Lectures: Condensed Matter II
1 – Electronic Transport in Quantum dots
2 – Kondo effect: Intro/theory.
3 – Kondo effect in nanostructures

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Lecture 1: Outline

- Introduction: From atoms to “artificial atoms”.
- What are Quantum Dots?
- Confinement regimes.
- Transport in QDs: General aspects.
- Transport in QDs: Coulomb blockade regime.
- Transport in QDs: Peak Spacing.
“More is Different”

“The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of simple extrapolation of the properties of a few particles.

Instead, at each level of complexity entirely new properties appear and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other.“

Phillip W. Anderson, “More is Different”, Science 177 393 (1972)
“More is Different?”

“If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words?

I believe it is the **atomic hypothesis** that **All things are made of atoms-little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.**“

In that one sentence, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

R. P. Feynman – *The Feynman Lectures*
Can you make “atoms” out of atoms?

Energy

[Ar] 3d^{10} 4s^2 4p^1[Ar] 4s^2 3d^{10} 4p^3

Atomic Energy levels

Band structure

M. Rohlfing et al. PRB 48 17791 (1993)

GaAs crystal

Band gap

Many Atoms!

M. Rohlfing et al. PRB 48 17791 (1993)
Making “artificial atoms”(?) out of atoms

Many Atoms! 4p 3d 4s

Confinement in 2D:

Confine in 2D!

Band gap

E F

Conduction

Valence

2D Electron gas

AlGaAs/GaAs heterostructure

AlGaAs

GaAs

Crystal (3D)

Density of states

lowest subband

E F

2D
Confined in 1 direction: 2D system

If a thin enough 2D plane of material (containing free electrons) is formed the electrons can be confined to be two dimensional in nature. Experimentally this is usually done in semiconductors.

$$\rho_{2D}(E) = \frac{m}{\pi \hbar^2} \sum_i \Theta(E - E_i)$$

e.g. by growing a large band gap material with a smaller band gap material you can confine a region of electrons to the interface - TWO DIMENSIONAL ELECTRON GAS (2DEG).

Confined in 1 direction: 2D system

Provided the electrons are confined to the lowest subband the electrons behave exactly as if they are two-dimensional i.e. obey 2D Schrödinger equation etc.

2DEG: Rich source of Physics.

- Nobel Prizes in Physics:
  - in 1985 to von Klitzing for the Quantum Hall Effect (QHE),
  - In 1998 to Tsui, Stormer and Laughlin for the Fractional QHE
- Semiconductor heterostructures, lithography
- Applications (lasers, QHE, etc.)
If the confinement length is of the order of the Fermi wavelength ($L \sim \lambda_F$ or $E_1 < \sim E_F$) then electrons confined in one quantum mechanical state in two directions, but are free to move in the third: 1D confinement.

A truly 1D DoS *diverges* at some energy values: these are van Hove singularities.

Electrons interact differently in 1D compared to 2D and 3D (e.g. Luttinger liquid physics).

Analogy: think of cars (electrons) moving along a single track lane. They “interact” differently compared to cars on dual lane roads, for instance.

Examples include carbon nanotubes, nanowires, lithographically defined regions of 2DEGs etc.
Making “artificial atoms” (?) out of atoms

Many Atoms!

Many Atoms!

Confine in 2D!

Confine in 0D

4p 3d 4s

Band gap

Conduction

Valence

E_F

density of states

lowest subband

2D Electron gas

E

AlGaAs/GaAs heterostructure

AlGaAs

GaAs

2D Quantum dot

0 D Quantum dot

CRystal (3D)
Confinement: Particle in a box

1-d box: wavefunction constrained so that

\[ L = N \lambda / 2 \] or
\[ k = 2 \pi / \lambda = N \pi / L \]

Energy of states given by Schrodinger Equation:

\[ \hat{H} \psi = E \psi \]

\[ E_N = \frac{\hbar^2 k_N^2}{2m} = \frac{\hbar^2 \pi^2 N^2}{2mL^2} \]

Typical semiconductor dots:

- L in nm, E in meV range

As the length scale decreases the energy level spacing increases.

\[ \Delta E = E_{N+1} - E_N \propto \frac{1}{L^2} \]
Can you make “atoms” out of atoms?

