$$\beta_1 = \frac{(\mu'-1)}{b^3\left(1+\frac{\mu'}{2}\right)} \cdot \frac{3m}{4\pi} \frac{\left(1+\frac{\mu'}{2}\right)b^3}{\left[(1+2\mu')\left(1+\frac{\mu'}{2}\right)b^3 - (\mu'-1)^2 a^3\right]}$$

$$\boxed{\beta_1 = \frac{3m}{4\pi} \frac{(\mu'-1)}{\left[(1+2\mu')\left(1+\frac{\mu'}{2}\right)b^3 - (\mu'-1)^2 a^3\right]}}$$

Then we return to α_1:

$$\alpha_1 = \beta_1 b^3 + \gamma_1 = \frac{3m}{4\pi} \frac{(\mu'-1)b^3 + \left(1+\frac{\mu'}{2}\right)b^3}{\left[(1+2\mu')\left(1+\frac{\mu'}{2}\right)b^3 - (\mu'-1)^2 a^3\right]}$$

$$= \boxed{\frac{3m}{4\pi} \frac{\frac{3}{2}b^3}{\left[(1+2\mu')\left(1+\frac{\mu'}{2}\right)b^3 - (\mu'-1)^2 a^3\right]} = \alpha_1}$$

Finally: $\delta_1 = \frac{1}{a^3}\left(\beta_1 a^3 + \gamma_1 - \frac{m}{4\pi}\right)$

$$= \boxed{\frac{m}{4\pi a^3}\left[\frac{3\left[(\mu'-1)a^3 + \left(1+\frac{\mu'}{2}\right)b^3\right]}{\left[(1+2\mu')\left(1+\frac{\mu'}{2}\right)b^3 - (\mu'-1)^2 a^3\right]} - 1\right] = \delta_1}$$

(c) Consider now the limit of large μ i.e. we keep the dominant term in this limit.

$$\boxed{\begin{aligned}
\gamma_1 &\approx \frac{3m}{4\pi\mu'}\frac{(b^3/2)}{(b^3-a^3)} \\
\beta_1 &\approx \frac{3m}{4\pi\mu'}\frac{1}{(b^3-a^3)} \\
\alpha_1 &\approx \frac{3m}{4\pi\mu'}\frac{\left(\frac{3}{2}b^3\right)}{(b^3-a^3)} \\
\delta_1 &\approx \frac{m}{4\pi a^3}\left[\frac{3}{\mu'}\frac{(a^3+b^3/2)}{(b^3-a^3)} - 1\right]
\end{aligned}}$$

$$\gamma_1, \beta_1, \alpha_1 \longrightarrow 0 \quad \text{as } \mu' \to \infty$$

$$\delta_1 \longrightarrow -\frac{m}{4\pi a^3} \quad \text{as } \mu' \to \infty$$

But we have to be careful! The field that matters is $\vec{B}$, but Φ_M generates $\vec{H}$.

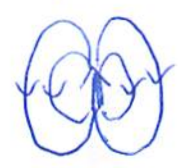
To answer (c), let us focus on α_1, the coefficient that matters outside the shell.

$$\Phi_M^{r>b} = \frac{\alpha_1}{r^2} P_1(\cos\theta) \overset{\text{large } \mu'}{\approx} \frac{3m \quad \frac{3}{2} b^3}{4\pi (b^3-a^3)\frac{\mu}{\mu_0}} \; \frac{\cos\theta}{r^2} =$$

$$= \underbrace{\frac{\frac{9}{2} b^3}{(b^3-a^3)\frac{\mu}{\mu_0}}}_{\substack{\text{dimensionless factor in front}\\ \text{that brings down the}\\ \text{intensity of this dipole}\\ \text{at large } \mu.}} \cdot \underbrace{\frac{m}{4\pi}\frac{\cos\theta}{r^2}}_{\substack{\text{standard potential}\\ \text{of a dipole}}}$$

$$\boxed{\vec{B} \overset{r>b}{=} -\mu_0 \nabla \Phi_M = -\mu_0 \left[\frac{\frac{9}{2} b^3}{(b^3-a^3)\frac{\mu}{\mu_0}} \right] \underbrace{\nabla\left(\frac{m}{4\pi}\frac{\cos\theta}{r^2}\right)}_{\substack{\text{usual gradient}\\ \text{that leads to}\\ \text{the canonical}\\ \text{shape}}}}$$

Coefficient that reduces the magnitude of the dipole for $r > b$.



(d) Inside the shell we have to be careful, since Φ_M may be reduced as $\frac{1}{\mu}$ at large μ but to get $\vec{B}$ we need to multiply by μ.

$$\vec{B} = -\mu \nabla \Phi_M$$

$\uparrow$ important!

$$\underset{\text{at large } \mu}{\Phi_M^{a<r<b}} = \left(\beta_1 r + \frac{\gamma_1}{r^2}\right)\cos\theta \overset{\text{large } \mu}{\cong} \frac{3m}{4\pi\mu'}\left[\frac{r}{(b^3-a^3)} + \frac{b^3/2}{(b^3-a^3)r^2}\right]\cos\theta$$

$$= \frac{3m}{4\pi\mu'(b^3-a^3)}\left(r + \frac{b^3}{2r^2}\right)\cos\theta$$

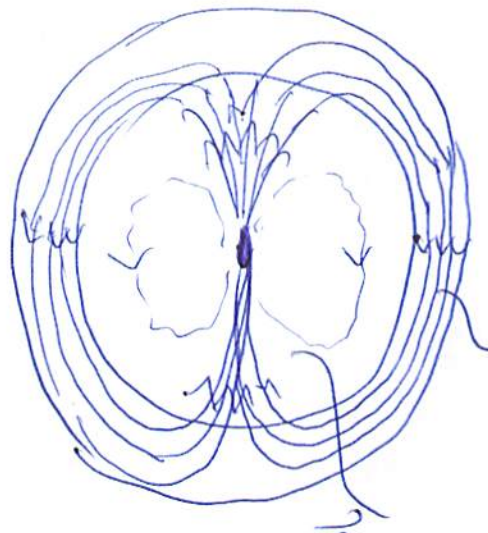
$$\vec{B} = -\mu\nabla\Phi_M = \frac{-\cancel{\mu}3m}{4\pi\underset{\mu_0}{\cancel{\mu}}(b^3-a^3)}\nabla\left[\left(r+\frac{b^3}{2r^2}\right)\cos\theta\right]$$

$$\boxed{\vec{B} = -\frac{3m\mu_0}{4\pi(b^3-a^3)}\nabla\underbrace{\left[\left(r+\frac{b^3}{2r^2}\right)\cos\theta\right]}_{\text{Combination of a constant } \vec{B} \text{ (because } r\cos\theta = z\text{) and a dipole}}}$$

$\vec{B}$ is independent of μ inside the shell.

$\boxed{\vec{B} \neq 0 \text{ even if } \mu \to \infty}$

For completeness, in the center hole the field $\vec{B}$ is also non zero. Drawing lines basically you get at $\mu \to \infty$ the following lines of $\vec{B}$:



$\vec{B} = 0$
outside

$\vec{B} \neq 0$
inside
even if $\mu \to \infty$

$\vec{B}$ combination
of center dipole
plus a constant
even as $\mu \to \infty$

This is only a crude sketch to show that as $\mu \to \infty$, the outside has no lines, the "shell" has dense lines and the inside also has nonzero lines of $\vec{B}$.