Lectures: Condensed Matter II

1 – Quantum dots

2 – Kondo effect: Intro/theory.

3 – Kondo effect in nanostructures

Luis Dias – UT/ORNL

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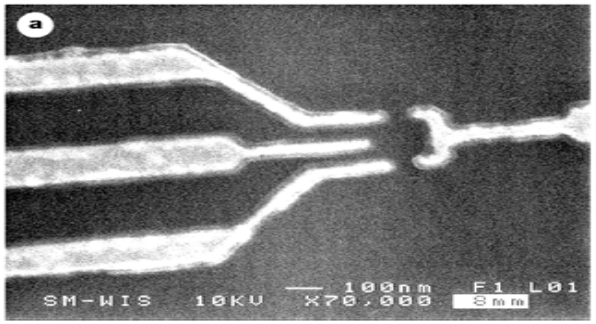
Lecture 3: Outline

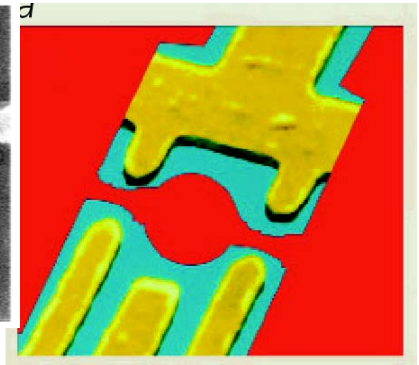
- Quantum Dots: brief review.
- Kondo effect: Review.
- Kondo effect in quantum dots.
- Kondo effect in Single Molecule Transistors.
- Kondo effect in Surfaces (STM, “quantum mirage”).
- Kondo effect in carbon nanotubes.

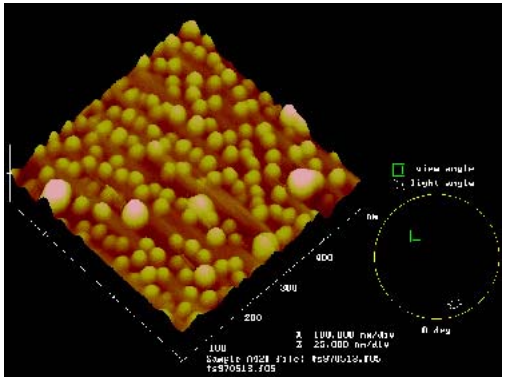
What are *Quantum Dots*?

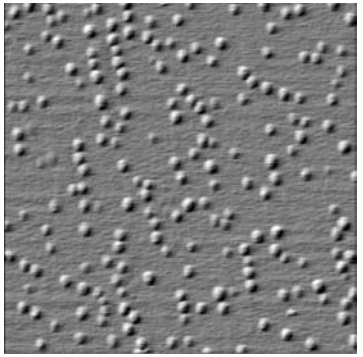
Semiconductor Quantum Dots:

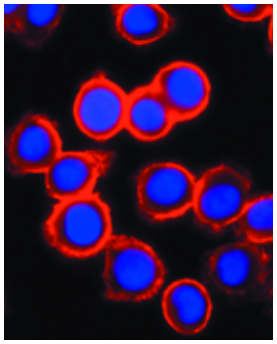
- Devices in which electrons are **confined** in nanometer size volumes.
- Sometimes referred to as "artificial atoms".
- "Quantum dot" is a generic label: lithographic QDs, self-assembled QDs, colloidal QDs have different properties.





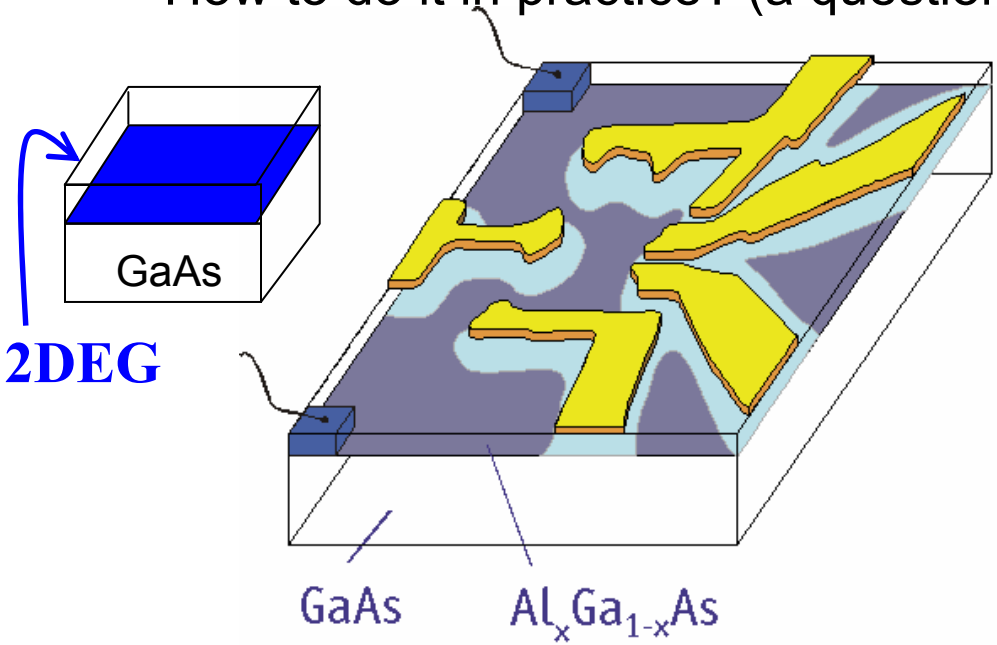


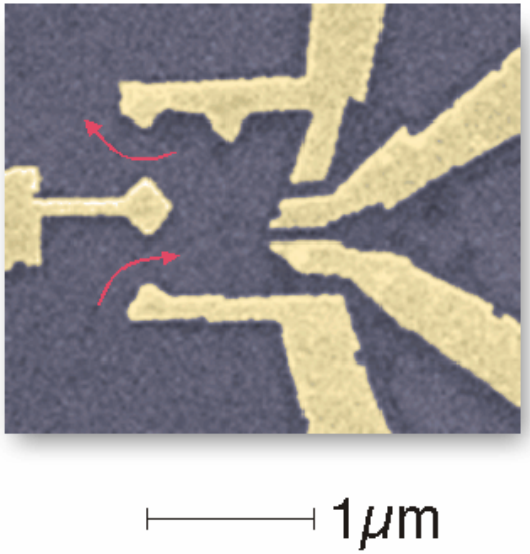




Lithographic Quantum Dots

How to do it in practice? (a question for the experimentalists…)



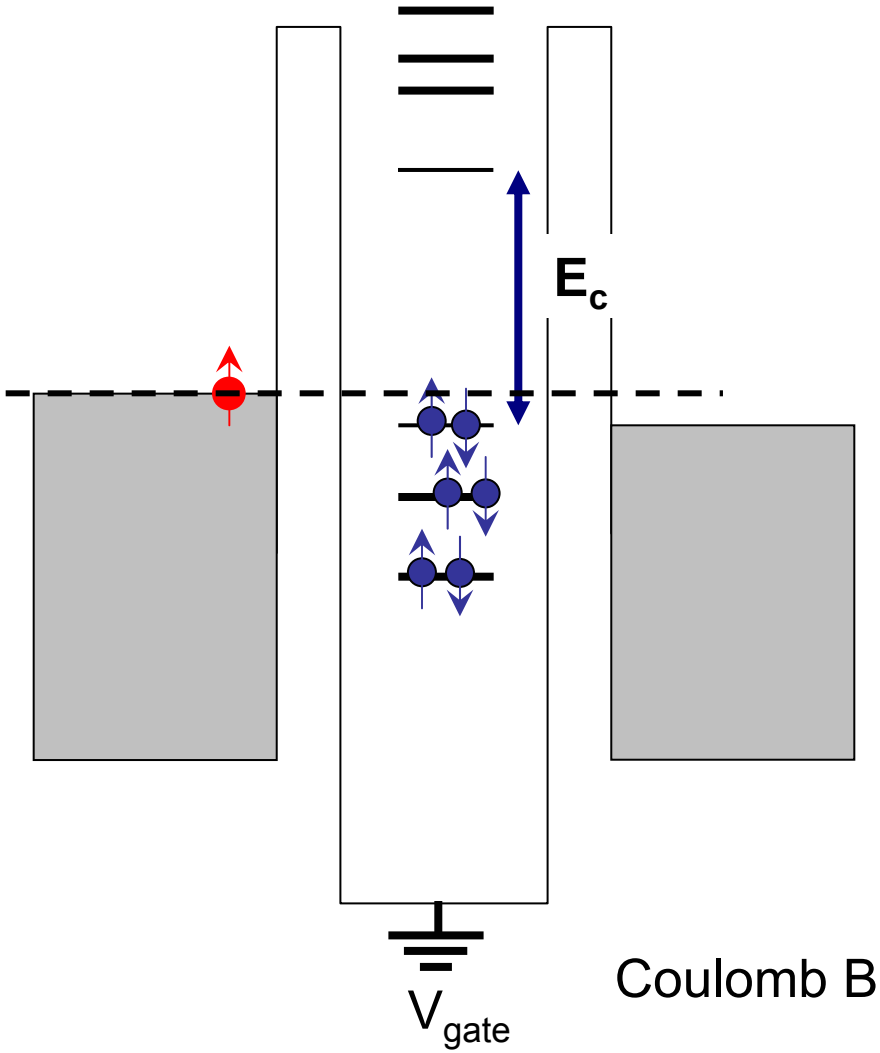


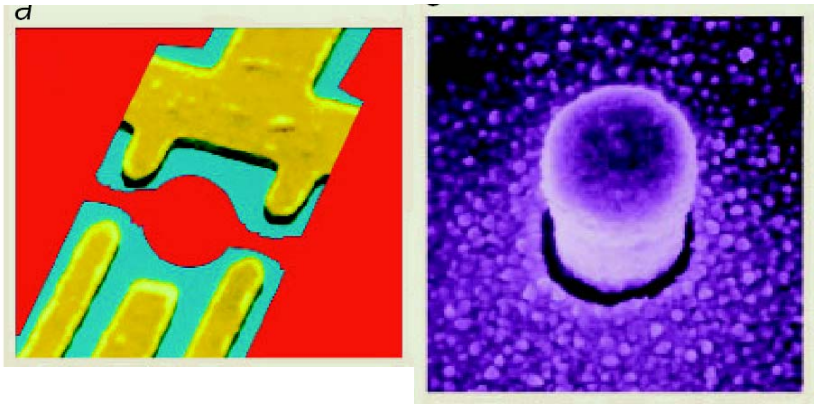
from Charlie Marcus' Lab website (marcuslab.harvard.edu)

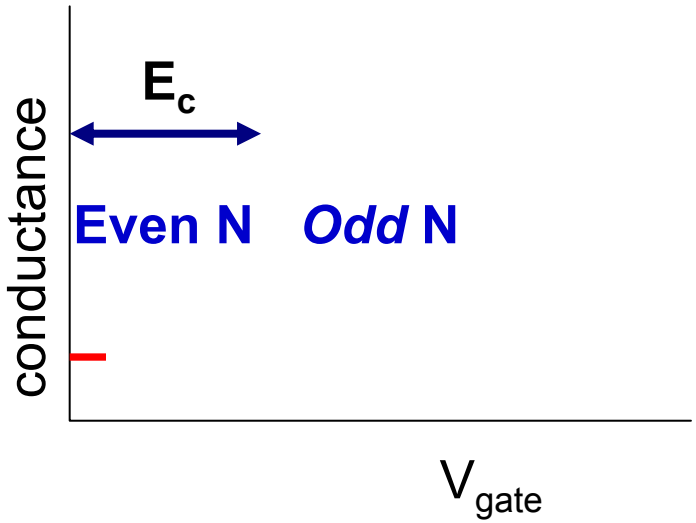
Ingredients:

- Growth of heterostructures to obtain the 2DEG
  - (good quality, large mean free-paths)
- Metallic electrodes electrostatically deplete charge: confinement
- Sets of electrodes to apply bias etc.
- LOW TEMPERATURE! (~100 mK)

