Stripe ordering in the Cuprates

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I. SUPERCONDUCTIVITY: AN OVERVIEW

Upon cooling, many materials undergo an electronic phase transition leading to infinite conductivity. This state of matter, known as superconductivity, was first discovered in 1911 by Kamerlingh Onnes while working with liquefied helium [Onnes]. At 4.2K he discovered that the resistivity of mercury dropped below detectable values. Shortly after this discovery, many other elements, such as tin, were also observed to exhibit zero resistance at extremely low temperatures. Indeed, the discovery of superconductivity in simple systems was necessarily delayed until refrigeration techniques advanced sufficiently to reach temperatures close to absolute zero.

The discovery of superconductivity came as a shock to the scientific community. As mercury cools, scientists expected that the resulting state would consist of bound immobile electrons; leading to the expectation that mercury would act as an insulator as temperatures were suppressed towards absolute zero. This unexpected deviation from theory sparked interest in understanding the underlying mechanism responsible for this new phase of matter. However, decades passed before a satisfactory explanation was supplied by Bardeen, Cooper, and Schriefer in thier famous paper outlining BCS Theory in 1957 [Bardeen]. This theory identified the charge carriers as a coherent state of electron pairs, known as Cooper pairs, bound together by phonon interactions. The binding energy of these Cooper pairs was shown to be proportional to

$$\Delta \propto \frac{1}{e^{1/V} - 1} \tag{1}$$

where V is the phonon mediated interaction energy [Kittel]. Writing the energy of two electrons at the Fermi surface as

$$\epsilon = 2\epsilon_F - \Delta \tag{2}$$

where ϵ_F is the Fermi Energy, we see that a positive interaction V lowers the total energy of the system. This state of lower free energy is responsible for the second order transition to the superconducting phase.

BCS theory came to be the paradigm that successfully described superconducting systems over the next three decades. However, in 1986 Bednorz and Muller discovered a class of compounds known as cuprates that superconducted at temperatures far higher than those achieved in pure elements or simple compounds [Bednorz]. Moreover, conventional phonon mediated BCS theory is inadequate for describing many of the properties of these new systems. For instance, conventional BCS theory predicts isotropic s-wave pairing. This pairing results in a clean energy gap of 2Δ that pushes the Cooper pair density of states above the gap. Predictions for measurable quantities such as electronic specific heat and magnetic penetration depth under such a pairing scheme are in excellent agreement with theory. However, many high temperature superconductors exhibit d-wave pairing which leads to an unclean energy gap that deviates in magnitude for different samples. As a result, many of the measurable properties of these systems do not follow the isotropic BCS predictions that worked so well for low T_c superconductors [Annett].

Aside from cuprates, other classes of compounds, such as heavy fermions, and more recently the Fe-Pnictides have also been discovered. Despite intensive study by the scientific community, the complexity of these new classes of systems has prevented the construction of a theory that can accurately describe the pairing mechanism responsible for their superconductivity. Experiments have shown, however, that the charge carriers in these new systems have a magnitude of 2e with no momentum [Esteve]. Consequently, the current belief is that Cooper pairs do form in these materials but may possibly be held together by a non-phonon interaction. Indeed, much of BCS theory can be be applied quite successfully to a wide range of nonconventional superconductors if one simply inserts the observed T_c value, the natural anisotropy due to the planar crystal structure, and estimates of the Fermi velocity and density of states [Tinkham].

