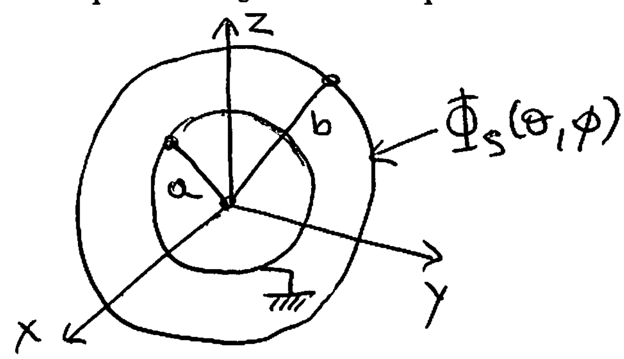
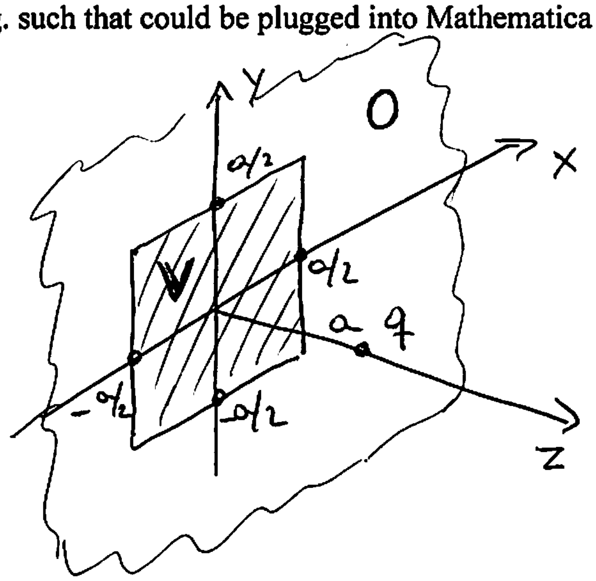
Midterm Exam 1. E&M, Spring 2016. Instructor: E. Dagotto. Delivered: Feb. 23. Deadline: March 1 (before or at the end of class, or in my mailbox by 11 AM). Maximum # of points: 20.

(1) Expansion in spherical harmonics (7 points). Consider two concentric spheres, as shown in the sketch. The outer sphere has radius b and the inner sphere radius a. The outer sphere is kept at a potential $\Phi_S(\theta,\phi) = V_0 \sin(\theta) \cos(\phi)$, where θ and ϕ are the canonical angles of spherical coordinates. The inner sphere is metallic and grounded. (a) Write the potential of the outer sphere $\Phi_S(\theta,\phi)$ in terms of spherical harmonics. (b) Using the general expansion of the scalar potential in spherical coordinates, find this potential $\Phi(r,\theta,\phi)$ in the empty region between the two spheres. Confirm that the result satisfies the boundary conditions at $r=a$ and b. (c) Find the density of charge σ at the inner sphere induced by the outer sphere. (d) Provide a crude sketch by hand of the sign and magnitude of σ in the x-y plane at z=0 and analyze if this is compatible with the sign of the potential Φ_S of the outer sphere in the same plane.



(2) Green functions (6 points). Consider a point charge q located at $\mathbf{a}=(0,0,a)$. In addition, the x-y plane at z=0 is kept fixed at potential 0 with the exception of a square of side also a (see drawing) that is at potential V. Using the Dirichlet Green function $G_D(\mathbf{x},\mathbf{x}')$ obtained by the method of images (no need to re-derive it; you can extract this function directly from lectures or the book), find the potential $\Phi(\mathbf{x})$ at an arbitrary point $\mathbf{x}$ in the region z>0. The surface contribution can be left expressed as an integral, but it must have a clearly developed integrand (e.g. such that could be plugged into Mathematica if needed).



(3) Dielectrics (7 points). Consider a hollow sphere of radius a immersed in a material with dielectric constant ε that covers all of space. The sphere is in the presence of an electric field of constant magnitude E_0 pointing along the z axis (see sketch). At the center of the hollow sphere there is a dipole of strength p also pointing along the z axis. (a) Via the expansion in Legendre polynomials, calculate the potential $\Phi(\mathbf{x})$ both inside and outside the sphere. (b) From the result in (a) check that if ε becomes ε_0 then the potentials $\Phi(\mathbf{x})$ inside and outside become identical because there is no longer a boundary. (c) From the result in (a), can you select the value of the dipole strength p such that outside the total dipolar contribution cancels? Explain intuitively why outside the sphere the dipolar contributions caused by the constant electric field and that caused by the dipole at the center can have different signs.