Coulomb Blockade in Quantum Dots

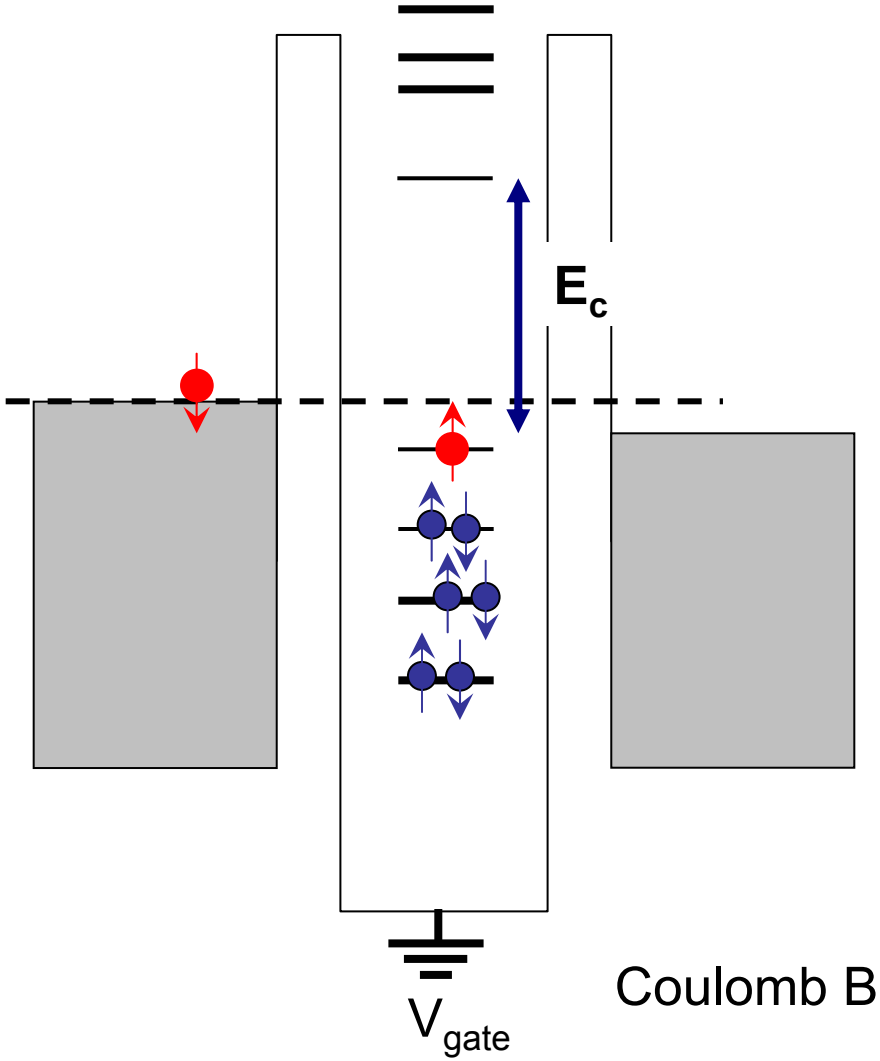


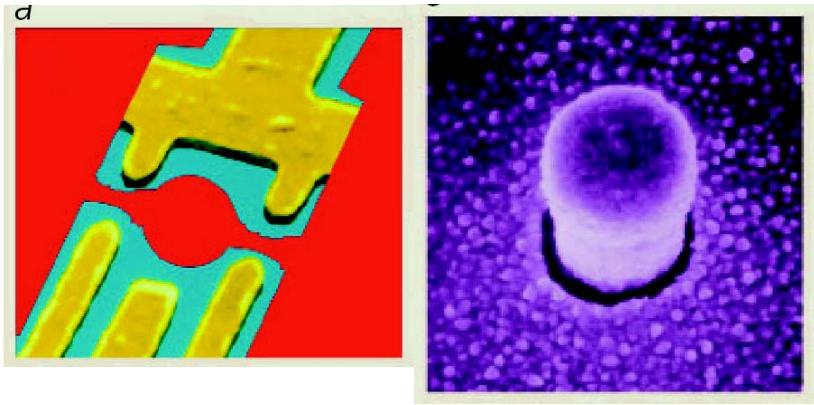


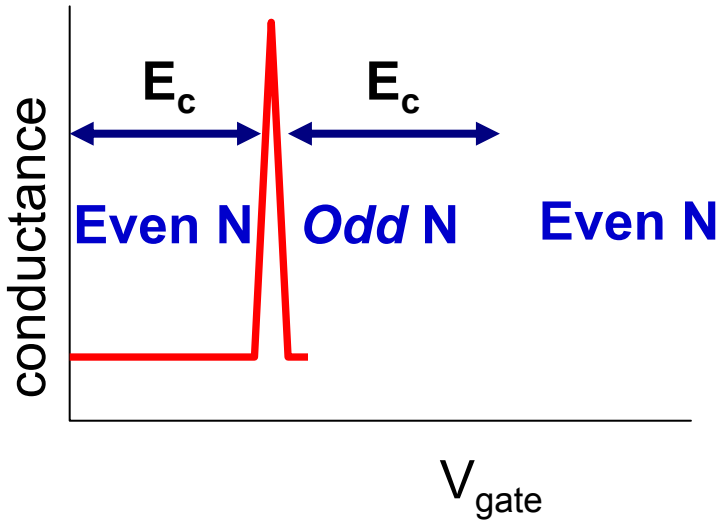


Coulomb Blockade in Quantum Dots

Coulomb Blockade in Quantum Dots

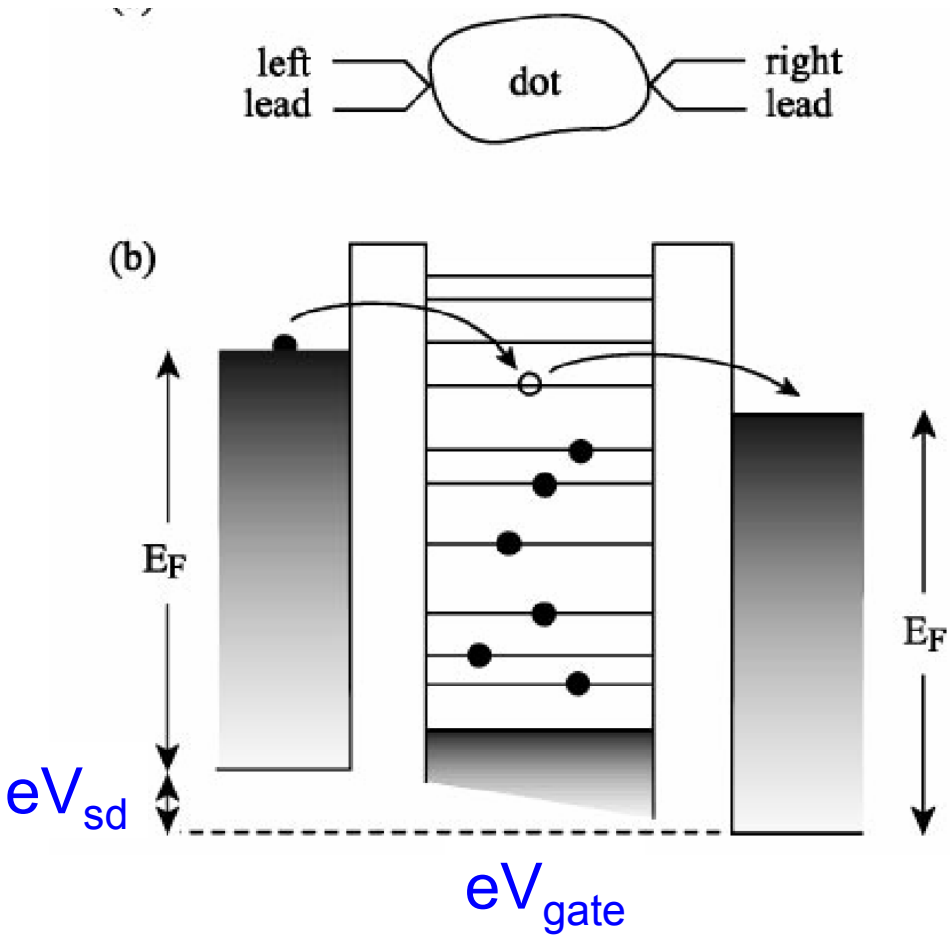




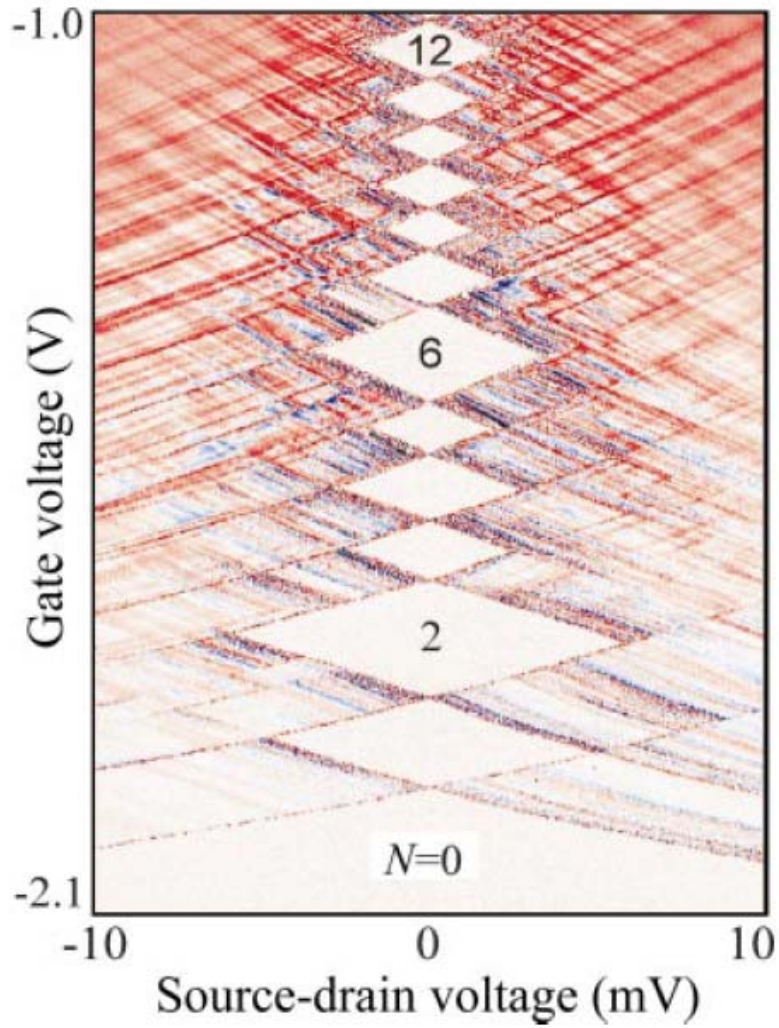


Coulomb Blockade in Quantum Dots

“Coulomb Diamonds” (Stability Diagram)



Coulomb Blockade in Quantum Dots



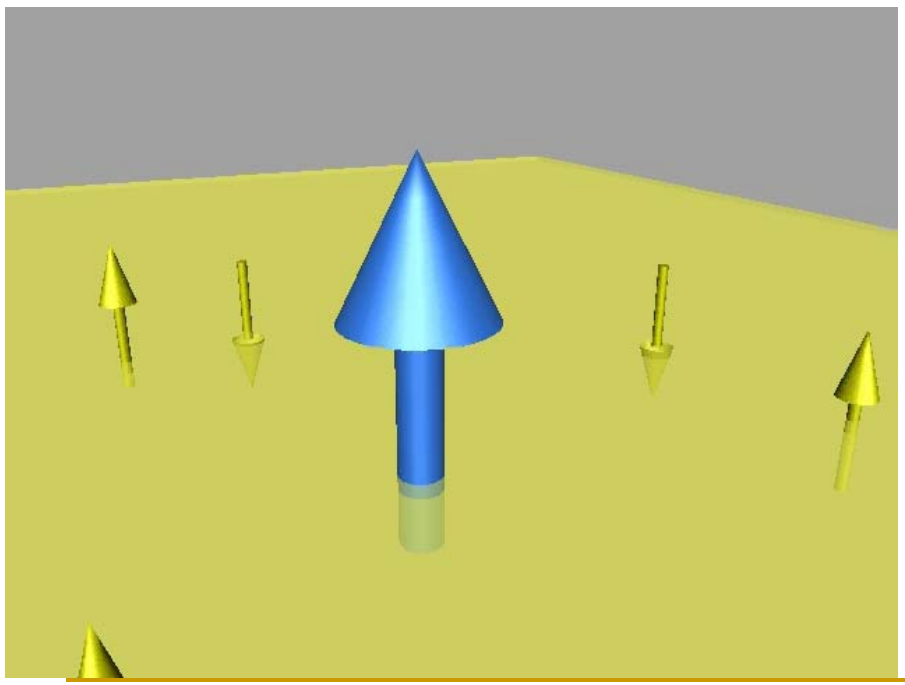
L. P. Kouwenhoven et al. *Science* **278** 1788 (1996).

Kondo's explanation for T_{min} (1964)

- Perturbation theory in J^3:
  - Kondo calculated the conductivity in the linear response regime

$$R_{\text{imp}}^{\text{spin}} \propto J^2 \left[1 - 4J\rho_0 \log\left(\frac{k_B T}{D} \right) \right]$$

$$R_{\text{tot}}(T) = aT^5 - c_{\text{imp}} R_{\text{imp}} \log\left(\frac{k_B T}{D} \right)$$



$$T_{\min} = \left(\frac{R_{\text{imp}} D}{5ak_B} \right)^{1/5} c_{\text{imp}}^{1/5}$$

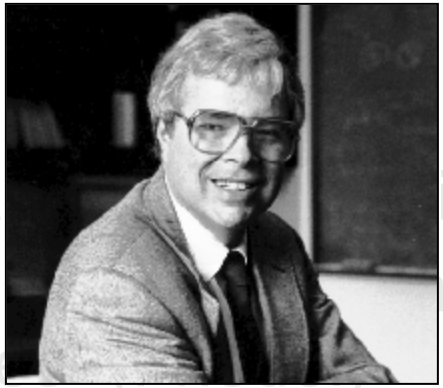
- Only *one* free paramenter: the Kondo temperature T_K
  - Temperature at which the perturbative expansion ***diverges.***

$$k_B T_K \sim D e^{-1/2J\rho_0}$$

A little bit of Kondo history:

- Early ‘30s : Resistance minimum in some metals
- Early ‘50s : theoretical work on impurities in metals “Virtual Bound States” (Friedel)
- 1961: Anderson model for magnetic impurities in metals
- 1964: s-d model and Kondo solution (PT)
- 1970: Anderson “Poor’s man scaling”
- 1974-75: Wilson’s Numerical Renormalization Group (non PT)
- 1980 : Andrei and Wiegmann’s exact solution

A little bit of Kondo history:



Kenneth G. Wilson – Physics Nobel Prize in 1982
"for his theory for critical phenomena in connection with phase transitions"