At the heart of the difficulty in understanding unconventinal superconductors is their ground state. Conventinal superconductors are typically metals that can be described quite accurately within the well worked out frame work of Fermi gases in the presence of a mean field electon-electron perturbation. However, the cuprates parent compounds do not exhibit metallic behavior or superconductivity. Rather, they are Mott insulators, ie: their insulating properties can only be understood in terms of a more thorough Fermi Liquid Theory. Moreover, in cuprates, the crystalline structure of the compounds favored the formation of a two dimensional CuO_4 planes that carried a long range antiferromagnetic order in the ground state. This long range order gives way to short range correlations when superconductivity is induced by doping the charge layers between the CuO_4 planes. As a consequence, magnetic interactions within cuprates quickly became a dominating feature in these systems. Magnetism already played a prominent role in BCS superconductors. For instance, in 1933, shorly after the discovery of perfect conductivity in superconductors, Meissner and Ochsenfeld discovered that a material would also expel all magnetic field when in the superconducting state [Meissner]. If the magnetic field was turned on after cooling below T_c, then the expulsion of all magnetic flux could be explained as a consequence induced critical currents due to perfect conductivity. However, this phenomenon was observed to occur even if the field was switched on before cooling the system down and is now known as the Meissner effect. In this case we would expect that flux would become trapped in the sample. Moreover, if the applied magnetic field is increased sufficiently, then the superconducting state breaks down suddenly, allowing the magnetic field to fully penetrate the sample. However, it turns out that both of these magnetic phenomenon can be quickly accounted for. The Meisner effect can be understood if one begins with the Lagrangian for a non-relativistic particle in an external magnetic field [Cohen-Tannoudji]

$$L = \frac{1}{2}mv^2 + \frac{\mu_0 e}{4\pi}\mathbf{v} \cdot \mathbf{A} - e\Phi \tag{3}$$

In the usual way, we can obtain the canonical momentum by taking the partial derivative with respect to velocity

$$\mathbf{p} = m\mathbf{v} + \frac{e\mu_0 \mathbf{A}}{4\pi} \tag{4}$$

Bloch demonstrated (though he never published it) that in the absence of an applied field the expectation of the particles ground state momentum will be zero. Thus we get that

$$\langle v_s \rangle = -\frac{e\mu_0 \mathbf{A}}{4m\pi} \tag{5}$$

London proposed that electrons that took part in the formation of Cooper pairs retained this ground state property $\mathbf{p} = 0$. By substituting this, as before, into $\mathbf{J}_s = n_s e\mathbf{v}$ we find

$$\mathbf{J}_s = -\frac{n_s e^2 \mu_0 \mathbf{A}}{4m\pi} \tag{6}$$

Taking the time derivative of both sides yields the first London equation. If, instead, we take the curl of (6) then we obtain the second London equation

$$\mathbf{B} = -\mu_0 \lambda_L^2 \nabla \times \mathbf{J}_s \tag{7}$$

where n_s is the number of electrons that take part in Cooper pair formation and λ_L^2 is a new parameter defined as $\lambda_L^2 = m/\mu_0 n_s e^2$. If we now take the curl of Ampere's Law $\nabla^2 \mathbf{B} = \mu_0 \mathbf{J}$ and substitute (6) we obtain

$$\nabla^2 \mathbf{B} = \mathbf{B}/\lambda_L^2 \tag{8}$$

with a solution of the form

$$\mathbf{B} = \mathbf{B}_0 e^{-x/\lambda_L} \tag{9}$$

The value of λ_L is always very large and leads to only a small penetration depth. The reason that superconductivity is suppressed upon applying a large magnetic field is even simpler to understand. From electromagnetic theory we know that the work per unit volume "stored" in this magnetic field is

$$\frac{W_{mag}}{V} = \frac{B^2}{2\mu_0} \tag{10}$$

This energy is equivalent to the increase in free energy density $F_S(\tau)$ due to the application of the external field. If a field is applied while the material is in a normal conducting state then there is no comparable change in the free energy $F_N(\tau)$ since screening does not occur. As a result we get that

$$\left[F_N(\tau) - F_S(\tau)\right]/V = \frac{B_c^2(\tau)}{2\mu_0} \tag{11}$$

where B_c is the critical magnetic field necessary to kill superconductivity at a given temperature τ. Unlike BCS superconductors though, the magnatism in cuprates appears more fundamental. Specifically, the superconducting phase in cuprates resides in close proximity to the long range AFM order of the parent compound. Neutron studies of inelastic magnetic scatterings have also revealed that there exists a gap in the spin excitation spectrum that disappears abruptly upon crossing T$_c$. Moreover, the d-wave pairing of Cooper pairs in these systems fits well with a mediating interaction that is magnetic in origin [Scalapino]. However, probably the most convincing peice of evidence is that an excitation within the energy spectrum, termed resonance, has been shown to be directly related to T$_c$ linearly [Fig. 1].

