(a) This is very similar to the problem solved in class and in the book, but replacing Eq. (9.25) by

$$I(z) = I_0 \cos\left(\frac{\pi z}{d}\right) \; ; \; |z| \leq d/2 .$$

As in the book, let us obtain $\rho(z)$ via the continuity equation assuming a simple $e^{-i\omega t}$ time dependence.

$$\rho(z) = \frac{1}{i\omega} \frac{d}{dz}\left[I_0 \cos\left(\frac{\pi z}{d}\right)\right]$$

$$\rho(z) = \frac{I_0}{i\omega}\left[-\sin\left(\frac{\pi z}{d}\right)\right]\frac{\pi}{d} = \frac{I_0 i \pi}{\omega d}\sin\left(\frac{\pi z}{d}\right)$$

This corresponds to a dipole since $\sin\left(\frac{\pi z}{d}\right) = \begin{cases} >0 , & z>0,\ z<d/2 \\ <0 , & z<0,\ z>-d/2 \end{cases}$

$$p_z = \int_{-d/2}^{d/2} z \left(\frac{I_0 i\pi}{\omega d}\right)\sin\left(\frac{\pi z}{d}\right) dz = \left(\frac{I_0 i\pi}{\omega . d}\right)\int_{-\pi/2}^{\pi/2} \frac{d . u}{\pi} \sin(u)\, \frac{d . du}{\pi} =$$

$$\frac{\pi z}{d} = u$$

$$dz = \frac{d}{\pi} du$$

$$= \frac{I_0 i}{\omega} \frac{\pi}{d} \frac{d^2}{\pi^2} \int_{-\pi/2}^{\pi/2} u \sin(u) du$$

$$\int_{-\pi/2}^{\pi/2} u \sin(u)\, du \underset{\substack{\uparrow \\ \text{integration} \\ \text{by parts}}}{=} -u\cos(u) + \sin(u) \Big|_{-\pi/2}^{\pi/2} = 2 .$$

Then, $\boxed{p_z = \dfrac{i I_0 d\, 2}{\omega \pi}}$.

(b) From Eq. (9.23):

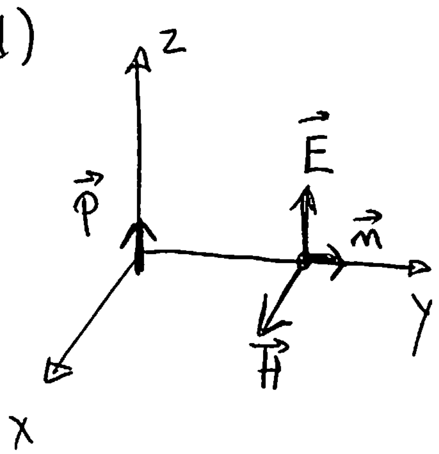
$$\omega^2 = k^2 c^2$$

$$\frac{dP}{d\Omega} = \frac{Z_0 c^2 k^4}{32\pi^2} |p_z|^2 \sin^2\theta = \frac{Z_0 c^2 k^4}{32\pi^2} \cdot \frac{I_0^2\, 4 d^2}{\omega^2 \pi^2} \sin^2\theta \overset{\downarrow}{=}$$

$$= \boxed{\frac{Z_0 I_0^2}{8\pi^4} (kd)^2 \sin^2\theta .}$$

(c) $P = \dfrac{Z_0 I_0^2 (kd)^2}{8\pi^4}\, 2\pi \cdot \dfrac{4}{3} = \boxed{\dfrac{Z_0 I_0^2 (kd)^2}{3\pi^3}}.$

(d)



Along y-direction, $\vec{n} = \hat{e}_y$. From (9.19), $\vec{H} \sim \vec{n} \times \vec{p}$. Then, $\vec{H}$ is along the x-direction. Since $\vec{E} \sim \vec{H} \times \vec{n}$, then $\vec{E}$ points along the z-direction.