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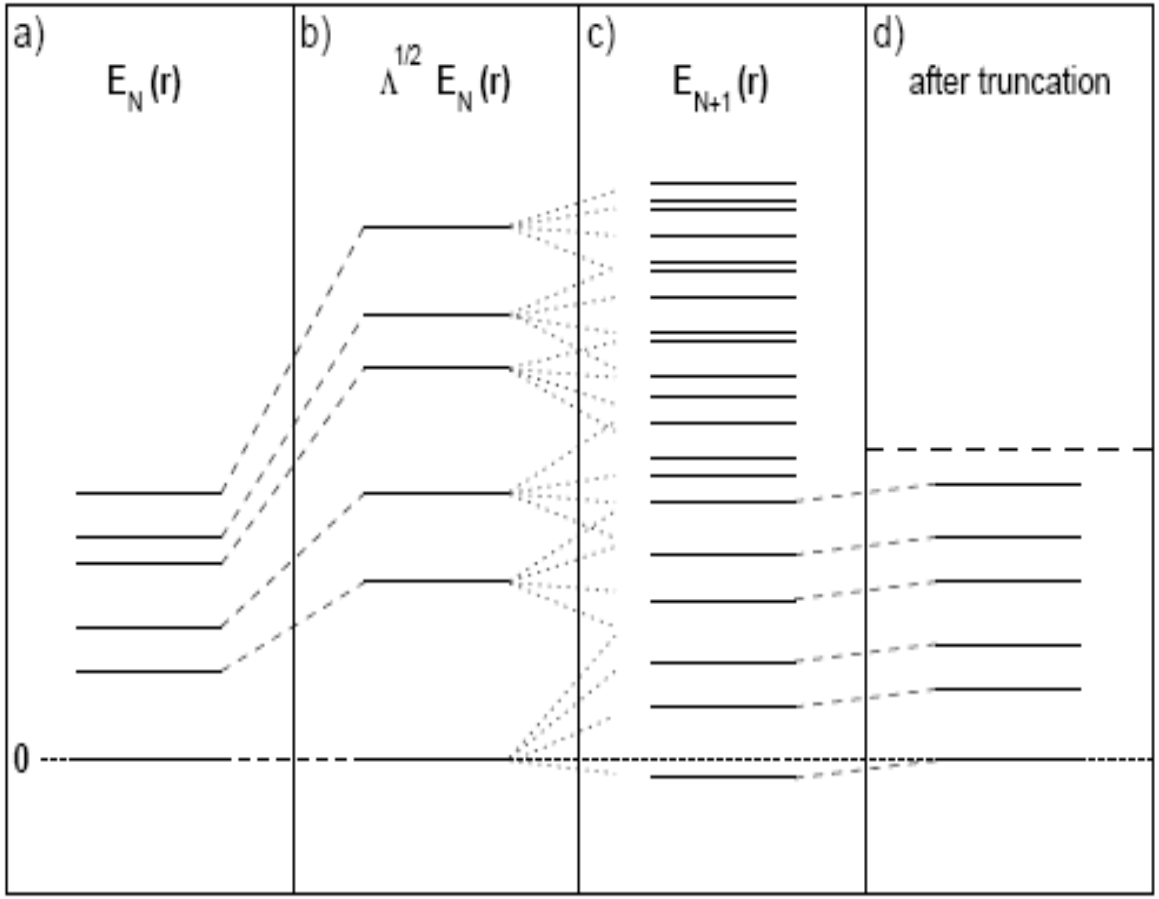
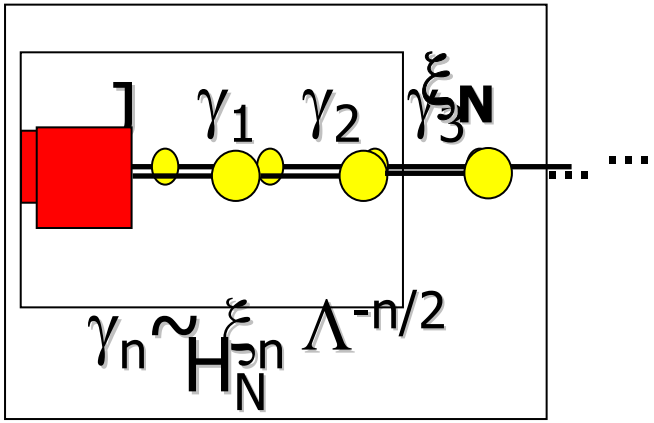
Renormalization Procedure

$$H_{N+1} = \sqrt{\Lambda} H_N + \xi_N \sum_\sigma f^\dagger_{N+1\sigma} f_{N\sigma} + f^\dagger_{N\sigma} f_{N+1\sigma}$$



- Iterative numerical solution.
- Renormalize by $\Lambda^{1/2}$.
- Keep low energy states.



H_{N+1}

Renormalization Group Transformation

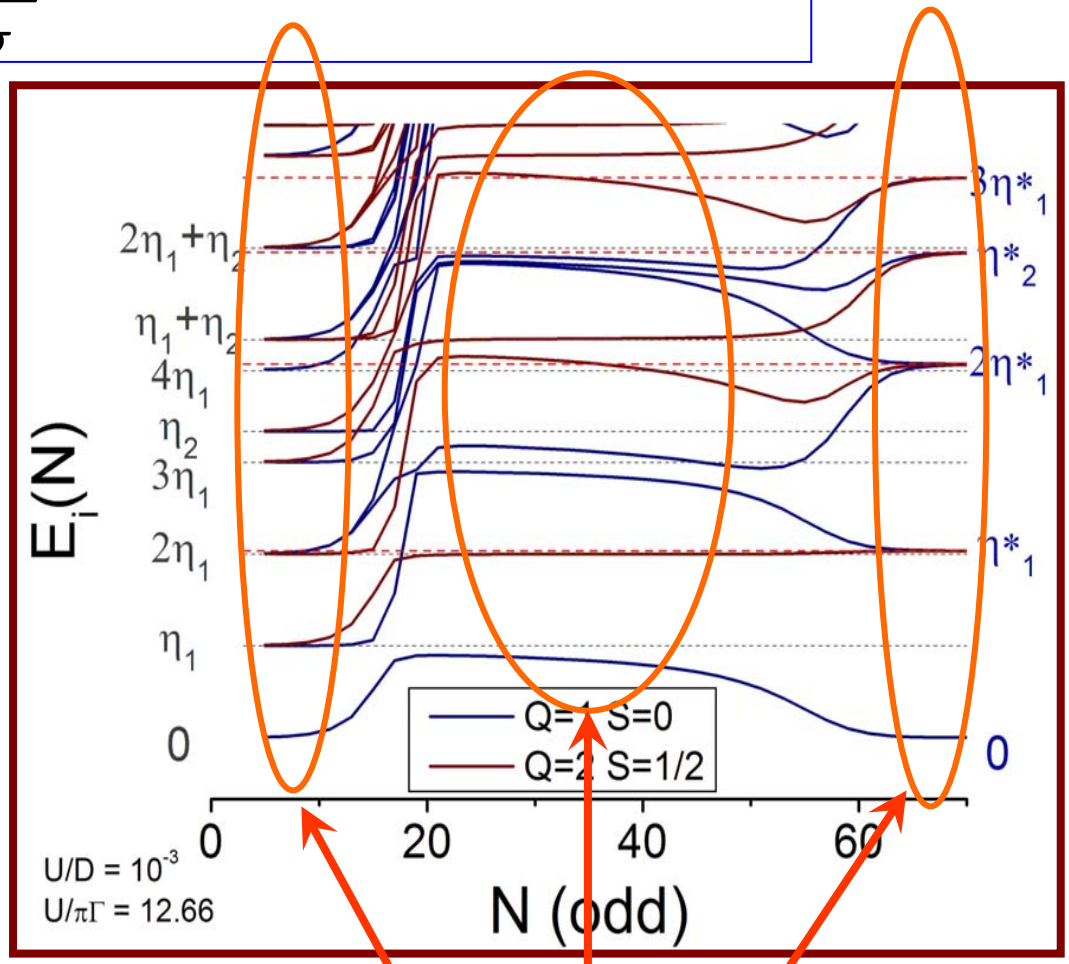
$$H_{N+1} = \sqrt{\Lambda} H_N + \xi_N \sum_\sigma f^\dagger_{N+1\sigma} f_{N\sigma} + f^\dagger_{N\sigma} f_{N+1\sigma}$$

- **Renormalization Group transformation**: (Re-scale energy by $\Lambda^{1/2}$).

$$H_{N+1} = R(H_N)$$

- **Fixed point H***: indicates *scale invariance.*

$$H^* = R^2(H^*)$$

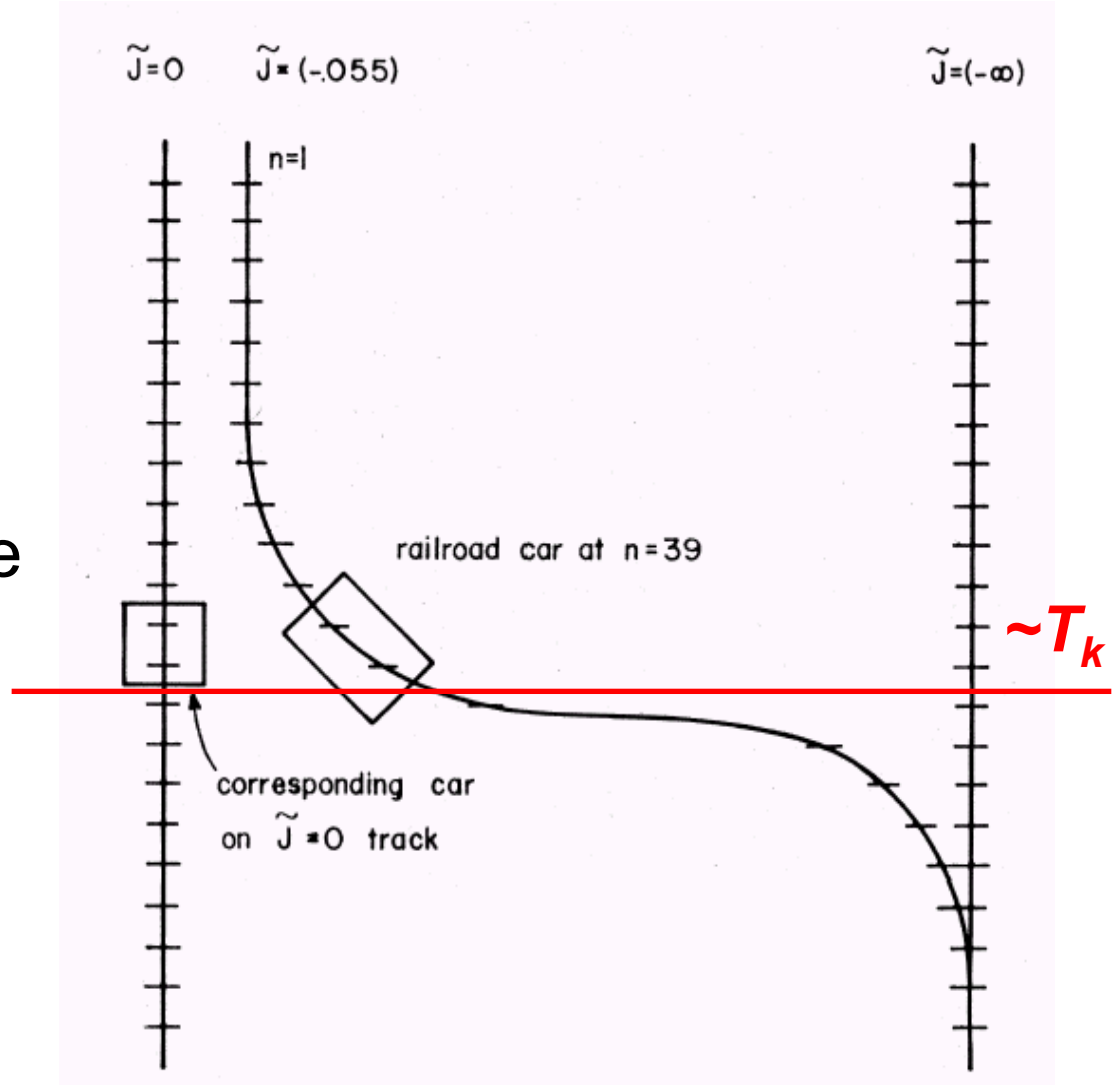


Fixed points

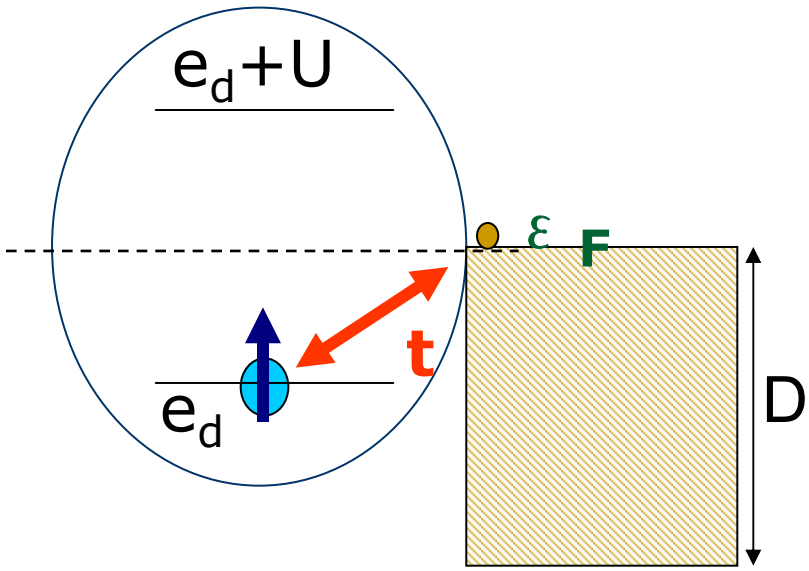
Numerical Renormalization Group

What can you do?