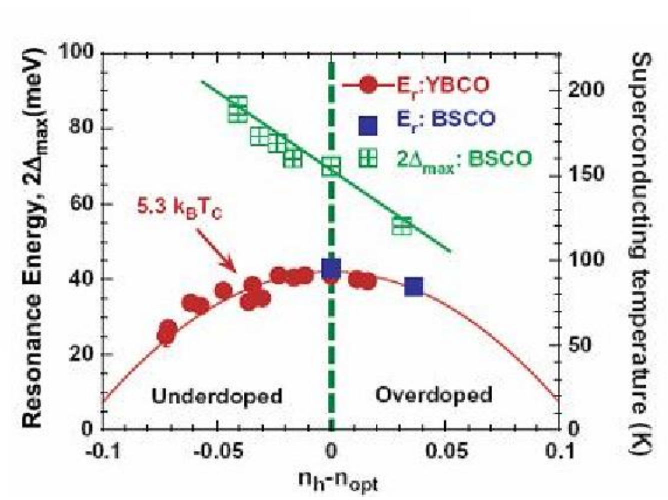

FIG. 1: Doping dependence of the resonance energy at (π, π) in YBCO and BSCCO as a function of various doping levels referenced to the optimal doping level. The red full curve show the doping dependence of the superconducting temperature times 5.3 [Bourges]

Unfortunately, the complexites of cuprates make it difficult to determine if the magnetic excitations seen can be considered sure evidence that spin excitions are the correct agent responsible for pairing or if they are simply a correlated phenomenon associated with the AFM ground state and lowered dimensionality of these systems. Another phenomenon, stripe ordering, associated with the lowered dimensionality and magnetic order of the cuprates has also been observed. Like the resonance and superconduction gap, the exact relationship of stripe order with superconductivity is unsure. There has been alot of debate in the literature arguing that stripes are fundamental for the formation of superconductivity while others have claimed that it suppresses it. The goal of this paper is to lay out a foundation for why stripes form and what they look like, how we experimentally determine stripe ordering, and how this order is known to be associated with superconductivity.

II. THE FORMATION OF STRIPES

The parent compounds of the high T_c cuprates are not themselves superconducting. In order to induce superconductivity it is necessary to inject holes or electrons into the CuO_4 planes by doping the inert "'charge resevoir"' layer that exist between them. In so doing, the extra charges disrupt the long range antiferromagnetic order that exists in the CuO_4 planes. This disruption comes at a cost of energy. If a single injected charge is left isolated then it will affect eight neighboring spins. However, if two injected charge coallase then the end result is that only six neighboring spins are affected. Under this basic scheme it would seem that all of the injected holes or electrons would come together in whatever manner would affect the least spins. Such a geometry would be circular in nature. Due to the repulsive Coulombic interactions between the charges this configuration is not the best suited for aquiring the lowest energy state. Instead, the charges will form in long lines known as charge stripes. Moreover, each copper in a stripe does not even carry its own hole. Rather, experiments have shown that the density is one doped charge carrier for every two copper ions. A result that has been born out by experiment but which initially disagreed with theortical calculations that predicted stripe ordering before any observations were made of their existence [Zaanen],[Schulz]. These charge stripes also act as phase boundaries for the sourrounding antiferromagnetic spin lattice. Specifically, the up-down repetition of the spins becomes out of phase by π upon crossing a charge stripe [Fig 2]. The disruption

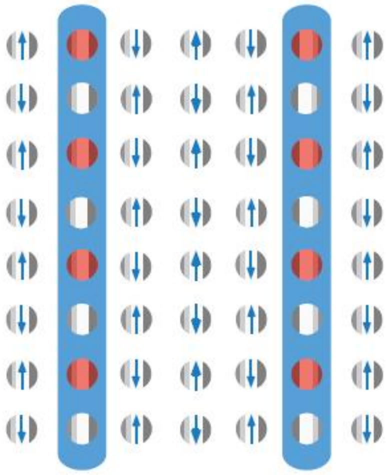

FIG. 2: Schematic illustration of stripe ordering. Charge is largely confined to the channels shaded in blue. Red and silver circles indicate charge stripes; one hole/Cu. Blue arrows indicate magnitude of the magnetic moment. [Orenstein]

of the long range AFM order results in the formation of magnetic stripes whose distance before repetition is twice that of the charge charge stripes.

