$$\underset{\text{surface}}{V(\theta,\phi)} = V \sin\theta\,(1-\cos\theta)\,e^{i\phi}$$

From table of spherical harmonics:

$$Y_{11} = -\sqrt{\frac{3}{8\pi}}\,\sin\theta\,e^{i\phi}$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}}\,\sin\theta\cos\theta\,e^{i\phi}$$

Then, $\sin\theta\,e^{i\phi} = -\sqrt{\frac{8\pi}{3}}\,Y_{11}$,

and $\sin\theta\cos\theta\,e^{i\phi} = -\sqrt{\frac{8\pi}{15}}\,Y_{21}$.

$$V(\theta,\phi) = V\sin\theta\,(1-\cos\theta)\,e^{i\phi} = V\left[-\sqrt{\frac{8\pi}{3}}\,Y_{11} + \sqrt{\frac{8\pi}{15}}\,Y_{21}\right]$$

The general term for the potential <u>outside</u> the sphere is:

$$\Phi(r,\theta,\phi) = \sum_{l,m} B_{lm}\,\frac{1}{r^{l+1}}\,Y_{lm}(\theta,\phi)$$

By continuity at the surface:

$$\Phi(a,\theta,\phi) = \sum_{l,m} B_{lm} \frac{1}{a^{l+1}} Y_{lm}(\theta,\phi) = V(\theta,\phi) =$$

$$= V\left[-\sqrt{\frac{8\pi}{3}}\, Y_{11} + \sqrt{\frac{8\pi}{15}}\, Y_{21}\right].$$

Then, simply, $\frac{B_{11}}{a^2} = -\sqrt{\frac{8\pi}{3}}\, V$ and $\frac{B_{21}}{a^3} = \sqrt{\frac{8\pi}{15}}\, V$

$$\Phi(r,\theta,\phi) = -V\sqrt{\frac{8\pi}{3}}\, \frac{a^2}{r^2} Y_{11}(\theta,\phi) + V\sqrt{\frac{8\pi}{15}}\, \frac{a^3}{r^3} Y_{21}(\theta,\phi) =$$

$$= \frac{V a^2 \sin\theta\, e^{i\phi}}{r^2} - \frac{V a^3 \sin\theta \cos\theta\, e^{i\phi}}{r^3}$$

$$= \boxed{V \frac{a^2}{r^2} e^{i\phi} \sin\theta \left(1 - \cos\theta \cdot \frac{a}{r}\right)}$$ outside the sphere.