psi^0 = E^0\psi^0$$

Usually, the perturbation H' will break the degeneracy.



Each split state is a particular linear combination of the two original degenerate states.

We will repeat the same steps as before:

$$H = H^0 + \lambda H'$$

$$E = E^0 + \lambda E^1 + \lambda^2 E^2 + \cdots$$

$$\psi = \psi^0 + \lambda \psi^1 + \lambda^2 \psi^2 + \cdots$$

To lowest order we arrive to the same equation as before, just without the index n.

$$H^0\psi^1 + H'\psi^0 = E^0\psi^1 + E^1\psi^0$$

Again, follow the same procedure as before, namely take the inner product but using first ψ_a^0 and then ψ_b^0 :

$$\langle\psi_a^0|H^0\psi^1\rangle + \langle\psi_a^0|H'\psi^0\rangle = E^0\langle\psi_a^0|\psi^1\rangle + E^1\langle\psi_a^0|\psi^0\rangle$$

Via hermiticity of H^0, again these two cancel.

Gives only α

$$\alpha\langle\psi_a^0|H'|\psi_a^0\rangle + \beta\langle\psi_a^0|H'|\psi_b^0\rangle = \alpha E^1$$

$$\boxed{\alpha W_{aa} + \beta W_{ab} = \alpha E^1}$$

When the "b" degenerate state is used, we find a similar formula:

$$\alpha W_{ba} + \beta W_{bb} = \beta E^1$$

$$\alpha W_{aa} + \beta W_{ab} = \alpha E^1$$

It is like a 2x2 matrix problem. α and β are the components of the eigenfunction and E^1 the eigenvalue.

$$E^1_{\pm} = \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\,\right]$$