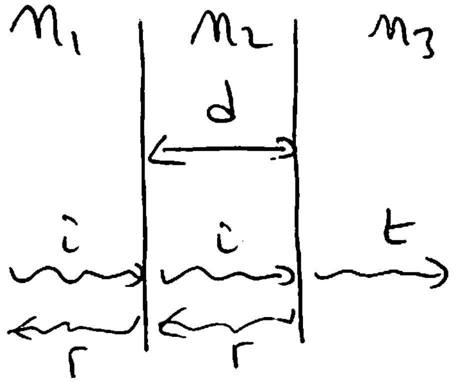
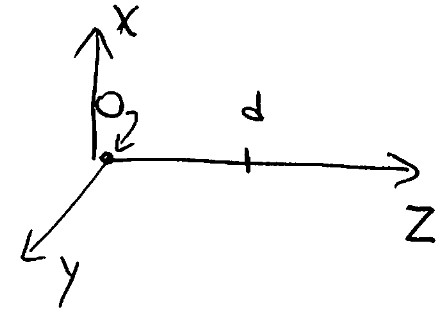
(a) This problem is very similar to Problem 7.2(b) of Jackson, that was given as a HW problem.





The electric field point, for example, along the x axis.

$$E_1 = E_1^i e^{ik_1 z} + E_1^r e^{-ik_1 z}$$

region 1

$$E_2 = E_2^i e^{ik_2 z} + E_2^r e^{-ik_2 z}$$

$$E_3 = E_3^t e^{ik_3 z}$$

But we know that $n_1 = n_3 = 1$ and $n_2 = 2$ from the start in this problem, different from Problem 7.2(b).

The B. Conditions are 4 at each surface according to Eq. (7.37) of Jackson.

However, in this case note that the $\vec{k}$ vectors all point along the z-axis, while all the electric field point along the x-axis. Then, for example $\vec{k} \times \vec{E}_1$ points along y-axis, and $\vec{n}$ points along z. Then, $(\vec{k} \times \vec{E}_1) \cdot \vec{n} = 0$

Also $\vec{E}_1 \cdot \vec{n} = 0$. Then, of the four B.C.'s only the bottom two of Eq. (7.37) Jackson are non-trivial. The third from the top of (7.37) is the continuity of the electric field at both interfaces:

$$\boxed{\begin{aligned} E_1^i + E_1^r &= E_2^i + E_2^r \\ E_2^i e^{ik_2 d} + E_2^r e^{-ik_2 d} &= E_3^t e^{ik_3 d} \end{aligned}}$$

The fourth eq. (7.37) (from the top) involves $(\vec{k} \times \vec{E}_1) \times \vec{n}$ that points along x. All the vectors in this fourth eq. point in the same direction.

Then, this fourth B.C. becomes:

$$\boxed{\begin{aligned}&\frac{k_1}{\mu_1}\left(E_1^i - E_1^r\right) = \frac{k_2}{\mu_2}\left(E_2^i - E_2^r\right)\\ &\frac{k_2}{\mu_2}\left(E_2^i e^{ik_2 d} - E_2^r e^{-ik_2 d}\right) = \frac{k_3}{\mu_3} E_3^t e^{+ik_3 d}\end{aligned}}$$

We have four eqs. and five unknowns ($E_1^i, E_1^r, E_2^i, E_2^r, E_3^t$) but in fact what we want to find are ratios using E_1^i as common denominator.

(b) Note that $\frac{k_2}{k_1} = \frac{n_2}{n_1}$ according to (7.36)

Then, $\frac{k_2}{\mu_2}\cdot\frac{\mu_1}{k_1} = \frac{n_2}{n_1}\frac{\mu_1}{\mu_2} = 2\cdot\frac{1}{2} = 1$

$\frac{k_3}{\mu_3}\cdot\frac{\mu_2}{k_2} = \frac{n_3}{n_2}\cdot\frac{\mu_2}{\mu_3} = \frac{1}{2}\cdot 2 = 1$

Thus, the eqs. are:

$$\boxed{\begin{aligned}&① \quad E_1^i + E_1^r = E_2^i + E_2^r \;; \quad E_1^i - E_1^r = E_2^i - E_2^r \quad ③\\ &② \quad E_2^i e^{ik_2 d} + E_2^r e^{-ik_2 d} = E_3^t e^{ik_3 d} \;; \quad E_2^i e^{ik_2 d} - E_2^r e^{-ik_2 d} = E_3^t e^{ik_3 d} \quad ④\end{aligned}}$$

Adding and substracting (1) and (3) we get:

$$2E_1^i = 2E_2^i \qquad (5)$$

$$2E_1^r = 2E_2^r \qquad (6)$$

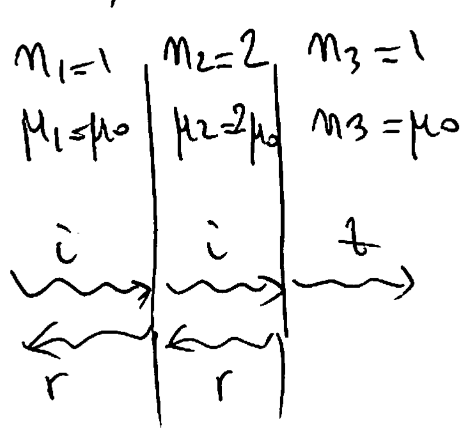
Adding and substracting (2) and (4) we get:

$$2E_2^i e^{ik_2 d} = 2E_3^t e^{ik_3 d} \qquad (7)$$

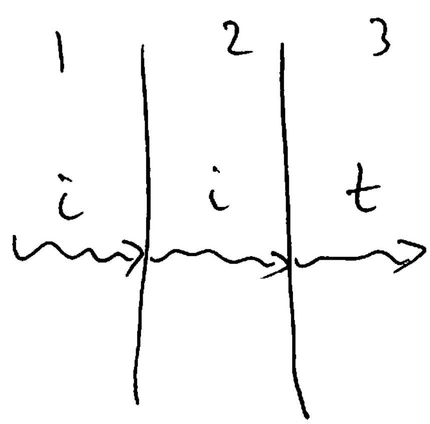
$$2E_2^r e^{-ik_2 d} = 0 \qquad (8)$$

Then, $E_2^r = 0$ from last equation (8). And from (6), $E_1^r = 0$. We then arrive to the interesting result that there is no reflection for the values of n_2 and μ_2 selected here.

Then,



becomes



(c) How do I achieve a $\pi/2$ phase shift between E_1^i and E_3^t ?

From Eqs. (5) and (7):

$$\boxed{E_1^i = E_3^t\, e^{i(k_3-k_2)d}} \quad \text{or} \quad E_3^t = E_1^i\, e^{i(k_2-k_3)d}$$

I need $(k_2-k_3)d = \pi/2$

$$d = \frac{\pi/2}{(k_2-k_3)} = \frac{\pi/2}{(\underset{\underset{2}{\uparrow}}{n_2} - \underset{\underset{1}{\uparrow}}{n_3})\,\omega_0\sqrt{\mu_0\epsilon_0}} = \boxed{\frac{\pi/2}{\omega_0\sqrt{\mu_0\epsilon_0}} = \frac{\pi c}{2\omega_0}}$$