Anderson Localization

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Anderson Localization

- What happens to various electronic properties when perfect crystalline symmetry is broken by introduction of impurities?
- What is the cause of metal-insulator transition in various materials?

Experimental Motivation

- In 1955, Feher et al. measured electron spin resonance of P and As doped in Si.
- They noticed that characteristic electron spin frequency of P an As persisted for a timescale on the order of seconds to minutes.
- Simple Fermi golden rule calculation gives the expected lifetime to range from 0.1 to 10^{-6}s.
- Anderson argued that the absence of spin diffusion was due to disorder arising from the random distribution of impurities.

Electrons in Periodic Lattice and Weak Disorder

- In periodic crystals, electrons are described by Bloch waves that are periodic up to a phase.
- In real life, there is always some amount of impurity and disorder.
- Traditionally, always take the impurity to be always very small.
- The electrons are still describe by Bloch waves, but now they scatter with the impurities.

Consequences of Weak Disorder Limit

- Low temperature resistivity has the form $\rho(T) = \rho_0 + AT^n$.
- Low temperature conductivity is $\sigma(T) = \sigma_0 - A\sigma_0^2 T^n$.
- n is a positive integer greater than or equal to two. It depends on the type of interaction.
- When electron-electron interactions prevail, $n = 2$.

Consequences Disorder

- It turns out that in disordered material, the Boltzmann description of electronic conductivity is wrong.
- The constant A could be positive or negative and n is usually $\frac{1}{2}$ for three dimensional systems.
- In three dimensions, there is a transition from metallic to insulating behaviour as the disorder is increased.
- We have to take into account the disorder from the outset.

Basic Concepts of Anderson Localization

- In the presence of sufficiently strong random potential, the envelope of the wave packet may exponentially decay.
  $|\psi(r)| = exp(|r - r_0|/\xi)$
- Orbitals that are nearby in space have very different energy. So the admixture is small because of big energy denominator.
- Orbitals that are far apart have very small overlap integral.

Basic Concepts of Anderson Localization

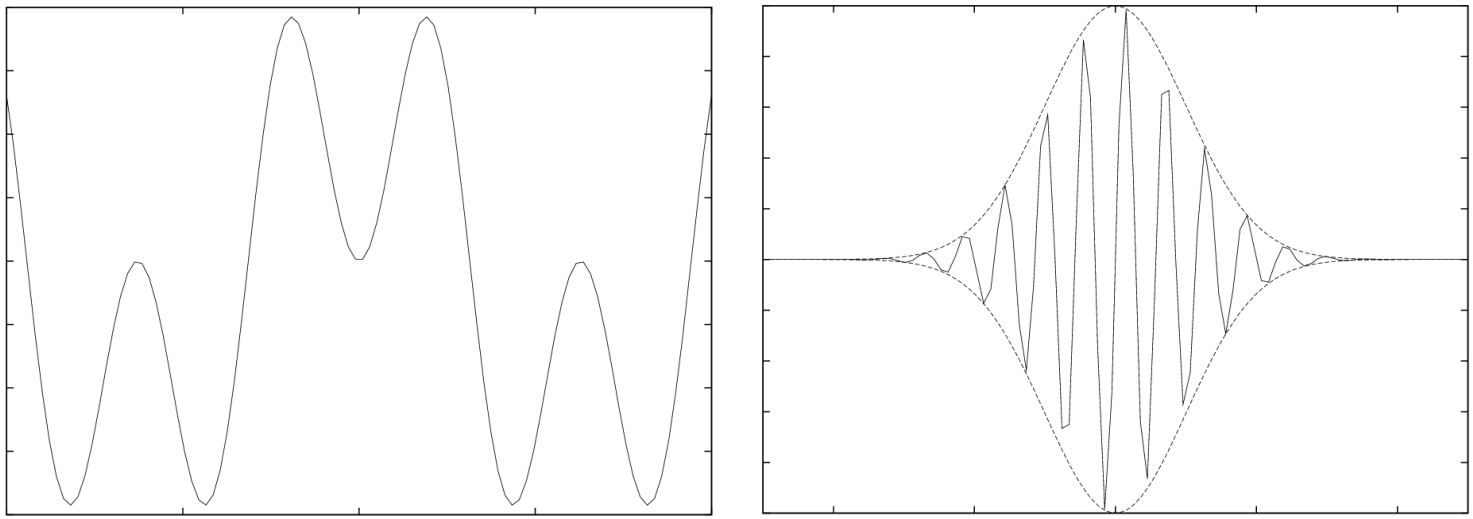
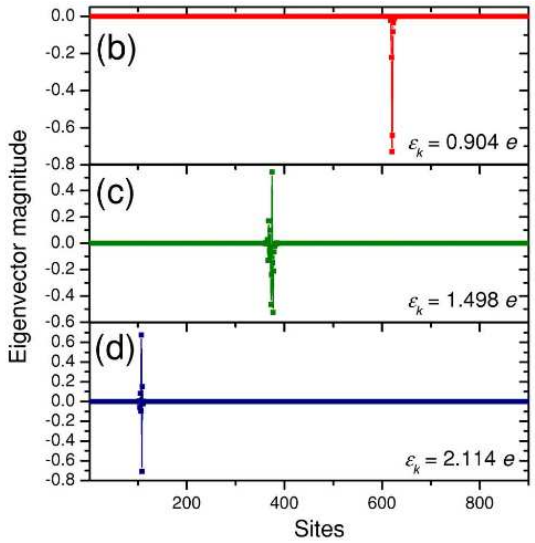


Figure: Typical delocalized (left) and localized (right) wavefunctions.

Computational Example

- Siber (2006) computed the probability amplitude of an electron in disordered one-dimensional tight-binding chain.
- Consider $H = \sum_i E_i |i\rangle\langle i| + \sum_i T_i [|i+1\rangle\langle i| + h.c.]$ with E_i in the range $[e, 2e]$.
- Siber calculated the eigenvectors of this matrix and found that all of them were localized.



Scaling Theory of Localization

- Main assumption: $g(L) = \frac{2\hbar}{e^2} G(L)$ is only a function of sample size.
- The scaling function $\beta(g) = d(\ln g)/d \ln L$ is a function of g only.
- In weak disorder limit $g(L) = \sigma L^{d-2}$.
- In the strong localization limit $g(L) = exp(-L/\xi)$.

Consequences of Scaling Theory of Localization

- In the weak disorder limit $\beta(g) = (d-2)$
- In the strong disorder limit $\beta(g) = -\infty$
- For 1D, in the weak disorder limit $\beta(g) = -1$. For 2D, $\beta(g) = 0$.
- For 3D, $\beta(g) = 1$. So in 3D there is metal to insulator transition.

Conclusions

- Existence of disorder may cause electrons to be localized.
- In one and two dimensions, any amount of disorder will cause localization
- In three dimensions, there is a continuous metal to insulator transition.