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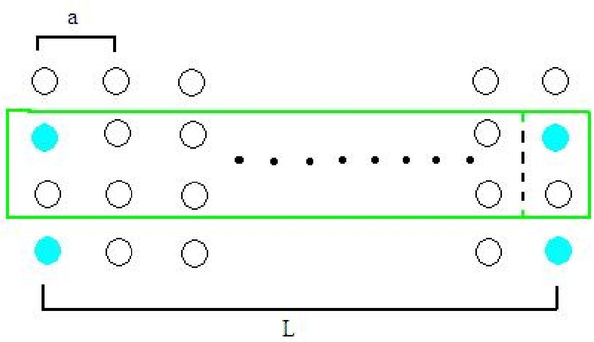

FIG. 3: A 'unit cell' of stripe ordering. Note that there is only one hole per N copper atoms in a cell.

The distance before a stripe repeats is directly linked to the level of doping within the system. Take as an example $La_{1-x}Sr_xCuO_4$. The electronegativity of La is 3+ wheras Sr is only 2+. Thus, with the introduction of each new Sr into the charge layer, one more electron is pulled into its valence from the neigboring CuO_4 plane. Thus we see that there is a one to one relationship between the number of holes injected into the superconducting layers and the number of Sr atoms doped into the charge layer. As a stochiometric consequence we can write a hole density as n_h = x holes/1 Cu ion = x. From Fig. 3 we see that the hole density may also be written as n_h = 1 hole/N Cu ions = 1/N where N is the number of copper atoms in the 'stripe unit cell'. By inspection we get that N = 2L/a. Combining these three results allows us to calculate the distance between charge stripes (L_c) and magnetic stripes (L_m) based on a given doping concentration:

$$L_c = \frac{a}{2x}, L_m = \frac{a}{x} \tag{12}$$

This periodicity of stripes has a direct impact on scattering measurements that map out the reciprocal space of the 2D copper oxide planes within the cuprates. The repricol space of th CuO_4 planes in a cuprate with a tegragonal unit cell is confined to the (H,K,0) plane and consist of nuclear bragg peaks at (N,M,0) where the sum of N and M must be even and N, M are integers. Thus, you would expect Bragg peaks at Q=(2,0,0), (1,1,0) (0,2,0) etc. but not at (1,0,0). This restriction can be determined from the nuclear structure factor. Likewise for magnetic scattering, the magnetic structure factor confines the scattering to Q=(1/2,1/2,0). Since a given distance L in the direct lattice gets mapped to 1/L in reciprocal space, then a quick estimate of the effect of stipe ordered periodicity of our reciprocal lattice would be that our Bragg peaks would shift out by $\delta_c \approx$ x and since the periodicity of our magnetic stipe ordering is twice that of our charge stripe ordering we get $\delta_m = \frac{\delta_c}{2}$. It turns out that $\delta_c = x$ almost exactly for a wide range of