- Describe the physics at different energy scales for arbitrary *J.*
- Probe the parameter phase diagram.
- Crossing between the "free" and "screened" magnetic moment regimes.
- Energy scale of the transition is of order T_k



Anderson Model



$$H = \epsilon_d \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \sum_k \epsilon_k \hat{n}_{k\sigma} + t \sum_k c^\dagger_{d\sigma} c_{k\sigma} + \text{h.c.}$$

with

$$\hat{n}_{d\sigma} = c^\dagger_{d\sigma} c_{d\sigma}$$

$$\hat{n}_{k\sigma} = c^\dagger_{k\sigma} c_{k\sigma}$$

- e_d: energy level
- U: Coulomb repulsion
- e_F: Fermi energy in the metal
- t: Hybridization
- D: bandwidth

"Quantum dot language"

- e_d: position of the level (V_g)
- U: Charging energy
- e_F: Fermi energy in the leads
- t: dot-lead tunneling
- D: bandwidth

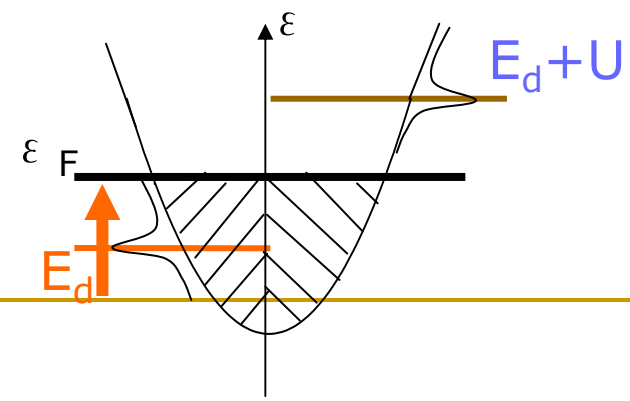
Schrieffer- Wolff Transformation

Anderson Model

Existence of localized moment

$$\left|V_{kd}\right| << U$$

Schrieffer-Wolff transformation



s-d Model

Schrieffer- Wolff Transformation

From: Anderson Model (single occupation)

$$\begin{aligned} H &= \epsilon_d \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} \\ &+ \sum_k \epsilon_k \hat{n}_{k\sigma} \\ &+ t \sum_k c^\dagger_{d\sigma} c_{k\sigma} + \text{h.c.} \end{aligned}$$

with

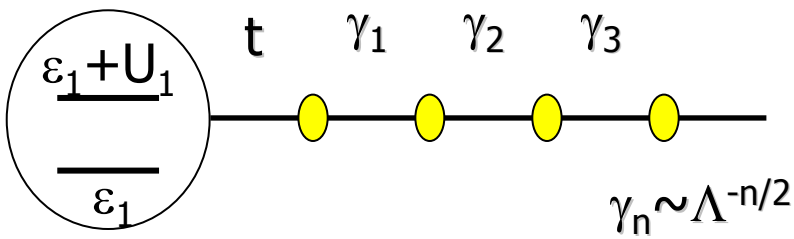
$$\begin{aligned} \hat{n}_{d\sigma} &= c^\dagger_{d\sigma} c_{d\sigma} \\ \hat{n}_{k\sigma} &= c^\dagger_{k\sigma} c_{k\sigma} \end{aligned}$$

$$J = t^2 \sum_{k,k'} \left\{ \frac{1}{U + \epsilon_d - \epsilon'_k} + \frac{1}{\epsilon_k - \epsilon_d} \right\}$$

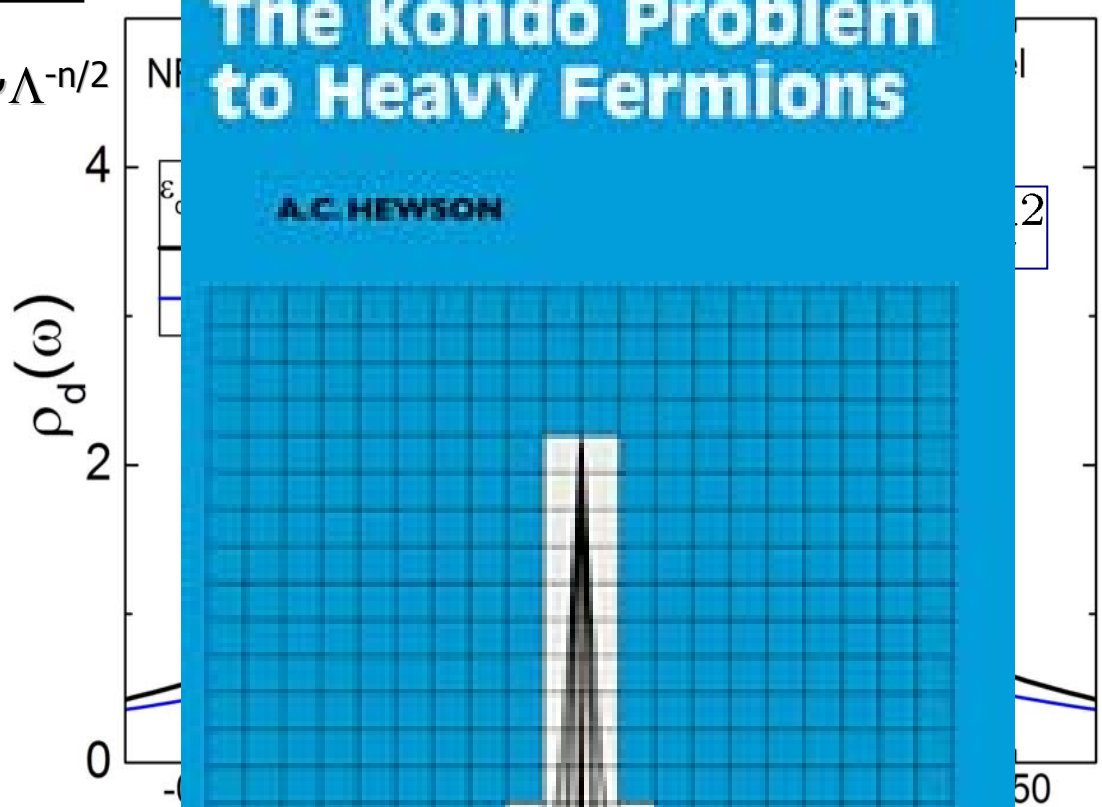
To: s-d (Kondo) Model

$$\begin{aligned} H_{\text{s-d}} &= J \sum_{kk'} S^+ c^\dagger_{k\downarrow} c_{k'\uparrow} + S^- c^\dagger_{k\uparrow} c_{k'\downarrow} \\ &+ S_z \left(c^\dagger_{k\downarrow} c_{k'\uparrow} - c^\dagger_{k\downarrow} c_{k'\downarrow} \right) \\ &+ \sum_k \epsilon_k \hat{n}_{k\sigma} \end{aligned}$$

NRG on Anderson model: LDOS



- Single-particle peaks at ε_d and ε_d+U.
- *Many-body* peak at the Fermi energy: ***Kondo resonance*** (width $\sim T_K$).
- NRG: good resolution at low ω (log discretization).



History of Kondo Phenomena

- Observed in the ‘30s
- Explained in the ‘60s
- Numerically Calculated in the ‘70s (NRG)
- Exactly solved in the ‘80s (Bethe-Ansatz)

So, what’s new about it?

Kondo correlations observed in many different set ups:

- Transport in quantum dots, quantum wires, etc
- STM measurements of magnetic structures on metallic surfaces (e.g., single atoms, molecules. “Quantum mirage”)
- ...

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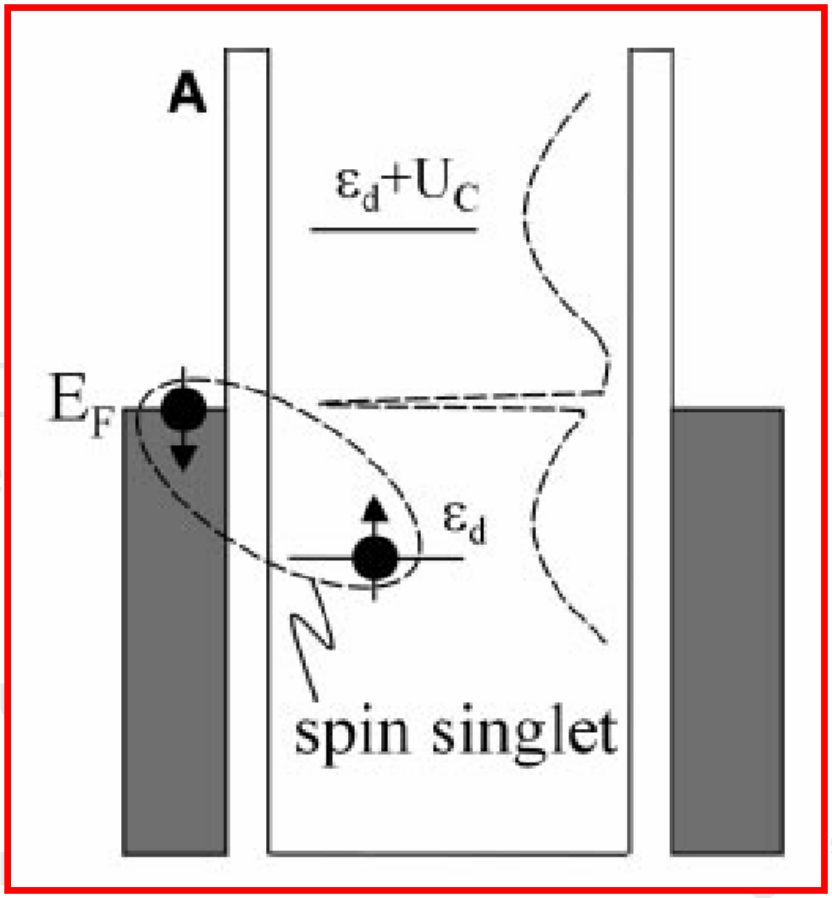


Kondo correlations observed

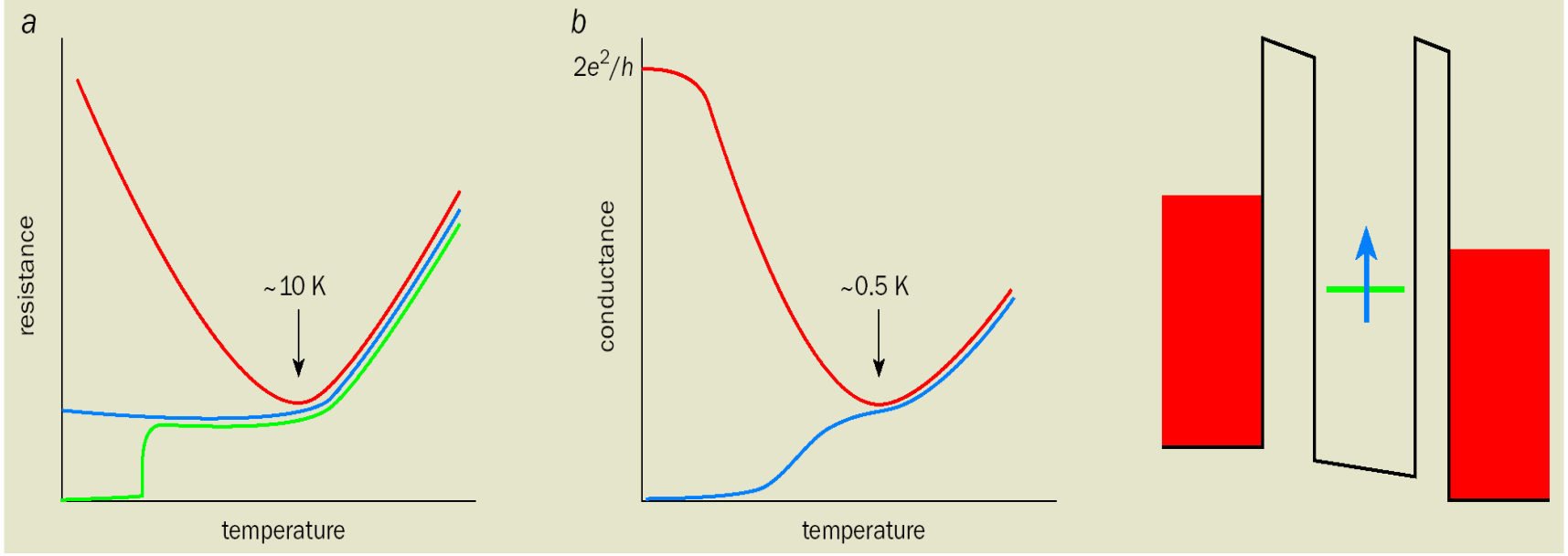
- Transport in quantum dots, q
- STM measurements of magn
  surfaces (e.g., single atoms,)
- ...

Kondo Effect in Quantum Dots

Revival of the Kondo effect

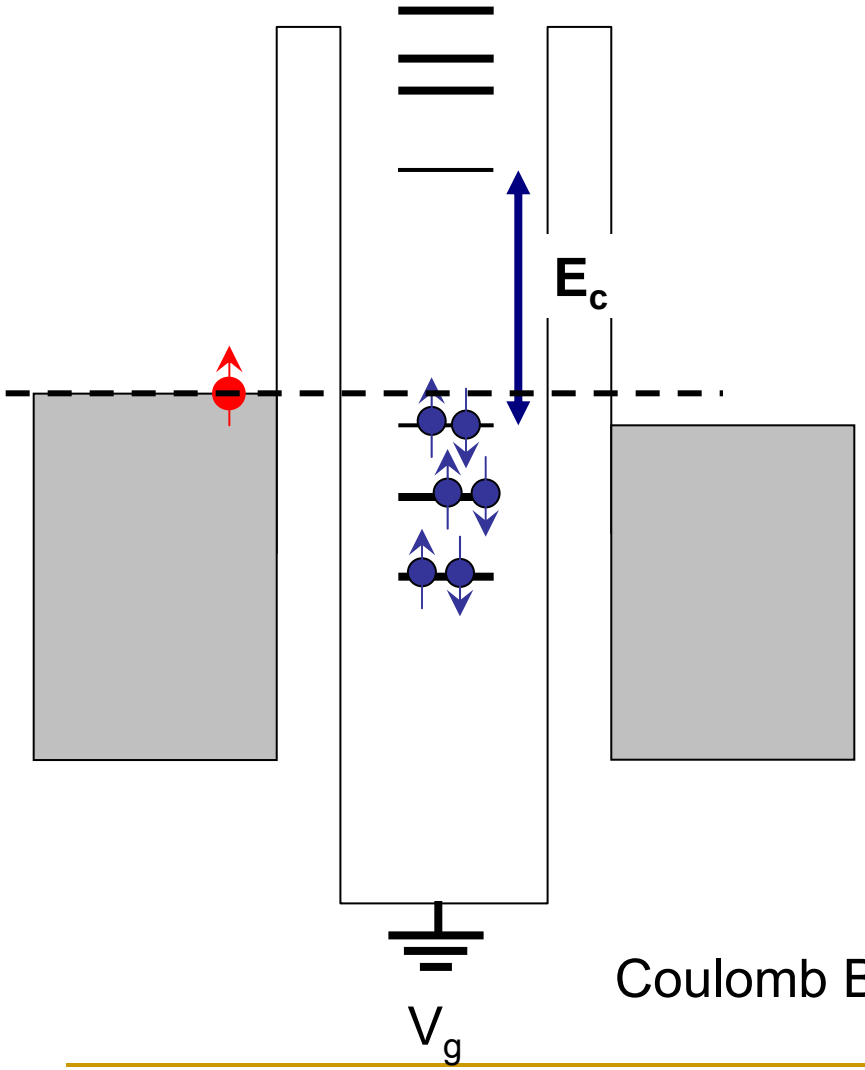
Leo Kouwenhoven and Leonid Glazman

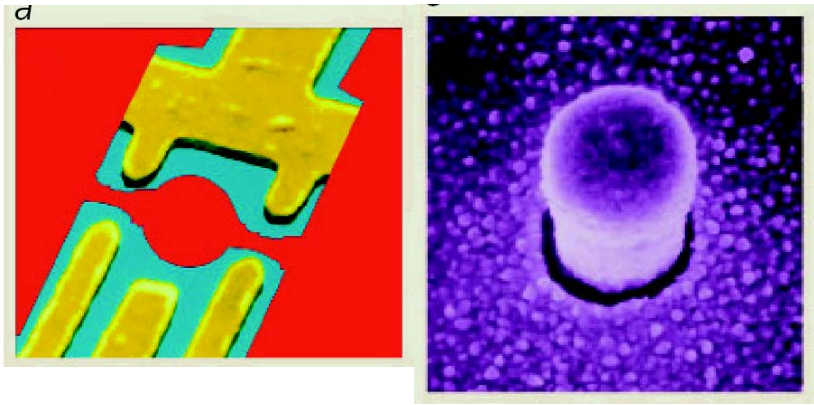
1 The Kondo effect in metals and in quantum dots

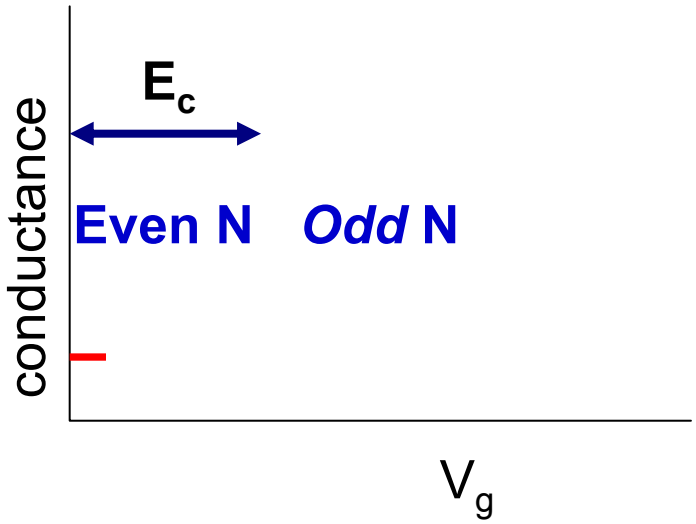


Kowenhoven and Glazman *Physics World* – Jan. 2001.

