This problem is very similar to Problem 7.2(b) of Jackson, that was given as a HW problem.

The electric field point, for example, along the x axis.

\[ E_1 = E_1 e^{i k_1 z} + E_1 e^{-i k_1 z} \]

Region 1

\[ E_2 = E_2 e^{i k_2 z} + E_2 e^{-i k_2 z} \]

\[ E_3 = E_3 e^{i k_3 z} \]

But we know that \( M_1 = M_3 = 1 \) and \( M_2 = 2 \) from the start in this problem, different from Problem 7.2(b).

The B Conditions are \( 4 \) at each surface according to Eq. (7.37) of Jackson.
However, in this case note that the $\mathbf{E}$ vectors all point along the $z$-axis, while all the electric field point along the $x$-axis. Then, for example $\mathbf{E}_1 \times \mathbf{E}_1$ points along $y$-axis, and $\mathbf{m}$ points along $z$. Then, $(\mathbf{E}_1 \times \mathbf{E}_1) \cdot \mathbf{m} = 0$.

Also $\mathbf{E}_1 \cdot \mathbf{m} = 0$. Then, of the four B.C.'s, only the bottom two of Eq. (7.37) Jackson are non-trivial. The third from the top of (7.37) is the continuity of the electric field at both interfaces:

\[
\begin{align*}
\mathbf{E}_1 + \mathbf{E}_1^r &= \mathbf{E}_2 + \mathbf{E}_2^r \\
\hat{\mathbf{e}}_{\pm k_2 d} + -\hat{\mathbf{e}}_{\pm k_2 d} &= \hat{\mathbf{e}}_{\pm k_3 d}
\end{align*}
\]

The fourth eq. (7.37) (from the top) involves $(\mathbf{E}_1 \times \mathbf{E}_1) \times \mathbf{m}$ that points along $x$. All the vectors in this fourth eq. point in the same direction.

Then, this fourth B.C. becomes:
\[
\frac{k_1}{\mu_1} (E_1^i - E_1^r) = \frac{k_2}{\mu_2} (E_2^i - E_2^r)
\]

\[
\frac{k_2}{\mu_2} (E_2^i e^{i\kappa d} - E_2^r e^{-i\kappa d}) = \frac{k_3 E_3}{\mu_3}
\]

We have four eqs. and five unknowns \((E_1^i, E_1^r, E_2^i, E_2^r, E_3)\) but in fact what we want to find are ratios using \(E_1^i\) as common denominator.

(b) Note that \(\frac{k_2}{k_1} = \frac{\mu_2}{\mu_1}\) according to (7.36)

Then, \(\frac{k_2 \cdot \mu_1}{\mu_2 \cdot k_1} = \frac{\mu_2}{\mu_1} \cdot \frac{1}{\mu_2} = 2 \cdot \frac{1}{2} = 1\)

\[\frac{k_3 \cdot \mu_2}{\mu_3 \cdot k_2} = \frac{\mu_3}{\mu_2} \cdot \frac{1}{\mu_3} = \frac{1}{2} \cdot 2 = 1\]

Thus, the eqs. are:

1. \(E_1^i + E_1^r = E_2^i + E_2^r\);
2. \(E_1^i - E_1^r = E_2^i - E_2^r\)
3. \(E_1^i e^{i\kappa d} + E_1^r e^{-i\kappa d} = E_3^+ e^{i\kappa d}\);
4. \(E_1^i e^{-i\kappa d} - E_1^r e^{i\kappa d} = E_3^+ e^{i\kappa d}\)
Adding and subtracting ① and ② we get:

\[ 2E_1^r = 2E_2^r \] ⑤

\[ 2E_1^c = 2E_2^c \] ⑥

Adding and subtracting ③ and ④ we get:

\[ 2E_2^c e^{ikzd} = 2E_3^c e^{ikzd} \] ⑦

\[ 2E_2^r e^{-ikzd} = 0 \] ⑧

Then, \( E_2^r = 0 \) from last equation ⑧. And from ⑥, \( E_1^r = 0 \). We then arrive to the interesting result that there is no reflection for the values of \( \mu_2 \) and \( \mu_3 \) selected here.

Then, \[ \begin{matrix}
M_1 = 1 & M_2 = 2 & M_3 = 1 \\
\mu_1 = \mu_0 & \mu_2 = \mu_0 & \mu_3 = \mu_0
\end{matrix} \]

becomes

\[ \begin{bmatrix}
\begin{array}{c}
1 \\
2 \\
3
\end{array}
\end{bmatrix} \]

\[ \begin{array}{ccc}
i & i & t
\end{array} \]
How do I achieve a $\pi/2$ phase shift between $E_i$ and $E_3$?

From Eqs. 5 and 7:

$E_i = E_3 e^{i(k_2 - k_3)d}$

or $E_3 = E_i e^{i(k_2 - k_3)d}$

I need $(k_2 - k_3)d = \pi/2$

$d = \frac{\pi/2}{(k_2 - k_3)} = \frac{\pi/2}{(n_2 - n_3)\omega/k_0\epsilon_0} = \frac{\pi/2}{\omega/k_0\epsilon_0} = \frac{\pi c}{2\omega}$