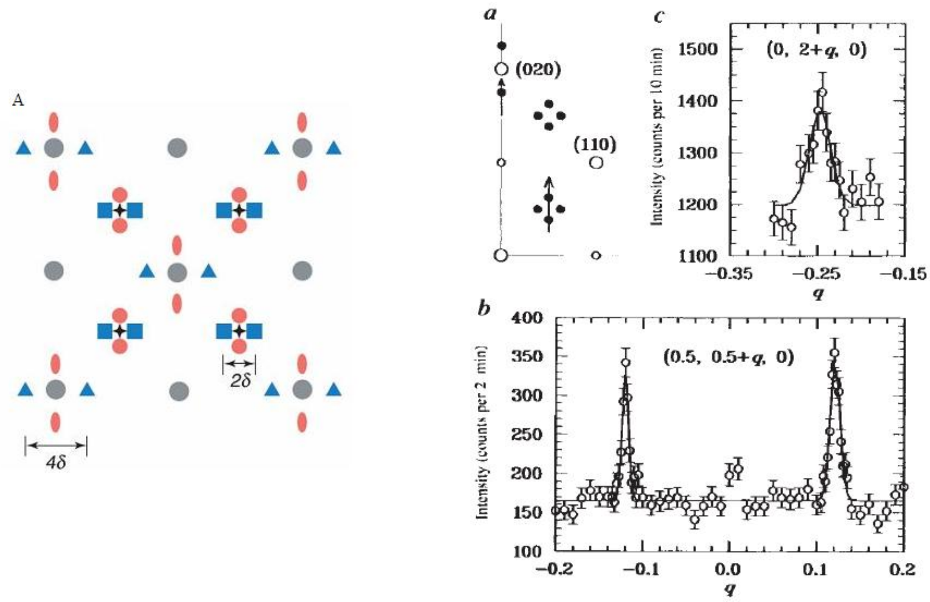

FIG. 4: A) The reciprocal space for a CuO_4 plane in the presence of stripe ordering. Large circles are due to Bragg scattering off of the nuclear structure, small plusses are magnetic scatterings off of the copper ions. The smaller symbols around these two primary scattering events represent the displacement of the scattering due to stipe ordering within the system [Orenstien]. Second figure block is for data collected on $a_{1.48}Nd_0.4Sr_{0.12}CuO_4$ a) Diagram of the (HK0) zone of reciprocal space. b) Scan along (1/2,1/2+q,0) through the (1/2,1/2$\pm\epsilon$,0) peakes measured with a neutron energy of 13.9 meV. The small peak width indicates that the inplane correlation length is greater than 150 $\AA$. c) Scan along (0,2+q,0) through the (0,2-2ϵ,0 peak using 14.7 meV neutrons. The lines is b and c are the result of least-squares fits to gaussian peak shapes plus a flat background [Tranquada (Nature)].

doping as we will see. With all of this in mind we can build a diagram of a stripe ordered reciprocol space as shown in Fig. 4.

IV. EXPERIMETALLY MEASURING STRIPE ORDER

Under the assumption that the diagram of reciprocal space in Fig. 4 is correct, it is possible to test for stripe ordering by using a triple axis spectrometer. This instrument is ideal for carrying out diffraction measurements along slices in Q space. By scanning along the K-direction, at Q = (0,2+q,0) there should exist a peak for some value of q approximately equal to the doping level x. Likewise, one should also see a magnetic peak at Q = (1/2,1/2+q,0). In 1995, nuetron diffraction experiments on $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$ were performed and the results confirmed this picture of stripe order

