Anderson Localization

Dr. Dagotto's Condensed Matter II - Spring 2009

Andrew Nicholson

Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee 37996, USA

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Periodic Potentials

Blochs Theorem:

$$\psi_i(r) = \sum_G c_{i,k+G} e^{[i(k+G)\cdot r]}$$

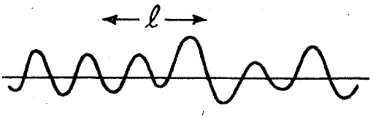


Figure: Extented wave function (Bloch Waves) [2]

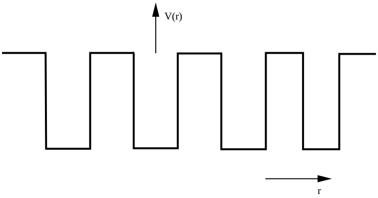


Figure: Periodic Potential approximated as a regular set of square wells

Electron wave functions characterized as Bloch waves existing throughout the lattice.

Conductivity

Boltzmann transport equation:

$$\sigma(T) = \sigma_0 - A\sigma_0^2 T^n$$

where σ_0 is the residual conductivity due to impurity scattering. It is important to note that this conductivity is a result of freely propogating electrons. As T increases scattering increases due to collisions so that $A > 0$ and n is a natural number.

However for strongly disordered systems this breaks down! A can be positive or negative and n is typically $\frac{1}{2}$.

Localization Part I

Localization

$$\Psi(r)| = e^{|r|/\xi}$$

ξ : Localization Length

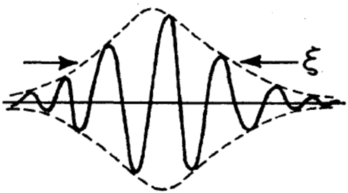


Figure: Wave function of a localized state with localization length ξ [2]

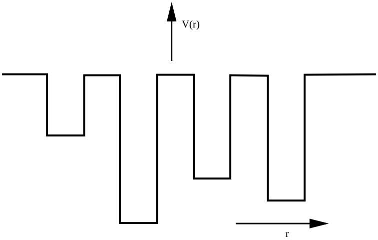


Figure: Strongly varying potential wells [8]

Localization Part II

- Two wells close to one another(large wavefunction overlap) have will usually have a large difference in energy
- Two wells with similar energy will be far apart (little wavefunction overlap)
- As the system increases in size localization increases

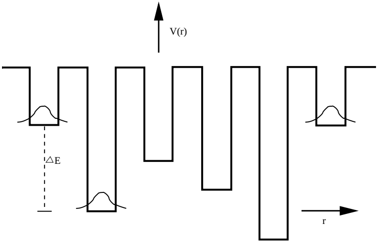


Figure: Schematic of a random potential

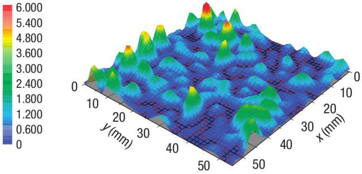
Experimental Verification

Localization of ultrasound in a three-dimensional elastic network. Used 0.3-3.0 MHz ultrasound frequencies through brazed aluminium beads

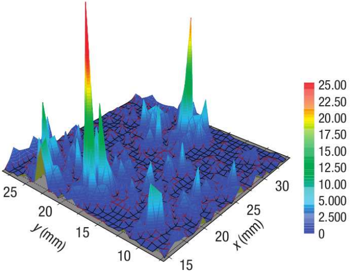
Figure: Brazed aluminium beads [7]

Figure: Brazed aluminium beads 4.011 ± 0.03 nm in diameter [7]

Experimental Verification



Diffuse waves at 0.20 MHz [7]



Localized waves at 2.4 MHz [7]

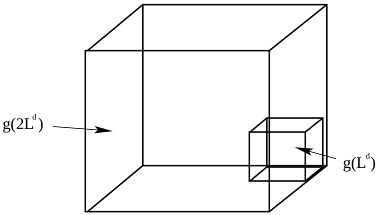
Notice that localization is scale dependent!

Introduction to Scaling Theory

Scaling theory uses conductance, G, as a measure of disorder. We define a dimensionless conductance g on scale size L

$$g = \frac{G}{e^2/\hbar}$$

Begin at the microscopic scale and build the system larger:



Scaling Theory

Two very different regimes:

Weak Disorder:
For $L >> l$

$$g(L) = \sigma L^{d-2}$$

where the conductivity follows Ohm's Law: $\sigma = \frac{nle^2}{\hbar k_F}$

Strong Disorder:
For $L >> \xi$

$$g(L) \propto e^{-L/\xi}$$

$g(L)$ evolves smoothly from one regime to the other. The limiting behavior reached depends on microscopic disorder $g_0 = g(l)$ which depends on the scale l and dimensionality d

The Scaling Function

The Scaling Function:

$$\beta(g) = \frac{d(lng)}{d(lnL)}$$

The change in disorder as a system gets bigger depends on previous length scales

Large Conductance (very weak disorder):

$$\beta(g) = (d - 2)$$

Notice that for $d \to 2$, $\beta \to 0$

Small Conductance (strong disorder):

$$\beta(g) = ln(g/g_c)$$

where g_c is a characteristic conductance. Notice that β is independent of dimensionality. In this limit $g < g_c$ so that $\beta < 0$.