Coulomb Blockade in Quantum Dots

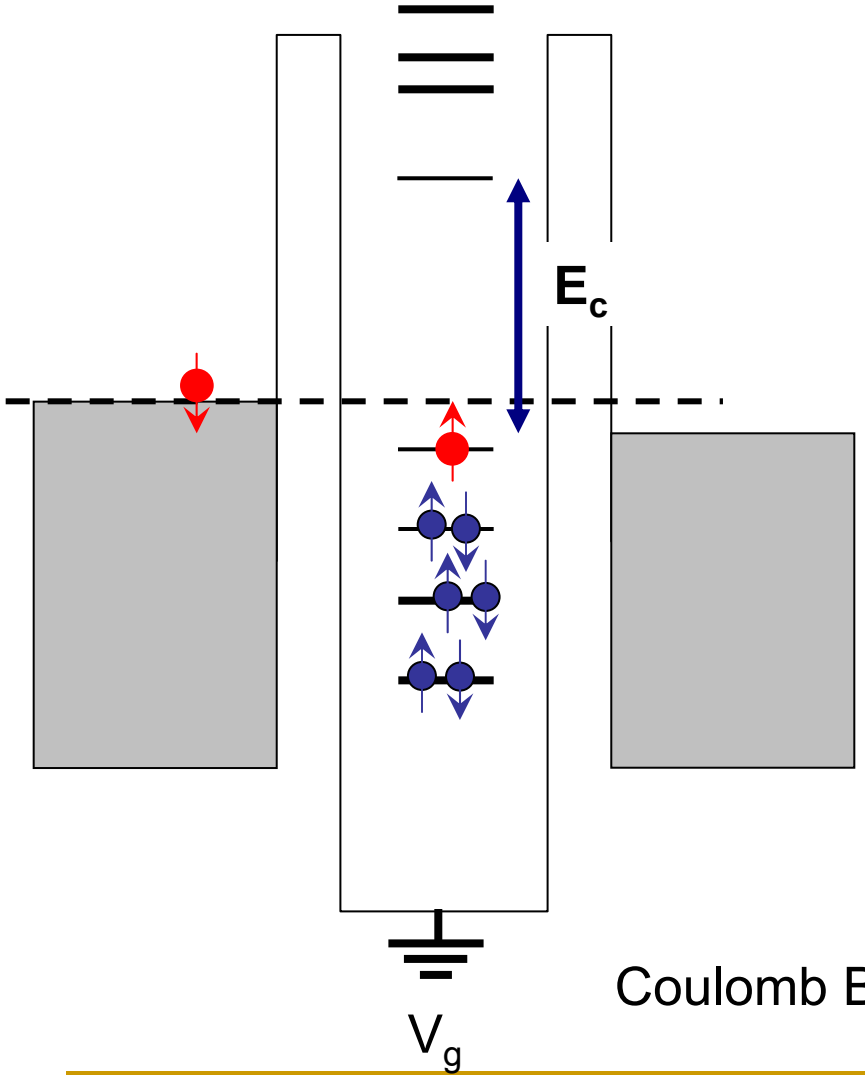


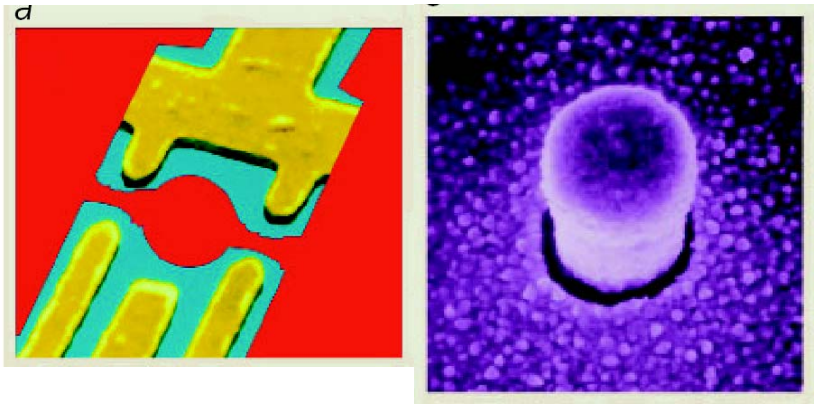


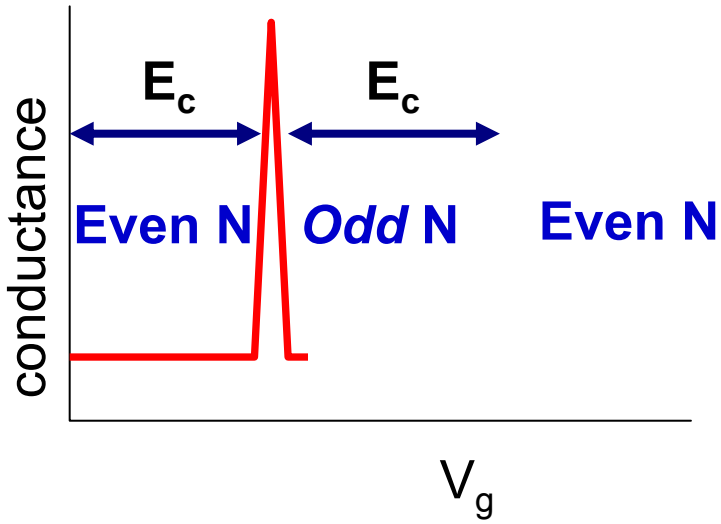


Coulomb Blockade in Quantum Dots

Coulomb Blockade in Quantum Dots



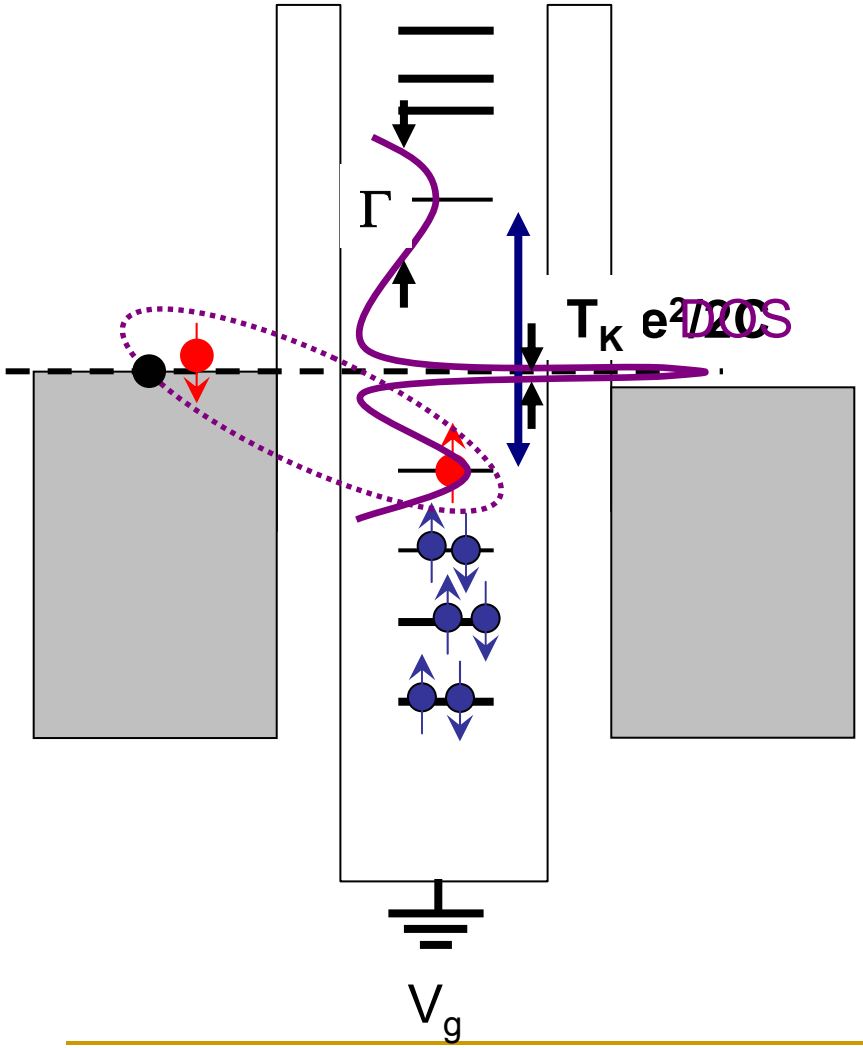


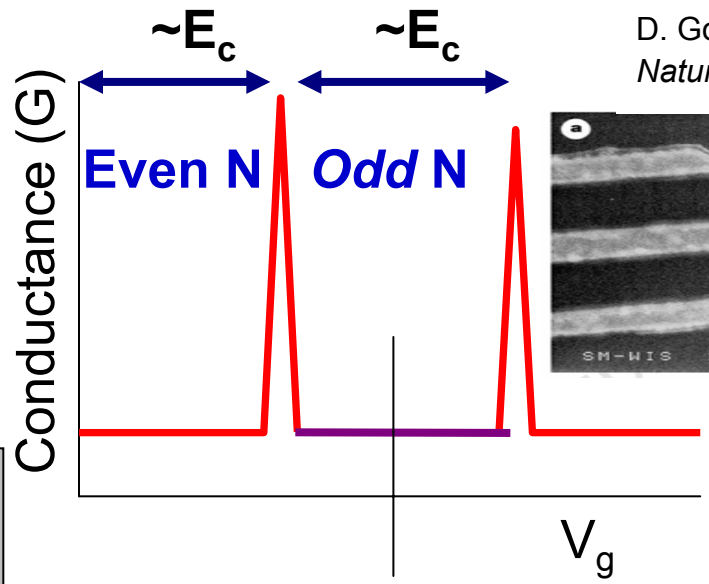


Coulomb Blockade in Quantum Dots

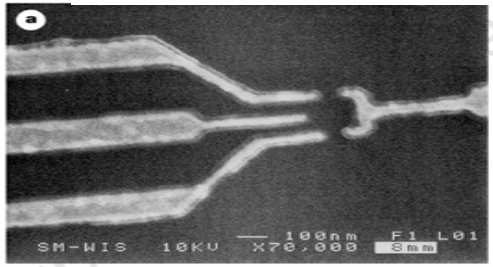
Y. Alhassid *Rev. Mod. Phys.* **72** 895 (2000).

Kondo Effect in Quantum Dots





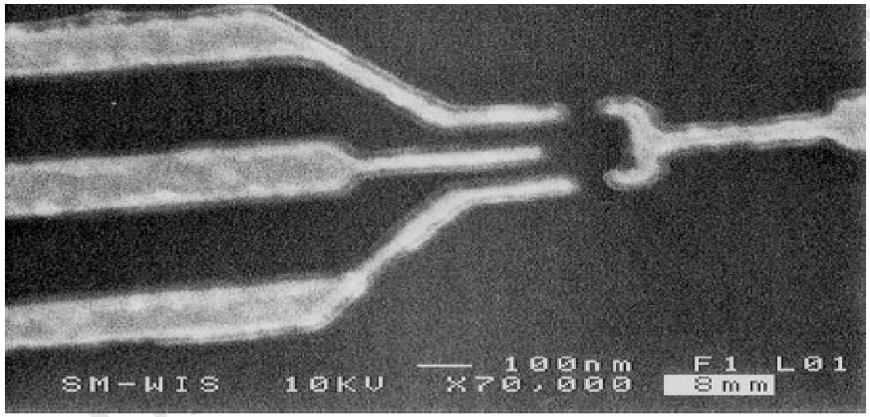
D. Goldhaber-Gordon et al
Nature **391** 156 (1998)

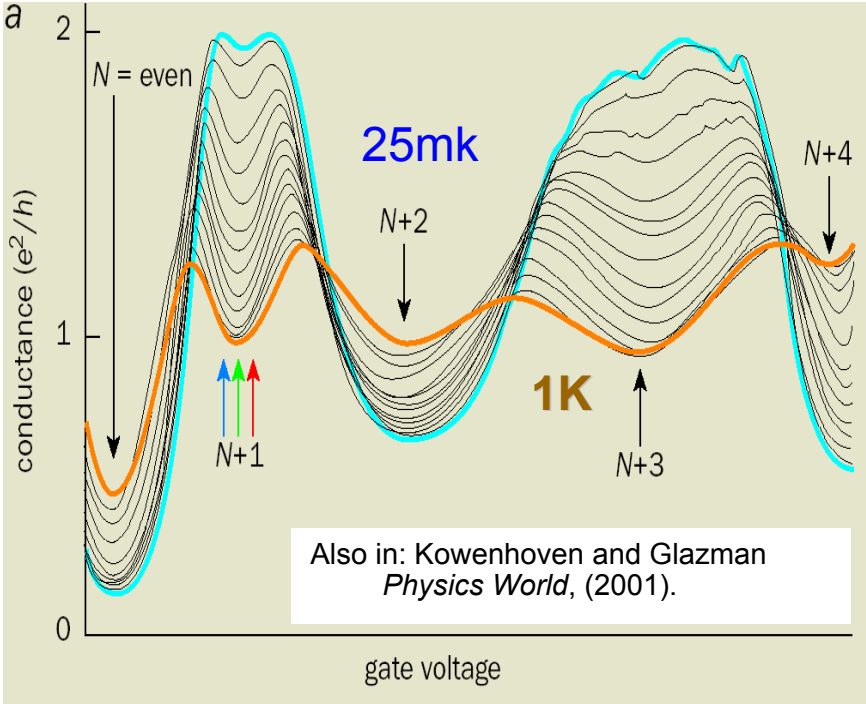


- $T>T_K$: Coulomb blockade (low G)
- $T<T_K$: Kondo singlet formation
- Kondo resonance at E_F (width T_K).
- New conduction channel at E_F:
  Zero-bias enhancement of G