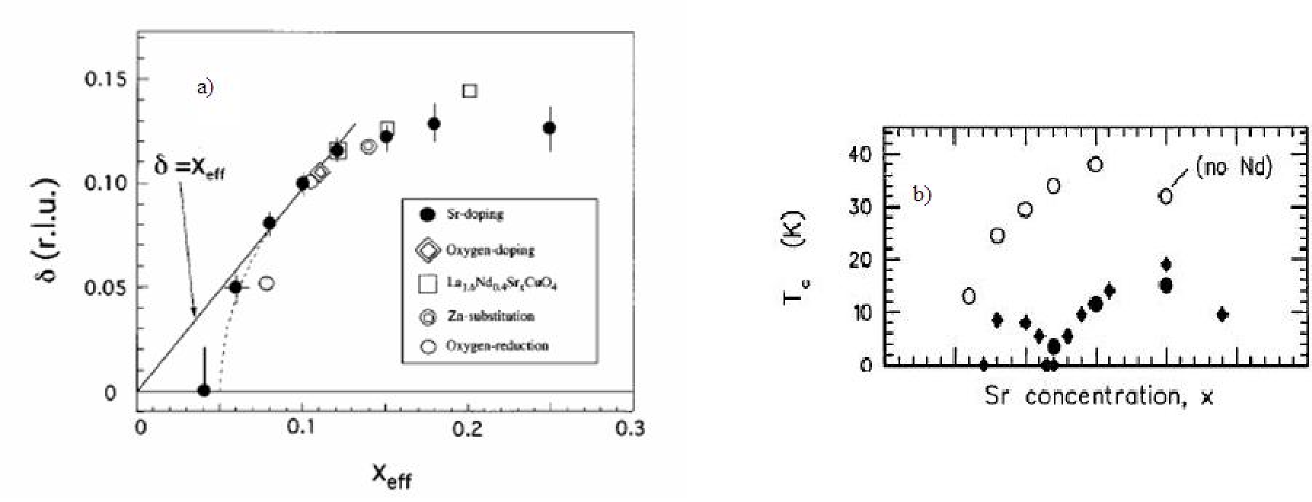

FIG. 5: a) Sr-doping dependence of the incommensuarability δ of the spin fluctuations. Data from electrochemically oxygen-doped La_2Cu0_{4-y} [Wells], $La_{1.6}Nd_{0.4}Sr_xCuO_4$, Zn-substituted $La_{1.86}Sr_{0.14}Cu_{1-y}Zn_y0_4$ with y=0.012 and deoxygenated $La_{1.85}Sr_{0.15}Cu_{1-y}$ [Yamada] b) Phase diagram of the superconducting transition temperature, the open circle represent $La_{2-x}Sr_xCuO_4$ without Nd doping. The filled circles are with doping [Tranquada (PRL)].

[Fig. 4] with the incommensurate splitting of the Bragg peak almost exactly equal to the doping [Tranquada (Nature)]. Later experiments showed that this doping dependence did deviate from the simple linear relationship that was originally confirmed by Tranquada [Fig. 5a] [Yamada]. Aside from being the first direct observation of stripe ordering in cuprates via neutron diffraction, Tranquada's experiment was made more important by the fact that the superconductivity in this system became anomolously suppressed at dopings around x = 1/8 [Fig. 5b] [Tranquada (PRL). However, the loss of superconductivity could now be related to a static stripe order that resulted due to pinning induced by the Nd. Thus the suppression of supeconductivity could be correlated the the suppression of inelastic magnetic excitations related to the fluctating stripes within the system. Since this orginal work, a growing consensus has concluded that static stripe order within a system would compete directly with superconductivity. The debate on the role of fluctating stripe order and its relationship with superconductivity still remains very cloudy. Research on some com-

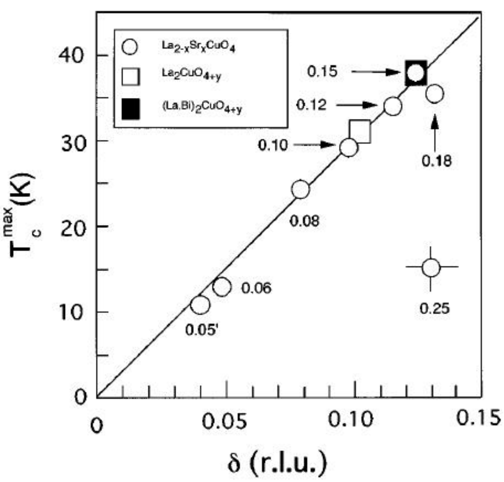

FIG. 6: Supeconducting transition temperature as a function of incommensurability due to stripe ordering. [Yamada]

pounds such as $La_{2-x}Sr_xCuO_4$ have revealed that stipe order and superconductivity appear in the system at exactly the same level of doping and that with further doping, the incommesurate splitting of the Bragg peaks correlates linearly to the T_c of the sample up to optimum doping. However, in the overdoped regime this relationship is quickly lost.

V. CONCLUSSION

The anti-ferromagnetic Mott insulating groundstate of the cuprates make this system much more complex than the simple Type I metallic superconductors that were orignally discovered and understood within the framework of BCS Theory. The fact the the superconductivity of the cuprates is confined to two dimensions makes the physics of these systems even richer. Stripe ordering and superconductivity are perfect examples of these physics playing out. However, despite over a decade of work on stripe ordering, the relationship of stipes with superconductivity is still an open question. Indeed, some even argue against the existence of stipes altogether, claiming instead that a careful analysis of the deformation of the Fermi surface due to doping can lead to the theortical predictions of observations already born out in experiments and attributed to stripes. It will be interesting to see how the story of stripes will change as time and discovory uncover a more complete picture of the cuprates.

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