Pertubative Regime

For weak disorder (large g) a correction can be added to our previous equation:

$$\beta(g) = (d-2) - \frac{a}{g}$$

From this we can see that β is always less than the very weak disorder regime (region where Ohm's Law is valid). In other words the introduction of the smallest ammount of disorder will decrease β.

Scaling Function for Different Dimensionalities

Strong Disorder (non-Ohmic)
$\beta(g) = ln(g/g_c)$

Weak Disorder (Ohmic)
$\beta(g) = (d-2)$

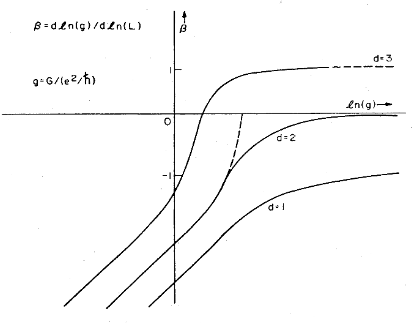
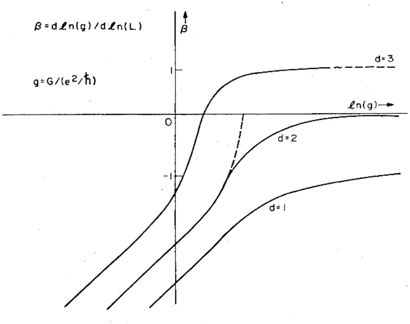


Figure: The β function vs conductance g for different dimensions d. [2]

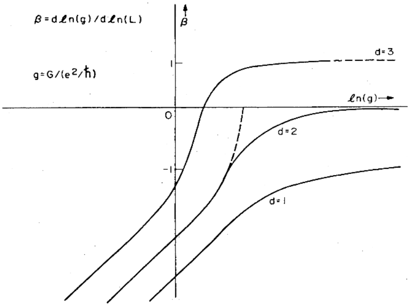
One and Two Dimensional Systems

For all g, $\beta(g) < 0$. So at large enough length scales only localized behavior is possible. Suppose you start with a system with large conductivity at the mean free path length scale $g(l) = g_0$. As the length scale increases one will move down the length curve to the strongly disordered limit $\beta(g) = ln(g/g_c)$.



Three Dimensional Systems

Let $\beta(g_3) = 0$
(1) $g_0 > g_3$: Increasing the length scale from l will increase g moving one up $\beta(g)$ to the limit of no disorder (Ohm's Law).
(2) $g_0 < g_3$: Increasing the length scale from l will decrease g so that one reaches the limit of strong disorder (localized states).
(1) $\rightarrow$ metal
(2) $\rightarrow$ insulator



Temperature Dependence

As temperature decreases so does conductance - a signature of localization

$$\sigma_{3D} = \sigma_0 + \frac{e^2}{\hbar\pi^3}\frac{1}{a}T^{p/2}$$

$$\sigma_{2D} = \sigma_0 + \frac{p}{2}\frac{e^2}{\hbar\pi^3}ln(\frac{T}{T_0})$$

$$\sigma_{1D} = \sigma_0 - \frac{ae^2}{\hbar\pi^3}T^{-p/2}$$

where p is an index depending on the scattering mechanism, dimensionality, ...

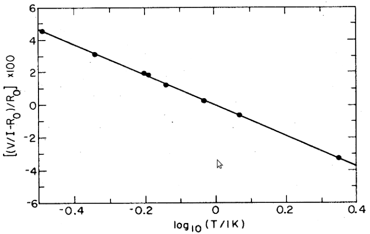


Figure: Resistivity vs. lnT for a PdAu film [2]

Important Points

- Ordered systems have electron wave functions characterized as Bloch Waves - freely propogating throughout the crystalline structure. Ohm's Law works well here.
- Disordred systems are very different. If disorder is strong enough and if the scale size is big enough electron wave functions will become localized.
- One popular way of describing disorder is using Scaling Theory which uses conductance as a measure of disorder.
- If a system is disordered at a small scale size then it will always be disordered.
- If a system is ordered at a small scale size the dimensionality and size of the system become important
- One and two dimensional systems always tend to the disordered limit as scale size increases.
- Three dimensional systems are dependent on the microscopic scale going to the ordered or disordered limits.

For Further Reading

- P. W. Anderson, Phys. Rev., 109, 1492 (1958)
- P. Lee, T. V. Ramakrishnan Rev. Mod. Phys., 57, 287 (1985)
- P. W. Anderson, Phys. Rev., 124, 41 (1961)
- P. W. Anderson, Science, 177, 4047 (1972)
- D. S. Wiersma et al, Nature, 390, 671 (1997)
- N. W. Ashcroft, N. D. Mermin *Solid State Physics*, 1976, (Holt Saunders, Philadelphia) p. 113
- H. Hu *et al*, Nature Physics, 4, 945 (2008)
- N. F. Mott E. A. Davis *Electronic Processes in Non-Crystalline Materials*, 1979, (Clarendon, Oxford) p. 16