Kondo effect in Quantum Dots

D. Goldhaber-Gordon et al. Nature **391** 156 (1998)



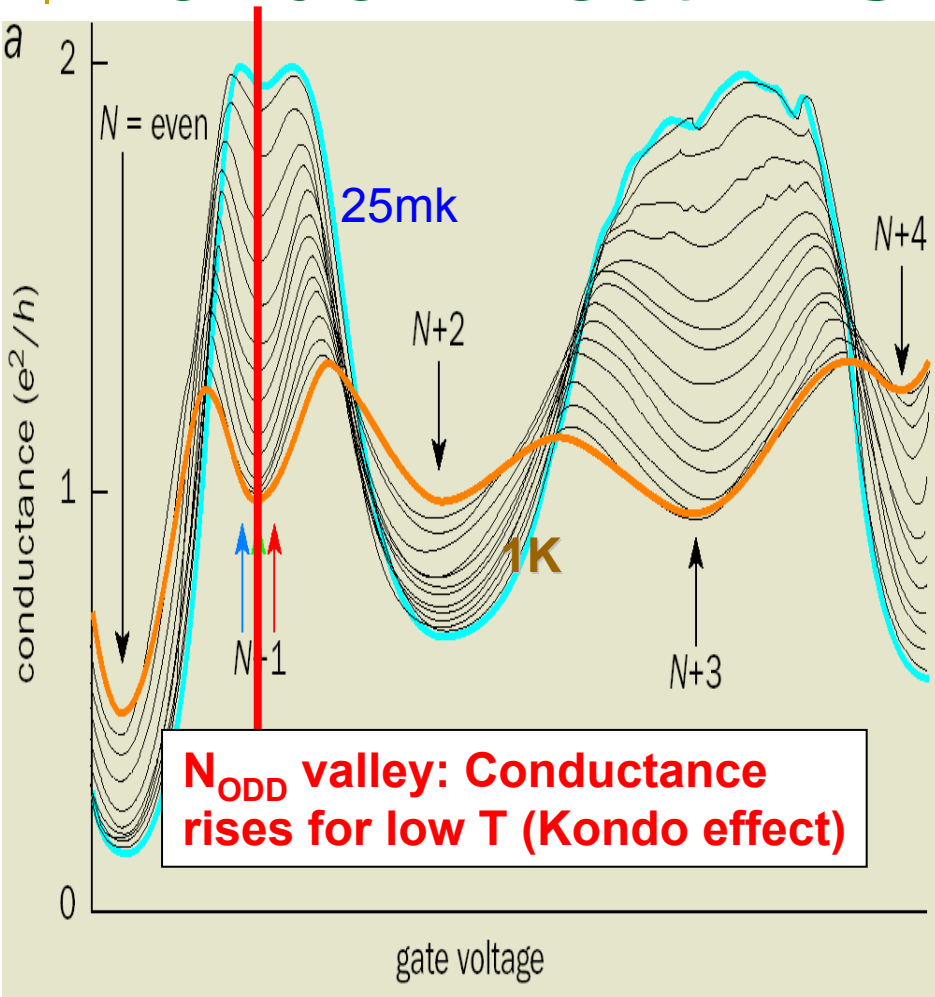


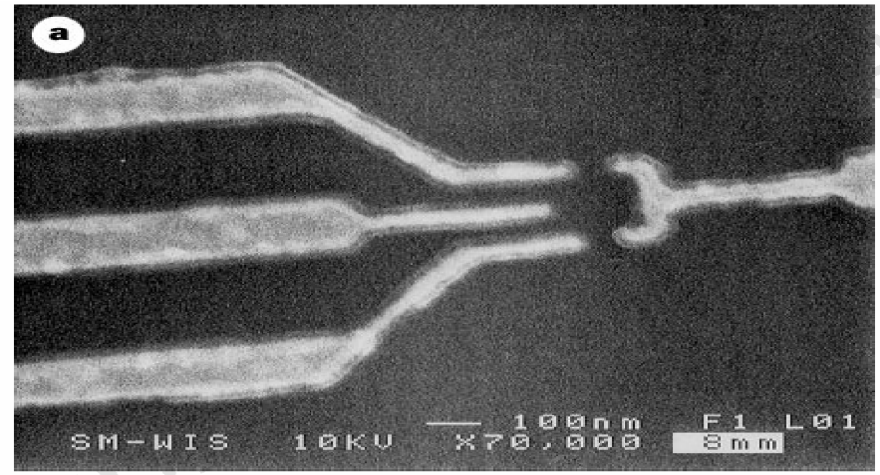
Also in: Kowenhoven and Glazman *Physics World*, (2001).

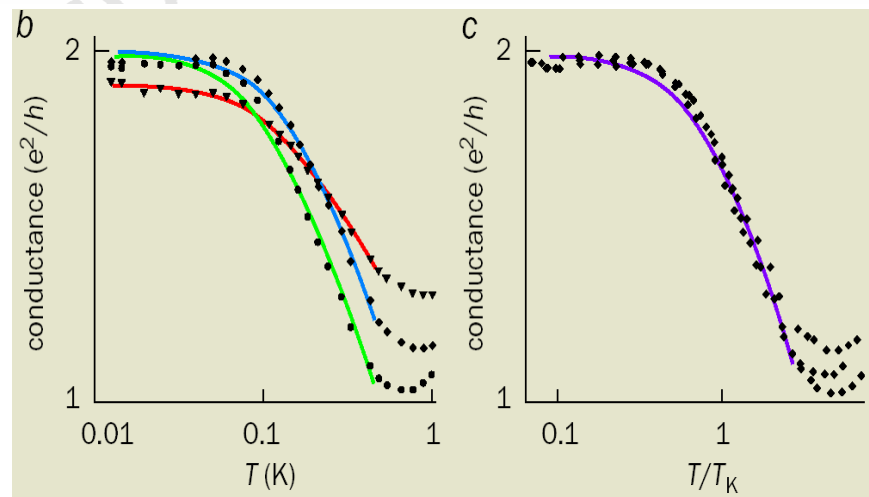
Semiconductor Quantum Dots:

- Allow for systematic and *controllable* investigations of the Kondo effect.
- QD in N_{odd} Coulomb Blockade valley: realization of the Kondo regime of the Anderson impurity problem.

Kondo Effect in CB-QDs





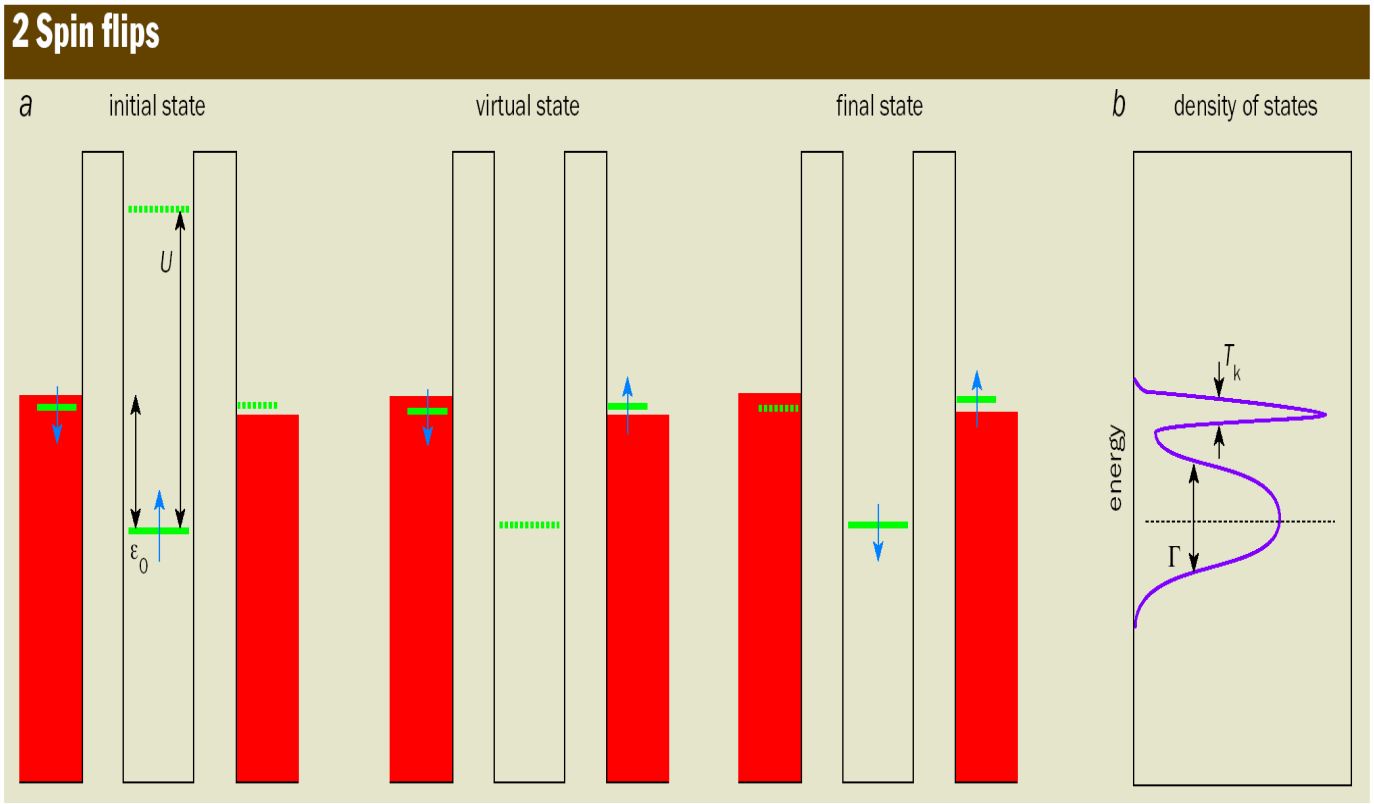


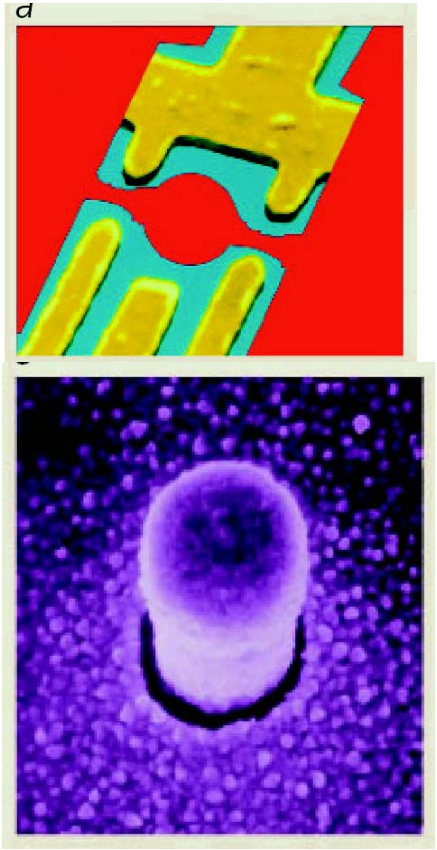
Kondo Temperature T_k : only scaling parameter (~0.5K, depends on V_g)

Kowenhoven and Glazman *Physics World* – Jan. 2001.

From: Goldhaber-Gordon *et al. Nature* **391** 156 (1998)

Kondo Effect in Quantum Dots



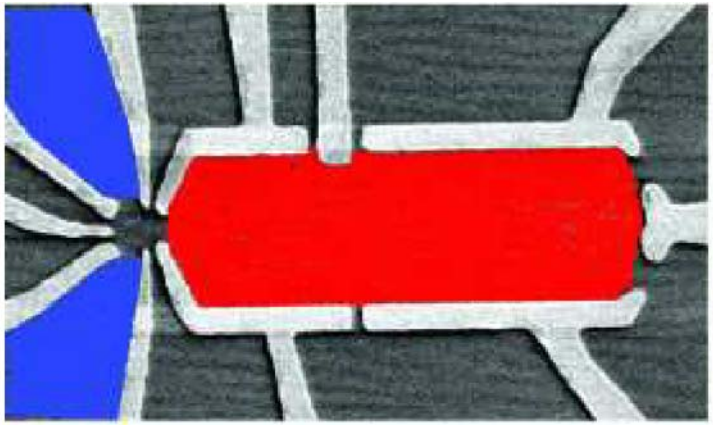


Basic mechanism of the Kondo effect in Coulomb Blocked quantum dots

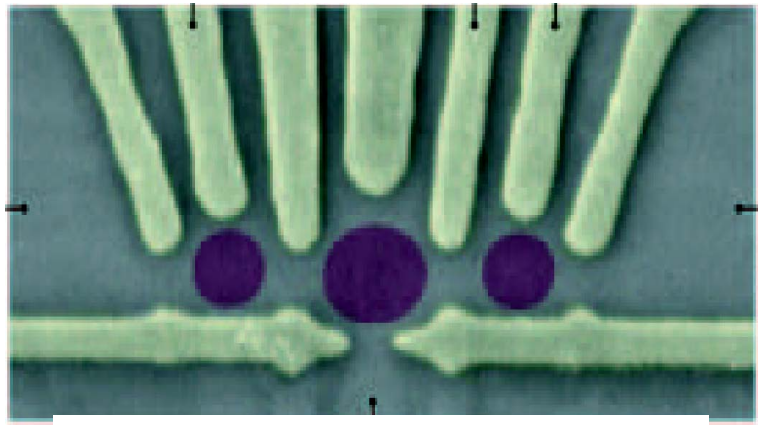
Kowenhoven and Glazman *Physics World* – Jan. 2001.

