Midterm Exam 3 ("a.k.a." Final Exam). E&M, Spring 2015. Instructor: E. Dagotto. Delivered: April 16. Deadline: April 27, noon, in my mailbox. Maximum # of points: 20.

(1) Antennas (10 points). This is a problem similar to the one in Section 9.4 of Jackson, which was explained in class. Consider a center-fed antenna as in Figure 9.3 involving a wire of total length d. The current density in the present case is given by

$$\mathbf{J}(\mathbf{x}') = I \sin(k|z'|)\, \delta(x')\, \delta(y')\, \mathbf{e_z}$$

where the product kd satisfies $kd=4\pi$.

(a) Graph the current along the wire vs. z`. Is the function even or odd?

(b) Write the vector field **A(x)** in terms of an integral in the radiation zone.

(c) Carry out the integration by using the identity

$$\sin(u) = (e^{iu} - e^{-iu})/2i \quad \text{(for arbitrary u)}$$

which leads to easy integrals. Provide the final result for **A(x)**.

(d) Using the generic formula (9.21) and also using the equation

$$\mathbf{H(x)} = i\, k\, \mathbf{n} \times \mathbf{A(x)} / \mu_0 \qquad (1)$$

discussed in Jackson to obtain the magnetic field **H**, find $dP/d\Omega$ as a function of μ_0, ε_0, the current intensity I, and the angle θ. *Note:* to get $dP/d\Omega$ only $|\mathbf{H}|$ is needed from equation (1).

(e) Make a sketch of the radiation pattern. Based on graph (a), intuitively why this result is different from the pattern created by a dipole?

(2) Radiation of a point charge (10 points). The main goal of this problem is for you to understand in detail the meaning of all the symbols used to calculate the pattern of radiation $dP/d\Omega$ of a moving point charge.

(i) Starting with Eq.(11.72) Griffiths, find the power radiated by a particle with charge q that is moving with velocity along the z axis $\mathbf{v} = v\, \mathbf{e_z}$, and with acceleration in the x-z plane

a = a [cos(δ) $\mathbf{e}_z$ + sin(δ) $\mathbf{e}_x$]. Express the results in terms of q, a, μ_0, c, β = v/c , the angle δ, and the angles θ and ϕ of the spherical coordinates. Use also the notation f(θ,ϕ,δ) = sin(θ) cos(ϕ) sin(δ) + cos(θ) cos(δ) to abbreviate the final result; this combination of angles appears in the solution.

(ii) Show that your result reduces to the result given in Problem 11.16 Griffiths for the special case when **a** points along the x axis (i.e. δ = π/2).

(iii) Show that your result reduces to the Larmor result for the special case when **a** points along the z axis (i.e. δ=0) and velocity v=0.

(iv) Finally, show that your result reduces to Eq.(11.74) of Griffiths for the case δ=0 i.e. when **v** and **a** are collinear.

Note: If you do not have the book of Griffiths third edition, you can find copies of the relevant chapters in the web page of this course.