**2D Electron gas**
Electrostatically confine electrons within a small (nanometer-size) region.

**“Quantum dot”**
Energy levels
Tunnel barriers

from Charlie Marcus’ Lab website (marcuslab.harvard.edu)
Making “artificial atoms” (?) out of atoms

Discrete levels: we might be tempted to call them “artificial atoms”.

BUT: different energy/length scales!

Strong charging (U) → Many-body correlations

Confine in 0D

\( \text{lowest subband} \)
“There is plenty of room at the bottom”.

“This field is not quite the same as the others in that it will not tell us much of the fundamental physics in the sense of, ‘What are the strange particles?’ But it is more like solid state physics in the sense that it might tell us much of the great interest about the strange phenomena that occur in complex situations.”

– Richard Feynman

Or, as Anderson might put it:

The rules of the game are different at the bottom.
What are Quantum Dots?

Semiconductor Quantum Dots:

- Devices in which electrons are confined in nanometer size volumes.

- Sometimes referred to as “artificial atoms”.

- “Quantum dot” is a generic label: lithographic QDs, self-assembled QDs, colloidal QDs have different properties.
Lithographic Quantum Dots

How to do it in practice? (a question for the experimentalists…)

Ingredients:
- Growth of heterostructures to obtain the 2DEG
  - (good quality, large mean free-paths)
- Metallic electrodes electrostatically deplete charge: confinement
- Sets of electrodes to apply bias etc.
- **LOW TEMPERATURE! (~100 mK)**
Lithography evolved quite a bit in the last decade or so. Allow different patterns: double dots, rings, etc.
Quantum Dots: transport

**Lithographic Quantum Dots:**

- Behave like small capacitors:
  \[ E_C = \frac{e^2}{C} \]
- Weakly connected to metallic leads.
- Energy scales: level spacing \( \Delta E \); level-broadening \( \Gamma \).
- \( E_C \) is usually largest energy scale:
  \[ E_C \gg \Delta E, \Gamma \]


Jeong, Chang, Melloch *Science* **293** 2222 (2001)
Electrical Transport

Gate Electrode

Insulator

Sample

Source → Sample → Drain

$E_F$  $E_F$  $E_F$

L  R  ?

METAL  METAL  METAL

e $V_{sd}$
Role of the Gate Electrode

$V_G = 0$

$V_G = +V$

Raise Fermi level – adds electrons

$V_G = -V$

Lower Fermi level – remove electrons
Ohm’s Law holds for metallic conductors => \( V = I R \)

We can also define a conductance which can be bias dependent. The **zero bias conductance**, \( G \), is conventionally quoted.

Electrical Transport: Ohm’s Law

Metallic conductor:

- Resistance due to scattering off impurities, mfp ~ 10 nm

![Diagram showing conduction band, filled region, and gate voltage change](image-url)
Semiconductor: tunneling through Schottky barrier or out of band gap.

Semiconductor - nonlinear $I$ - $V$ response

gate voltage $V_G$

(changes $E_F$)
Quantum dots contain an integer number of electrons.

Adding an electron to the QD changes its energy $\Rightarrow$ electrostatic charging energy

In order for a current to pass an electron must tunnel onto the dot, and an electron must tunnel off the dot.