Kondo Effect in *Double* QDs

"Side dot" configuration

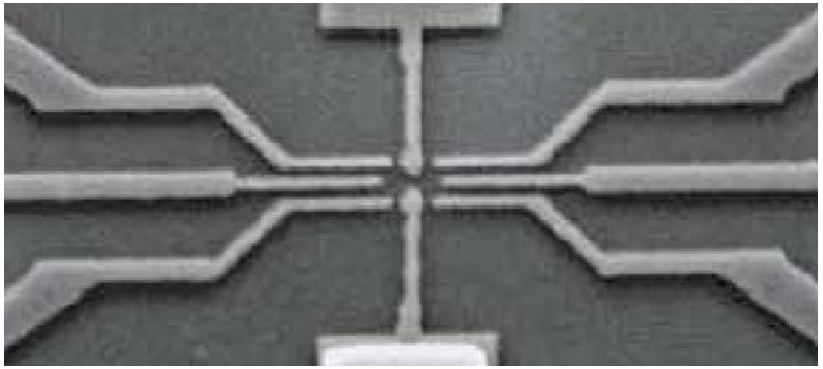


R. Potok *et al. Nature* 446 167 (2007).



Craig et al., *Science* **304** 565 (2004)

"Parallel" configuration



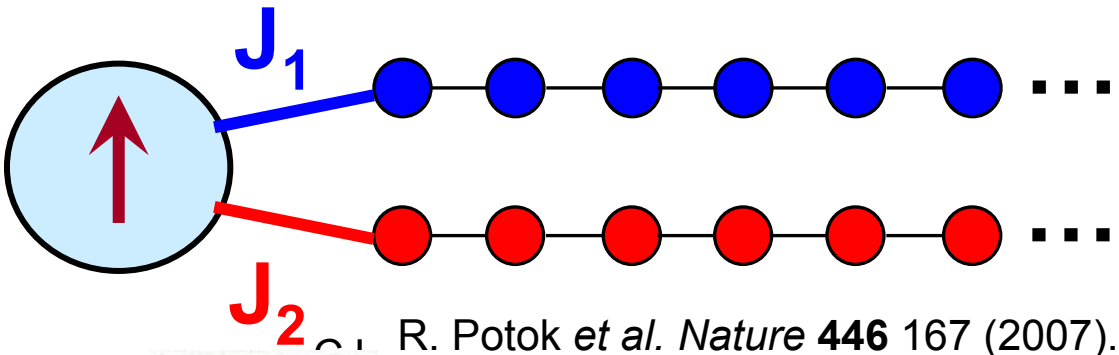
Chen, Chang, Melloch, *PRL* **92** 176801 (2004)

- Tunability of intradot and interdot parameters (couplings, gate voltage).

- Prospects for experimental probe of many-body phenomena, e.g:
  - SU(4) Kondo, RKKY interactions,...
  - Non-Fermi liquid physics (2-ch Kondo)
  - **Quantum phase transitions.**

Two-channel Kondo effect.

- Spin 1/2 coupled to two ***independent*** bands: 2-channel Kondo model ("overscreened").

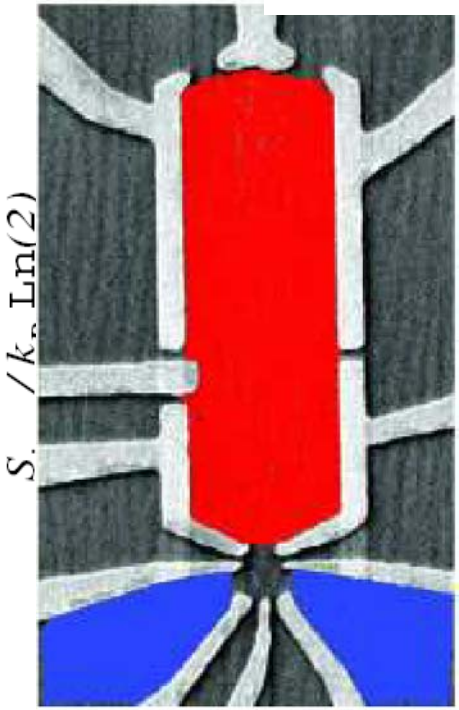


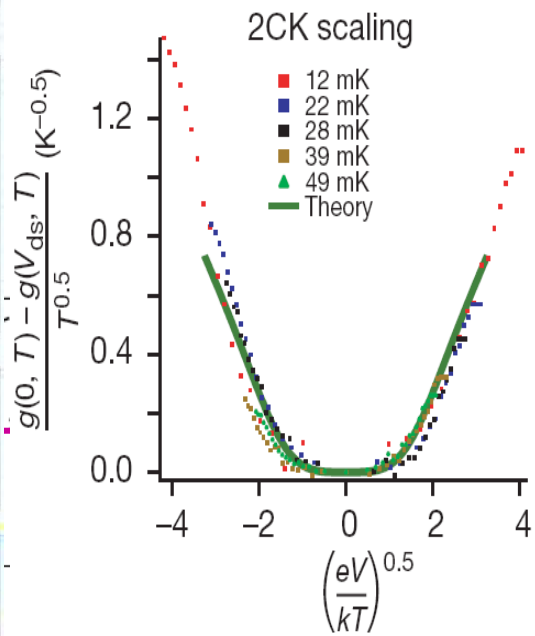
R. Potok *et al. Nature* **446** 167 (2007).

- Non-Fermi-liquid (NFL) behavior for $J_1 = J_2$.

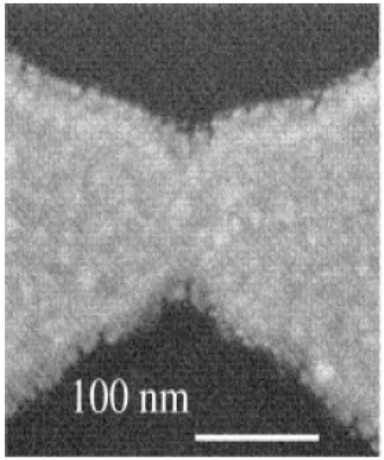
- Impurity entropy (NFL): $S_{imp} = k_B \log(\sqrt{2})$ (NRG, Bethe ansatz).

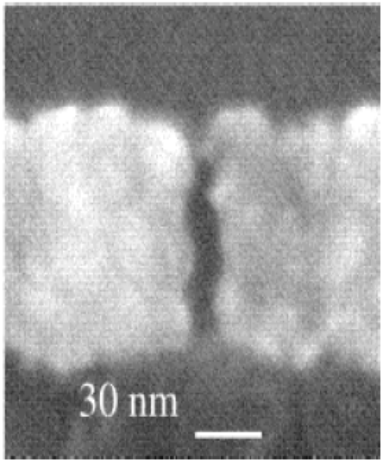
- Recent expts in q-dots.

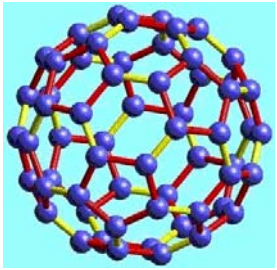




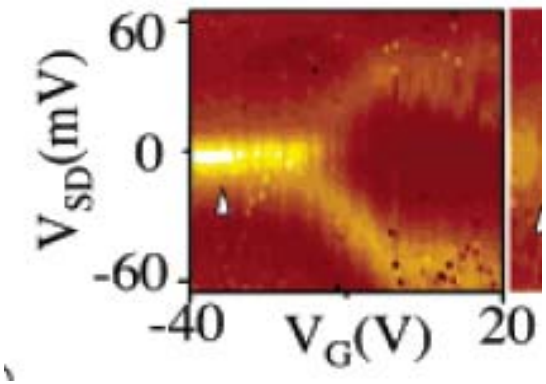
Kondo effect in Single Molecule Transistors

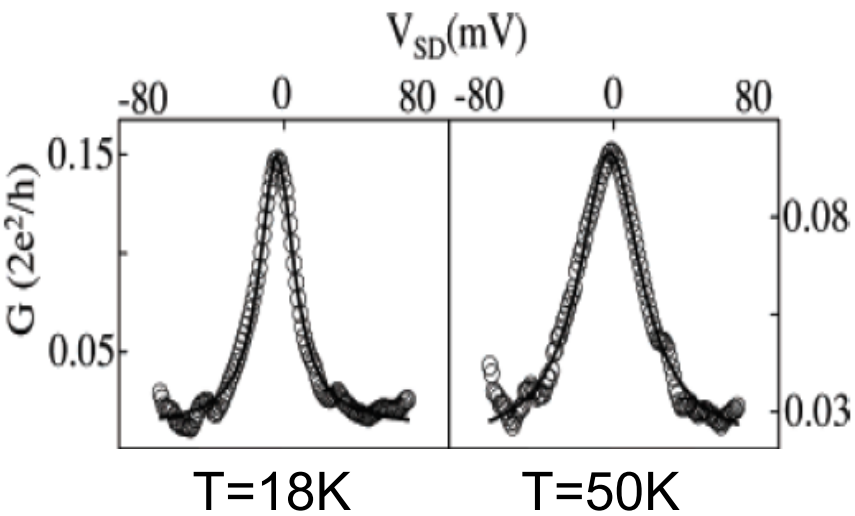






C_{60}



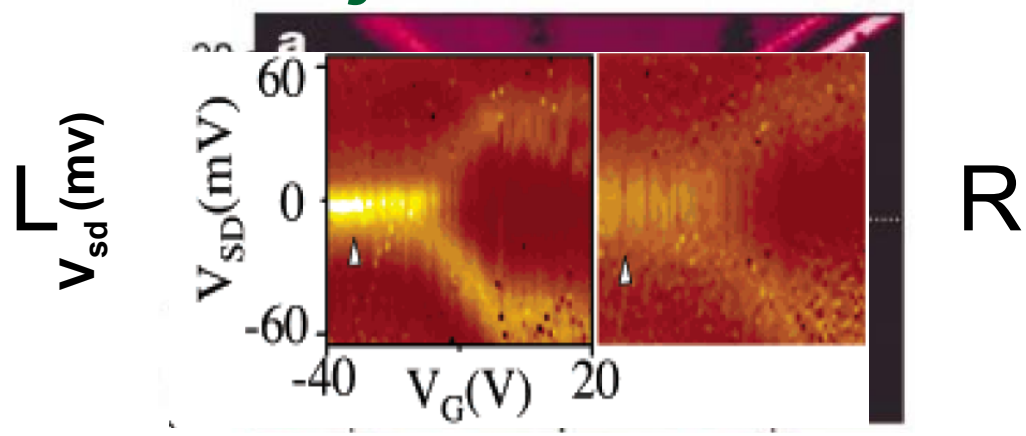


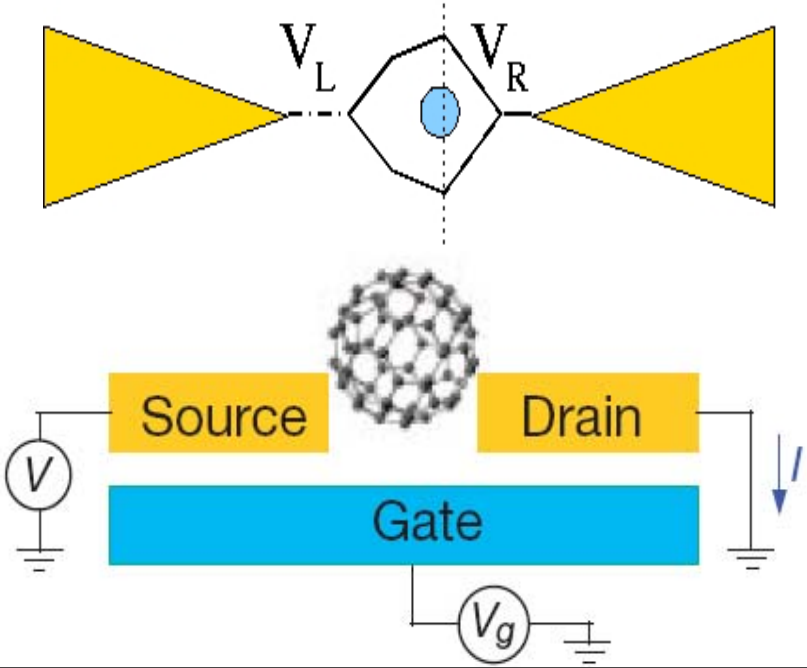
- Single molecule transistors: C_{60} molecules "caught" between electrodes (break junction).
- Zero-bias peak as a function of gate voltage: correct Kondo scaling.
- Correct behavior vs. Bias.
- T_K>50K .

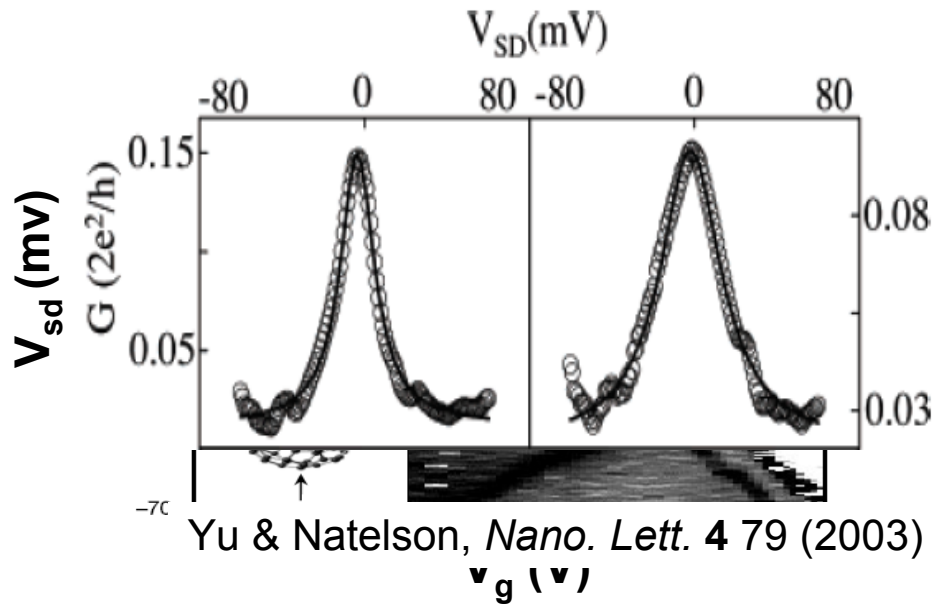
Yu, Natelson, *NanoLett.* **4** 79 (2004).

