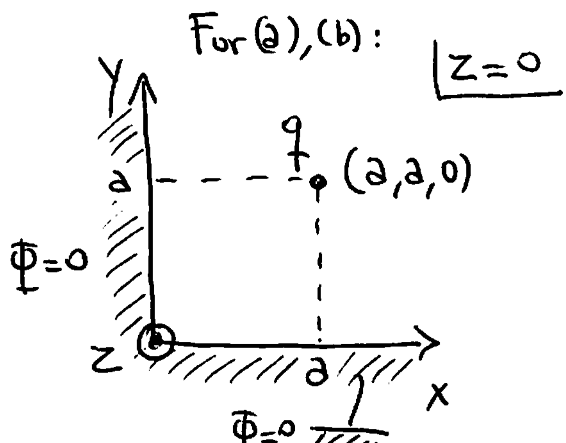
Midterm Exam 1. E&M, Spring 2015. Instructor: E. Dagotto. Delivered: Feb. 12. Deadline: Feb. 19 (at the beginning of class). Maximum # of points: 22.

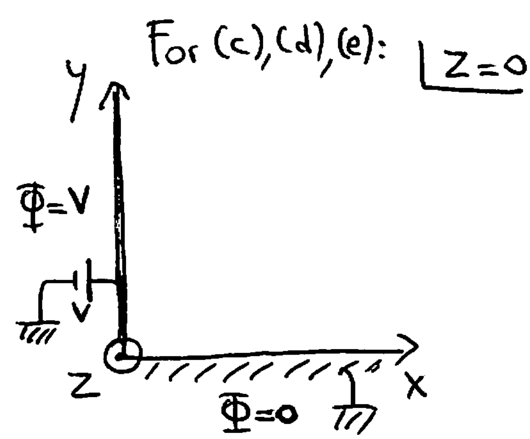
(1) Method of images and Green functions (10 points). Consider a conducting region defined by two half-planes connected to the ground (i.e. at zero potential), as shown below. These planes are perpendicular to one another i.e. one is defined by x=0 and y>0 and the other by y=0 and x>0. A charge q is located at position **x**=(a,a,0) as shown. The region where the charge resides is in the vacuum, and it is called the "first quadrant". Note that this is a 3D problem, not just 2D.

(a) (3 points) Using the method of images find the scalar potential $\Phi(\mathbf{x})$ in the first quadrant.
(b) (1 point) Find the force on the charge q induced by the planes (i.e. by the images).

Now assume that we remove the charge, we raise the x=0 half-plane to a fixed potential Φ=V>0, and we keep the y=0 half-plane fixed at Φ=0. Once again we want to know the scalar potential $\Phi(\mathbf{x})$ in the first quadrant, via the following steps:

(c) (2 points) Find the Dirichlet Green function $G_D(\mathbf{x},\mathbf{x}')$ in the first quadrant by the method of images employing the results of item (a).
(d) (2 points) Write formally the solution of the problem as a surface integral. Be as explicit as you can, i.e. calculate explicitly the derivative of $G_D(\mathbf{x},\mathbf{x}')$ that appears in the formula, write explicitly the limits of integration, etc, but do not integrate.
(e) (2 points) Using tables of integrals or internet searching for the integrals, integrate (d) and find $\Phi(\mathbf{x})$ in the first quadrant. After a tedious calculation, the final result is very simple. Without calculating, where in the first quadrant you expect the electric field to be the largest intuitively?





(2) Expansion in spherical harmonics (4 points). Consider a sphere of radius "a" kept at a fixed scalar potential given by $V(\theta,\phi) = V \sin\theta(1-\cos\theta)e^{i\phi}$, where V is a constant, and θ and ϕ are the usual spherical coordinates angles.

(4 points) Calculate the potential $\Phi(r,\theta,\phi)$ outside the sphere.

(3) Dielectrics (8 points). Consider a sphere of radius "a" made of a material with dielectric constant ε. The sphere has a dipole of magnitude "p" at the center, pointing along the *z* axis. In addition, there is an external field of magnitude E_0 also pointing along the *z* axis.

(a) (3 points) Find the scalar electric potential $\Phi(\mathbf{x})$ both inside and outside the sphere.
(b) (1 point) Check that for the case $\varepsilon=\varepsilon_0$, i.e. when the material becomes the same as vacuum and the sphere effectively disappears, you recover that $\Phi(\mathbf{x})$ inside and outside are identical.
(c) (2 points) Consider now $E_0 = 0$ for simplicity. Calculate the electric field $\mathbf{E}$ inside the sphere (no need to calculate outside), and the polarization $\mathbf{P}$.
(d) (2 points) From the result in (c) separate the contributions of the dipole at the center and that coming from the polarization of the medium. Explain the direction you found for the latter based on the expected polarization density of charge generated at the surface and its sign. No need to do any calculation, just a sketch and informal discussion.