For **conduction at zero bias** this requires the energy of the dot with $N$ electrons must equal the energy with $N+1$ electrons. i.e. charging energy balanced by gate potential.

\[
\frac{Q^2}{2C}
\]
Coulomb blockade

The addition of an electron to the dot is blocked by the charging energy as well as the level spacing \( \Rightarrow \) Coulomb blockade

The number of electrons on the QD is adjusted by the gate potential. Conduction only occurs when \( E_N = E_{N+1} \)
Coulomb Blockade in Quantum Dots

Coulomb Blockade in Quantum Dots

$V_{\text{gate}}$ $E_c$

Conductance

Even $N$ Odd $N$ $V_{\text{gate}}$
Coulomb Blockade in Quantum Dots

Coulomb Blockade in Quantum Dots

Even N  Odd N  Even N

conductance

E_c  E_c  E_c

V_{gate}
Electrical Transport: Coulomb staircase

Probing the energy levels in the ‘artificial atom’
Electrical Transport: Coulomb staircase

Probing the energy levels in the ‘artificial atom’
Coulomb Blockade in Quantum Dots: “dot spectroscopy”

“Coulomb Diamonds” (Stability Diagram)

Coulomb Blockade in Quantum Dots

“Carbon nanotube Quantum dots”.

- Carbon nanotubes deposited on top of metallic electrodes.
- Quantum dots defined within the carbon nanotubes.
- More structure than in quantum dots: “shell structure” due to orbital degeneracy.


Gleb Filkenstein’s webpage: http://www.phy.duke.edu/~gleb/
Charging Energy Model

Energy of $N$ particles on QD can be split into the energy levels and the charging energy

$$E_N = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_N + \frac{(Ne)^2}{2C}$$

An extra contribution is given by the gate electrode,

$$\alpha V_G N$$

$\alpha$ is the capacitive coupling of the dot to the gate.

The number of electrons on the QD is determined by the Fermi energy, $E_F$ if

$$E_N + \alpha V_G N < E_F < E_{N+1} + \alpha V_G (N + 1)$$

then there will be $N$ electrons on the QD.

The no. of electrons on the QD is adjusted by the gate potential.
Conductance peak spacing

The energy of \( N + 1 \) electrons is

\[
E_{N+1} = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_{N+1} + \frac{(N + 1)^2 e^2}{2C}
\]

The difference in energy between \( N \) and \( N + 1 \) electrons is

\[
\Delta E_{N \rightarrow N+1} = E_{N+1} - E_N = \varepsilon_{N+1} + (2N + 1) \frac{e^2}{2C}
\]

The separation between conductance peaks in this model is given by

\[
\alpha \Delta V_G = \Delta E_{N \rightarrow N+1} - \Delta E_{N-1 \rightarrow N} = \varepsilon_{N+1} - \varepsilon_N + \frac{e^2}{C}
\]
Conductance peak spacing II

\[ G \]

\[ \frac{e^2}{C} + \Delta \epsilon \]

N-1  N  N+1

no conductance - Coulomb Blockade
Conductance peak spacing III: spin degeneracy

In the event of degeneracy, e.g. spin degeneracy

\[ \varepsilon_N = \varepsilon_{N+1} \]

Level spacing statistics (\( \frac{e^2}{C} \) is const):

\[
P(E_{N+1} - E_N) = P(\Delta E) \quad \text{for even } N
\]

\[
P(E_{N+1} - E_N) = \delta(E) \quad \text{for odd } N
\]

no conductance - Coulomb Blockade
Peak spacing statistics: e-e interactions

\[ P(E_{N+1} - E_N) = P(\Delta E) \] for even \( N \)

\[ P(E_{N+1} - E_N) = \delta(E) \] for odd \( N \)

Level spacing distribution \( P(\Delta E) \):

**Theory:** CI+RMT prediction

Superposition of two distributions:
- **even \( N \):** large, chaotic dots:
  P(\Delta E) obeys a well known RMT distribution (*not* Gaussian)
- **odd \( N \):** Dirac delta function.

**Experiment:**
P(\Delta E) gives a Gaussian.

Possible explanations:
Spin exchange, residual e-e interaction effects*.