Transport in molecular junctions.

- Coulomb blockade effects .
- Features consistent with vibrational modes in dI/dV.
- Kondo signatures.





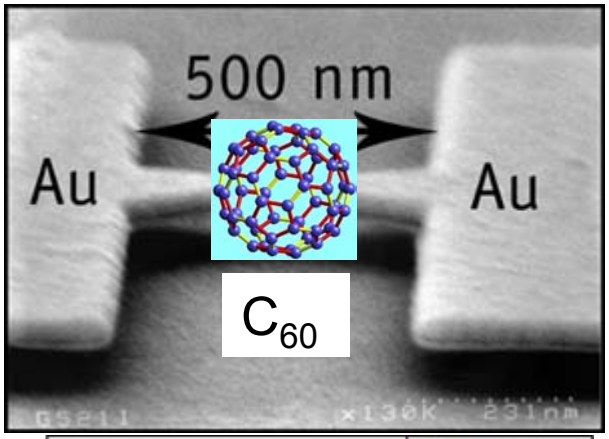


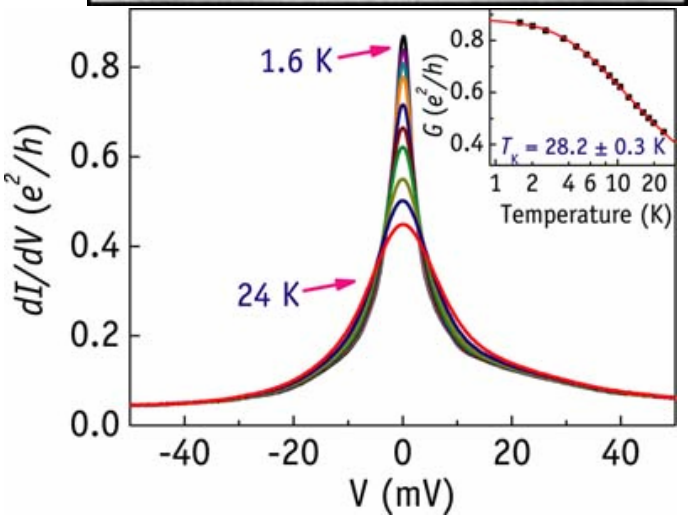
Yu & Natelson, *Nano. Lett.* **4** 79 (2003)

Park et al. *Nature* **407** 57 (2000)

Connection of vibrational modes with 2-channel Kondo:
LDS, E. Dagotto - arXiv:0902.3225 (*to appear in PRB*).

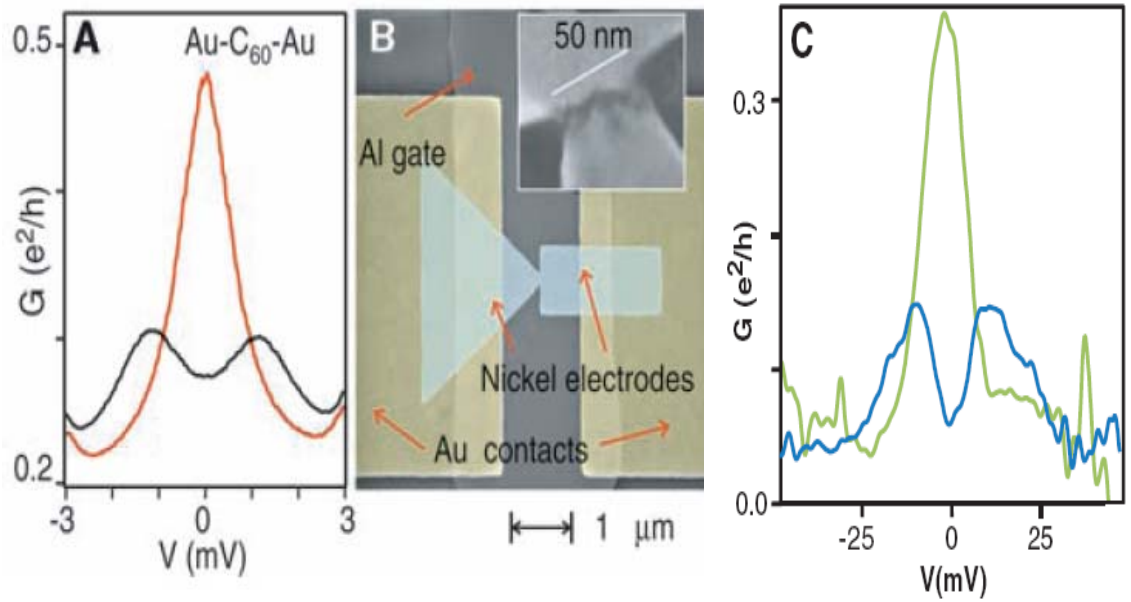
Kondo effect in Single Molecule Transistors





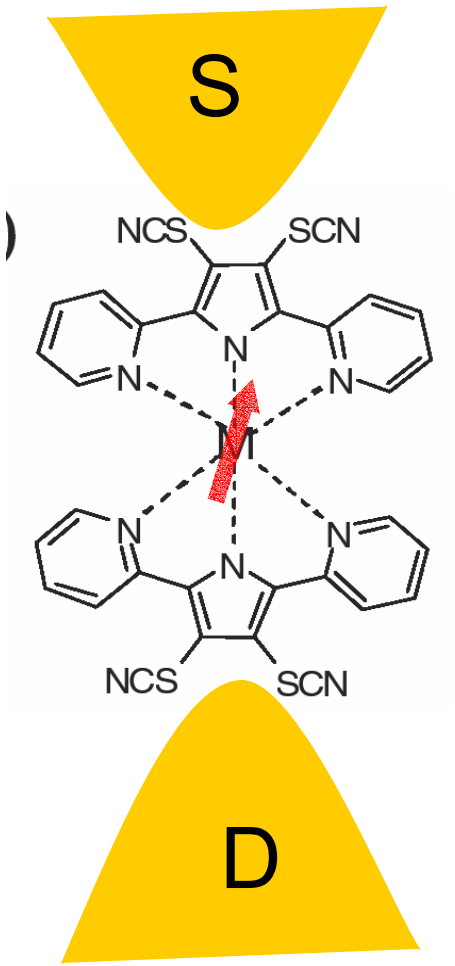
From Dan Ralph's webpage:
http://people.ccmr.cornell.edu/~ralph/

Pasupathy et al., *Science* **306** 86 (2004)

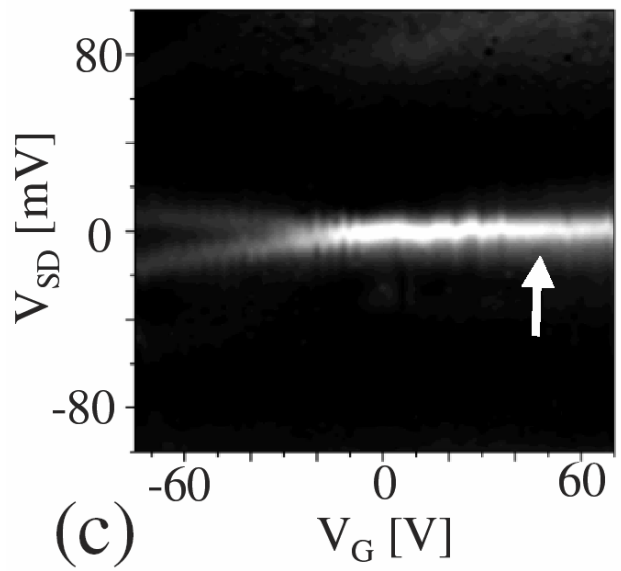


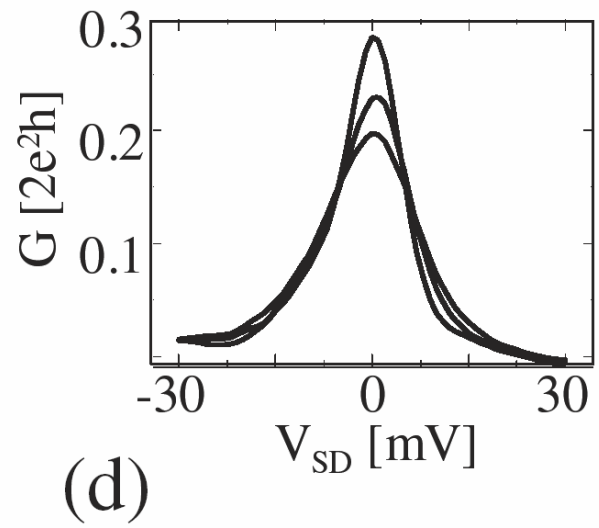
- Similar expts (D. Ralph's group).
- Supression of the Kondo resonance in the presence of a magnetic field (top left, black curve, B=10T) and *magnetic* leads (top right, parallel [green] and antiparallel [blue] magnetizations).

SMTs: Condutanace measurements



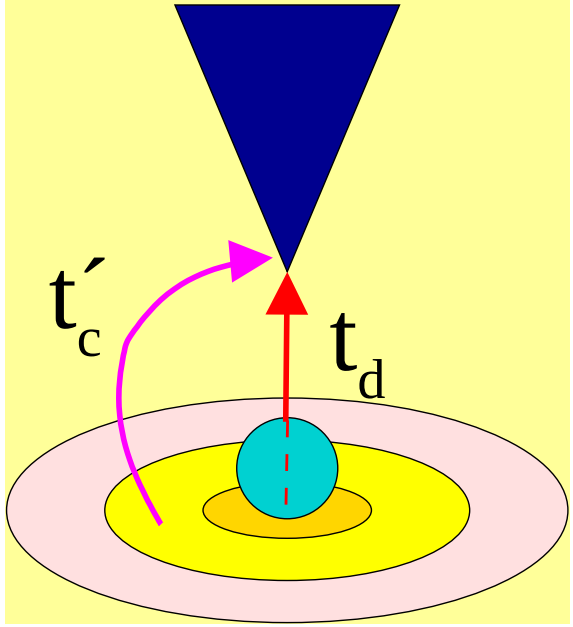
T=5K, 16K, 30K



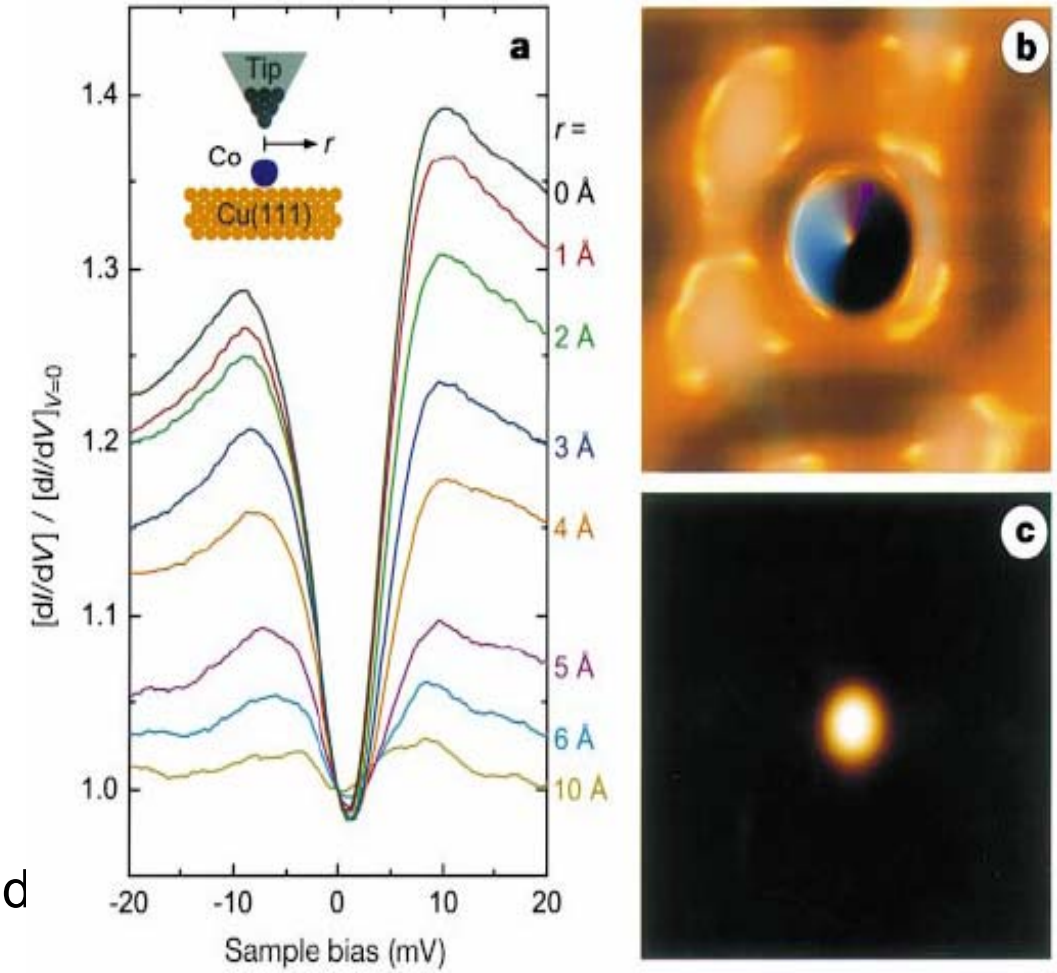


- Zero-bias peak at the conductance: Signature of Kondo correlations.
- Their range
- Peak height decreases with T and peak width increases with T (as expected).

Kondo effect in surfaces (STM images).



Manoharan et al., *Nature* **403** 512 (2000).



- Magnetic (Co, Fe) atoms on metallic *surfaces!* Right ingredients for Kondo.
- In this case, Kondo is marked by a *dip* at zero-bias conductance (dI/dV at V=0).

Summary: Lectures on strongly correlated phenomena in nanostructures.

- *Lecture 1*: Quantum Dots.
- *Lecture 2*: Kondo effect/NRG.
- *Lecture 3*: Kondo effect in nanostructures.
- **Nanostructures display an array of strongly correlated phenomena:** (Kondo and 2ch Kondo effects (= non-Fermi-liquid behavior), interplay of spin and vibrational effects… quantum phase transitions, SU(4) Kondo effect).
- Opportunity: *controlled* studies of all these features.
  - *Thanks for your attention.*